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Interest Rate Signals and Central Bank Transparency

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Abstract

The present paper extends the literature on central bank transparency that relies on information heterogeneity among private agents in four directions. First, it adds the interest rate to the list of signals that the central bank can reveal. Second, it allows for more than one economic fundamental. Third, it extends the range of uncertainties that matter. So far the literature has focused on uncertainty about the economic fundamentals, assumed to be estimated with known precision; we also allow for uncertainty about precision. Fourth, it derives results that are general in the sense that they do not depend on any particular social welfare criterion. Each extension sheds new light on the role of central bank transparency.

While uncertainty about the fundamentals results in the now familiar common knowledge effect, uncertainty about information precision creates a fog effect, which reduces the quality of decision taken by the central bank and the private sector. In the absence of the fog effect, full transparency is generally not desirable, because it deprives the central bank from the ability to optimally manipulate private sector expectations. When the central bank fog is large, full transparency is usually the best communication strategy, even when the private sector fog is large. We also find that it is usually desirable for the central bank to divulge some information, even if it is erroneous.

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1 Introduction

Central banks have become increasingly transparent, but just how transparent should they be? Some central banks strive to reveal just about everything that is relevant; this is the case of the Reserve Bank of New Zealand, of the Bank of Norway and of Sweden's Riksbank. Others are more circumspect; they consider that there may be too much transparency, see e.g. Bean (2005).[1] Likewise, the academic literature is divided about the welfare case for full transparency. Blinder (1998) argues that central banks should be as transparent as possible. As further elaborated by Svensson (2005) and Woodford (2005), the economic case for transparency rests on the dominant role played by expectations of private agents when they make decisions on prices, spending and production. When the main channels of monetary policy operate through expected inflation, long-term interest rates, asset prices and exchange rates, central banks are most effective when the private sector fully understands their intentions. Yet Cukierman (2007) observes that transparency may backfire, for instance when uncertainty about the economy, including our understanding of the economy, is large or because a high degree of transparency can provide a distorted view of what the central bank knows and intends to achieve.

At a very general level, in an Arrow-Debreu world with complete markets, transparency is always desirable (Hellwig, 2005). In a more realistic setting, second best arguments are bound to uncover cases where some degree of opacity welfare-dominates transparency. The literature has mostly focused on two generic departures from market completeness, building two influential cases for some degree of central bank opacity.

The first case for limiting transparency starts with the "constructive ambiguity" argument initially advanced by Cukierman and Meltzer (1986). The argument rests on two assumptions: 1) only unanticipated money matters (Kydland and Prescott, 1977) and 2) the central bank preferences are not precisely known by the public (Vickers, 1986). Under these combined assumptions, some degree of opacity enhances monetary policy effectiveness because a fully transparent central bank cannot create surprises.[2] These assumptions have become less appealing. New Keynesian models do not provide support to the "only unanticipated money matter" view, already convincingly criticized by McCallum (1995) and Blinder (1998). The view has also been undermined by central bank practice; far from concealing their preferences, today's central banks clearly specify their objectives, as is the case with the increasingly popular inflation targeting strategy.

Heterogeneous information provides the second influential case for limited transparency. Morris and Shin (2002, 2005) - henceforth referred to as M&S - argue that central banks should not reveal all the information at their disposal. Their argu-

[1] For the sake of completeness, we note that an important reason for transparency is democratic accountability. We do not pursue this argument further.

[2] For review of this literature, see Geraats (2002).

ment does not appeal to the assumptions of the constructive ambiguity literature. It rests instead on three different assumptions: 1) the information available to both the central bank and the private sector is noisy; 2) the central bank's signals are seen by everyone in the private sector; and 3) private sector agents form forecasts that are just as precise as possible but also as close as possible to the consensus forecast (a case of strategic complementarity). The last assumption, which goes back to Keynes' celebrated beauty contest effect, is meant to capture the basic principle that it is relative prices that matter in competitive markets. An implication of the beauty contest assumption is that everyone knows that everyone else observes the same central bank signals. A consequence is the common knowledge effect: relative to private information, central bank signals receive undue attention in the sense that their impact will not just reflect their quality. It follows that it may be desirable for the central bank to withhold releasing its information when the quality of its signals is not good enough. This influential result has been shown not to be robust. Svensson (2005) observes that, in practice, the quality of central bank signals is unlikely to be sufficiently poor to justify withholding information. Woodford (2005) observes that the result occurs because M&S use a welfare function that ignores the negative welfare effect of price dispersion. This general observation is further developed in Hellwig (2005) and Roca (2006).

The present paper extends the analysis of information heterogeneity in a number of directions. To start with, most of the literature contrasts just two regimes, opacity and transparency. One exception is Walsh (2007), which explores the optimum degree of transparency by allowing the central bank to release its information to subgroups of private agents; optimality refers to the size of the subgroups that receive and act upon the information. It seems to us that central banks take great pain to ensure that their information is strictly not preferentially distributed. Partial transparency, as we see it, refers to the share of information that is released. To that effect, we allow for more than one economic fundamental and to different types of information.

Publication of the interest rate is now common practice even though, as is well known, the Federal Reserve did not reveal its interest rate until 1994. That change represents a major step towards more transparency. But the extensive attention devoted by central bank watchers to policy announcements suggests that the interest rate acts a crucial signal that does not seem to have studied so far. In our model the interest rate is one element of the information set that a central bank may decide to reveal. This allows us to consider at least three transparency regimes: full opacity, when the central bank does not release any private information; partial transparency, when the central bank only reveals its interest rate decision; and full transparency, when the central bank tells it all, i.e. also publishes its signals on the fundamentals.

The interest rate is a special signal because, unlike information about the state of the economy, it can be used by the central bank to affect market expectations. In other words, it is a manipulable signal.[3] We push this logic to its end and assume

[3]Of course, the central bank can also manipulate its other signals by not being truthfull about

that the interest rate is only a signaling device and that it does not play any direct macroeconomic role. Admittedly, this is an extreme assumption, but it allows us to focus on this important aspect of interest rate decisions.

Another aspect of the literature is that, typically, the precision of the heterogeneous signals received by the central bank and private sector agents - the inverse of signal variance - is assumed to be known with certainty. Here we allow for imperfect knowledge of signal precision and we find that it makes an important difference.

As already mentioned, some controversies about the desirability of central transparency revolve around the choice of the social welfare criterion. Even though some authors derive this criterion from microfoundations, many assumptions creep in along the way. We deal with this problem in two ways. First, we adopt the general social welfare function proposed by Hellwig (2005), which encompasses some important special cases. In addition, whenever possible, we derive results that are general in the sense that they do not depend on any social welfare function.

Our main interest is not just to determine which transparency regime is best. Much of the emphasis is on how central bank transparency, or the lack thereof, affects the economy through private expectations. The story we tell is one where the interest rate allows the central bank to shape expectations. By optimally choosing the interest rate, the central bank can deal with the unavoidable common knowledge effect in a way that is welfare enhancing. That tends to make partial transparency preferable to full transparency because in the latter case the interest rate does not convey any additional information and cannot be used by the central bank to shape private sector expectations. If, however, the central bank misestimates the private sector signal precision, its optimally chosen interest rate may do more harm than good. This tends to make full transparency the best regime choice.

The paper is organized as follows. The next section presents our model, which extends much of the literature by allowing for any finite number of economic fundamentals. Beyond its generality, this extension is needed as we assume throughout that the central bank optimally sets the interest rate; with just one fundamental, the interest rate would fully reflect the central bank signal on that fundamental. Since the central bank optimally sets the interest rate to maximize social welfare, it must form a forecast of the private sector information precision. Section 3 considers the case when the precision of the central bank and private sector information is perfectly known to both the central bank and the private sector. In this case, partial transparency dominates full transparency - unless all signals are drawn form the same distribution - because the central bank can adequately influence private sector expectations. In Section 4, the precision of private sector signals is unknown to the central bank but known to the private sector. As a result, the central bank operates in a sort of fog, which reduces its ability to optimally shape private sector expectations. Full transparency may then be the most desirable regime. We next allow for the private sector itself to be uncertain about its own signal precision. As shown in Section 5,

its information. We ignore such a strategy since it is not sustainable in equilibrium.

this assumption does not radically change the previous conclusions. The last section briefly summarizes our results and discusses limits and potential extensions.

2 The Model

We follow the literature on heterogeneous information as we imagine an economy populated with a continuum of agents, each of whom makes one (static) decision based on her utility function. The desirability of central bank transparency is then assessed with a social welfare function that aggregates individual preferences. Part of the debate about the desirability of central bank transparency hinges on the form of the individual utility and social welfare functions. We borrow the model of Hellwig (2005) who proposes a general utility function that encompasses many other formulations. For illustration purposes, we interpret private agent actions as setting the price of the goods that they each produce.

Since we assume that the central bank may decide to announce its chosen interest rate, we need to allow for more than one fundamental. If there were only one fundamental, the interest rate decision would be fully revealing. We therefore assume that there exist n fundamentals θ_k, $k = 1, n \geqslant 2$, which are independently, identically and uniformly distributed so that $E(\theta_k) = 0 \; \forall k$ and $Var(\theta_k)$ is indefinite.[4] Their effect on the price level is given by $\mathbf{A}\boldsymbol{\theta}$ where $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_n)'$ and $\mathbf{A}$ is a conformable vector. The fundamentals are meant to capture all the exogenous factors that may affect the economy while $\mathbf{A}$ represents the true model of the economy. We assume that this model is known to all, an unsavory assumption that is further discussed in the concluding section.

2.1 The private sector

Each private agent $i \in [0, 1]$ decides on action p^i - which we illustratively call the price of her production - with two objectives: match the imperfectly known fundamental $\mathbf{A}\boldsymbol{\theta}$ and stay close to other agents action. This description of individual preferences can be rationalized in different ways, see M&S and Woodford (2005). Formally, the preferences of private agent $i \in [0, 1]$ are described by the following linear-quadratic loss function:

$$L_i = (1-r)\left(p_i - \mathbf{A}\boldsymbol{\theta}\right)^2 + r\left(p_i - \bar{p}\right)^2 - k_1 \int_j \left(p_j - \bar{p}\right)^2 dj - (1-r)\, k_2 \left(\bar{p} - \mathbf{A}\boldsymbol{\theta}\right)^2$$

where p_i is the (log) price of the good from producer i and $\bar{p} = \int_{j=0}^{1} p_j dj$ is the aggregate price index. The two first terms are a weighted average of the cost of setting the price away from its fundamental value and of the cost of deviating from

[4] This assumption, which is of no economic interest, simplifies results.

the average price. The relative weight $r \in [0, 1]$ thus captures the degree of strategic interaction among producers; it is the source of the beauty contest effect that lies at the heart of the common knowledge effect emphasized by M&S. The last two terms, with no sign restriction on $k_1 < 1$ and k_2, indicate how much each agent internalizes the dispersion of prices and aggregate volatility or mispricing.[5] These last two terms do not affect producer i own decision since they do not depend on her choice of p^i; they represent externalities. The central bank, on the other hand, can take these externalities into account when making its own decision. The loss function reduces to the one used by M&S when $k_1 = r$ and $k_2 = 0$ and to the loss function assumed by Woodford (2005) when $k_1 = -r$ and $k_2 = 0$.[6] For this reason, for simplicity we will henceforth assume that $k_2 = 0$.

Taking other agents's prices as given, agent i's optimal choice is:

$$p^i = (1-r)E^i(\mathbf{A}\boldsymbol{\theta}) + rE^i(\bar{p}) \tag{1}$$

where E^i is conditional on the agent's information set. The higher the interaction parameter r the more producers react to the expected aggregate price and the less they respond to the fundamentals. When setting her own price p^i, agent i must guess the aggregate price level, which depends on the prices set by all the other producers; she must therefore guess what the other producers will guess, etc., which leads to infinite iteration on guesses of guesses.

Each private agent is assumed to receive her own idiosyncratic signals about the fundamentals θ_k. These signals are unbiased but noisy. The simplest representation is to allow for an identically and independently distributed additive noise such that agent i's signal x_k^i about fundamental θ_k is:

$$x_k^i = \theta_k + \eta_k^i \qquad k = 1, ...n \qquad E(\eta_k^i) = 0 \qquad Var(\eta_k^i) = \frac{1}{\beta_k}$$

where β_k, the precision of private signal x_k^i, is assumed to be the same for all private agents.

Under these assumptions, we iterate (1) infinitely, and denoting $\bar{E}^n$ the n^{th} order expectation, we obtain the optimal pricing decision:

$$p^i = (1-r)\sum_{n=0}^{\infty} r^n E^i\left(\bar{E}^n(\mathbf{A}\boldsymbol{\theta})\right) \tag{2}$$

which exists when $0 < r < 1$.

Without any loss of generality, we normalize the fudamentals θ_k so that $A_k = 1$ $\forall k$ and $\mathbf{A}\boldsymbol{\theta} = \sum_{k=1}^{n} \theta_k$.

[5] Hellwig (2005) allows for a fifth term $-k_3 A\theta\,(\bar{p} - A\theta)$ in the loss function. This term captures the cost of mispricing due to the common knowledge effect.

[6] Thus M&S fully eliminate price dispersion from the social welfare function $\int L_i di$ while Woodford gives it a weight of r.

2.2 The central bank

Like each private agent, the central bank receives some noisy but unbiased information about the fundamentals:

$$\tilde{\theta}_k = \theta_k + \varepsilon_k \qquad k = 1, ..., n \qquad E(\varepsilon_k) = 0 \qquad Var(\varepsilon_k) = \frac{1}{\alpha_k}$$

where the noises ε_k are independently and identically distributed, and are also independent of the private noise signals. The the precision of central bank signal x_k^i is α_k.[7] The central bank disposes of an instrument, the short-term interest rate R. In principle, the interest rate has two effects: a macroeconomic effect, which affects prices in addition to the fundamentals θ_k, and a signalling effect. We ignore the macroeconomic effect because allowing for such a channel would greatly complicate the model, precluding a closed-form solution. The assumption is unrealistic but it has the advantage of focusing attention on the information content of the interest rate. It sets the present paper as a complement to the large literature on optimal monetary policy, which focuses on the macroeconomic effect of the interest rate with limited attention to its information content. Here the central bank uses the interest rate purely as a component of its communication strategy.[8] Of course, the assumption is not innocuous; we will indicate its implication where it matters.

The central therefore makes two decisions. It decides on its communication strategy and on the interest rate. Any signal released by the central bank is public, in the sense that all private agents receive it. Walsh (2007), instead, allows the central bank to inform subsets of the private sector; the optimal degree of transparency concerns the proportion of agents who are informed. Here the optimal degree of transparency concerns the amount of information that is simultaneously released to all agents.

In deciding what information to reveal, the central bank maximizes social welfare, i.e. it minimizes $E^{CB} \int_i L_i di$ where the expectation operator is conditioned on the central bank's information set. The social loss is evaluated as the unconditional average of private losses $E \int_i L_i di$. Thus the central bank preferences are well known and are the same as those of the private sector; this eliminates the creative ambiguity motive for limited transparency. We will examine the optimal choice of interest rate R by the central bank assuming that it follows a linear rule:

$$R = \sum_{k=1}^{n} \mu_k \tilde{\theta}_k \tag{3}$$

[7] In line with the literature, we treat $Var(\epsilon_\kappa)$ as exogenous. Obviously, central bank signals are based on variables that include private sector actions and, therefore, private sector signals. Ignoring this dependence is subject to a Lucas critique since the precision of central bank signals may vary with the policy regime and, in particular, on central bank transparency. We thank Hyun Song Shin for attracting our attention to this limitation of our paper.

[8] The assumption can be seen as an extreme characterization of the observation by Woodford (2005) that "the current level of the overnight interest rates *as such* is of negligible importance for economic decisionmaking".

with a normalization on R such that $\sum_{k=1}^{n} \mu_k = 1$. Note that, to make its decision, the central bank must forecast the p_i's, which requires guessing the private sector forecasts, see (2).

3 Known Information Precision

We consider first the case when the second moments of both private and central bank signals ($Var(\eta_k^i)$ and $Var(\epsilon_k)$), and therefore their precision β_k and α_k, respectively, are known. In this case, there are three possible degrees of transparency: full opacity - denoted OP - when the central bank does not reveal anything; partial transparency - denoted PT - when the central bank only reveals the optimally-chosen interest rate; and full transparency - denoted FT - when the central bank reveals both the interest rate and its signals $\widetilde{\theta}_k$. We limit our study to the binary choice of releasing all or none of the n signals.

3.1 Full opacity

The opacity case is trivial given that the interest rate, which by assumption only has a signalling role, is not published. Each private agent receives her own idiosyncratic signals x_k^i, $k = 1, n$ and has no further information. Her best estimate of the aggregate price level is therefore $E^i(\bar{p}) = 0$ and, using (2), we have:

$$p^i = \sum_{k=1}^{n} x_k^i \tag{4}$$

The optimal price is the unweighted sum of the signals. Part of the reason is that we have normalized them so that $A\theta = \sum_k \theta_k$. The other reason, which will soon become clear, is that each agent receives only one signal about each fundamental and thus has no better option than to take it at face value. The corresponding social loss L^{op} is shown in the Appendix.

3.2 Partial transparency

We now consider the case when the central bank reveals its interest rate R. Each private agent receives two kinds of signals: the interest rate, which they know is optimally set by the central bank according to (3), and its own signals x_k^i. Applying Bayes' rule, the optimum forecast of fundamental θ_k by agent i is:

$$E^i(\theta_k) = \gamma_k \left(\frac{R - \sum_{j=1, j\neq k}^{n} \mu_j x_j^i}{\mu_k} \right) + (1 - \gamma_k) x_k^i \tag{5}$$

where:

$$\gamma_k = \frac{\frac{1}{\beta_k}\mu_k^2}{\sum_{j=1}^n \mu_j^2 \left(\frac{1}{\alpha_j} + \frac{1}{\beta_j}\right)}$$

Then the Appendix shows that (2) implies:

$$p^i = \left(\sum_{j=1}^n \frac{\varphi_j}{\mu_j}\right) R + \sum_{k=1}^n \left(1 - \mu_k \sum_{j=1}^n \frac{\varphi_j}{\mu_j}\right) x_k^i \tag{6}$$

with

$$\varphi_k = \frac{\gamma_k}{1 - r(1 - \sum_{k=1}^n \gamma_k)}$$

The common knowledge effect is present: because each private agent observes R and knows that the others do as well, she tends to overweight this signal. This is due to the beauty contest assumption that each agent wishes to set her price close to those of her competitors. Indeed, when the beauty contest assumption is eliminated, $r = 0$ and $\varphi_k = \gamma_k$: the weight on R corresponds exactly to optimal Bayesian signal extraction. When $r > 0$, $\varphi_k > \gamma_k$ and φ_k increases with the interaction coefficient r. See the Appendix for the corresponding value L^{PT} of the social loss function.

3.3 Full transparency

Full transparency occurs when the central bank reveals both the interest rate and all its signals $\widetilde{\theta}_k$.In that case, the interest rate, which by (3) is just a linear combination of the signals, does not provide any additional information and becomes a useless instrument. Agent i now receives two signals about each fundamental θ_k: her own signal x_k^i, with precision β_k, and the central bank signal $\widetilde{\theta}_k$, with precision α_k. Applying Bayes rule, we have:

$$E^i(\theta_k) = \bar{\gamma}_k \widetilde{\theta}_k + (1 - \bar{\gamma}_k) x_k^i \tag{7}$$

where:

$$\bar{\gamma}_k = \frac{\alpha_k}{\alpha_k + \beta_k}$$

Using (2), in equilibrium the price level is:

$$p^i = \sum_{k=1}^n \left[\overline{\varphi}_k \tilde{\theta}_k + (1 - \overline{\varphi}_k)\, x_k^i\right] \tag{8}$$

with

$$\overline{\varphi}_k = \frac{\alpha_k}{\alpha_k + (1 - r)\, \beta_k}$$

Here again, because the information released by the central bank is common knowledge, it tends to receive an excessive weight in price setting. The Appendix displays the associated social loss L^{FT}.

3.4 Welfare comparisons

Formally, we can evaluate the losses under the three regimes of interest. We can achieve a more general and more revealing result, however. Recall that the central bank's choice of the interest rate only matters in the partial transparency regime. Under full opacity, the interest rate is not published and does not affect the economy; under full transparency it does not bring any additional information. It turns out that, in the transparency regime, the central bank can always choose the interest rate so as to replicate the two other regimes, which implies that it can do better by optimizing.

Comparing (4) and (6), we note that in the latter the coefficient of R is $\sum \frac{\varphi_j}{\mu_j}$. By choosing the policy coefficients μ_j such that $\sum \frac{\varphi_j}{\mu_j} = 0$, (6) reduces to (4). Noting that:

$$\left(\sum_{j=1}^{n} \frac{\varphi_j}{\mu_j} \right) = \frac{\sum_{j=1}^{n} \frac{\mu_j}{\beta_j}}{\left[1 - r(1 - \sum_{k=1}^{N} \gamma_k)\right] \sum_{j=1}^{n} \mu_j^2 \left(\frac{1}{\alpha_j} + \frac{1}{\beta_j} \right)} \tag{9}$$

we see that $\sum \frac{\varphi_j}{\mu_j} = 0$ when $\sum \frac{\mu_j}{\beta_j} = 0$. Since $\sum_{j=1}^{N} \mu_j = 1$, we can eliminate any one of the policy parameters, say μ_n, and the condition becomes:

$$\sum_{j=1}^{n-1} \left(\frac{1}{\beta_j} - \frac{1}{\beta_n} \right) \mu_j + \frac{1}{\beta_n} = 0 \tag{10}$$

When the β_js are not all equal , $\frac{1}{\beta_j} - \frac{1}{\beta_n} \neq 0$ for some values of β_j (we consider the symmetric case $\beta_i = \beta_j \; \forall i, j$ below), there exists an infinite number of combinations of the policy parameters μ_js such that $\sum \frac{\varphi_j}{\mu_j} = 0$. This means that a partially transparent central bank can always set the interest rate in a way that mimics the opacity case. It follows that, when it optimizes the choice of μ_j, a partially transparent central bank can always do at least as well as an opaque central bank.

When $\beta_i = \beta_j \; \forall i, j$, a partially transparent central bank can still mimic an opaque central bank. Since their various signals have the same precision, Bayesian private agents give the same weight in their forecasts to each fundamental. In that sense, the fundamentals are equivalent and the central bank can no longer use its policy parameters μ_k to manipulate private expectations.[9] Still, the central bank can set $\mu_j = \pm\infty$, which makes the interest rate uninformative (this is the solution to (10) when $\beta_j \to \beta_n$ for all $j = 1, n-1$). In this case, reproducing the opacity regime is optimal and the two regimes become equivalent as far as welfare is concerned.

[9] Formally, the central bank wants to choose the μ_j's such that $\sum_j \frac{\varphi_j}{\mu_j} = 0$. When $\beta_i = \beta_j \; \forall i, j$, (9) shows that $\frac{\varphi_j}{\mu_j}$ is proportional to $\frac{\gamma_j}{\mu_j}$, which is itself proportional to μ_j so $\sum_j \frac{\varphi_j}{\mu_j}$ is proportional to $\sum_j \mu_j = 1$ and the μ_j's cancel out.

We can apply the same logic to the comparison between the partial and full transparency regimes. Indeed, (6) reduces to (8) when $\frac{\mu_k}{\mu_j} = \frac{\overline{\varphi}_k}{\overline{\varphi}_j}$, which implies $\sum_{j=1}^{N} \frac{\mu_k}{\mu_j}\varphi_j = \overline{\varphi}_k$.[10] Since $\sum_{k=1}^{N} \mu_k = 1$, this condition determines a unique set of policy parameters μ_k. It follows that a partially transparent central bank can always choose the interest rate to reproduce the outcome under full transparency. When it optimizes, the partially transparent central bank stands to achieve at least the social welfare reached under full transparency, and it can possibly do better.

Proposition 1 *When the precision of central bank and private sector information is known, partial transparency dominates both opacity and full transparency. This result holds for any loss function (which preserves the price setting) and any number of fundamentals.*

The result is very general. It is independent of the welfare function since we do not even need to specify optimal policy under partial transparency. It also holds independently of the relative precision of central bank and private signals. It remains valid even if the central bank reveals only a subset of the signals $\tilde{\theta}_k$ that it has received.[11]

The intuition behind Proposition 1 is as follows. Under either opacity or full transparency, the interest rate does not convey any signal. The central bank can use the interest rate to optimally manipulate private expectations only in the partially transparency regime. Relative to opacity, it uses the interest rate to enlarge the private sector information set, but at the same time it creates a common knowledge effect, which could have adverse welfare consequences. However, a shrewd - i.e. optimizing - central bank can take this into account and make the interest rate a useless signal through infinite interest rate volatility so as to achieve the same outcome as under opacity. Similarly, in the case of full transparency, when the central bank reveals all its information, it creates a distortionary common knowledge effect with no

[10] To see this, we use (6) and (8) to write:

$$\sum_{j=1}^{N} \frac{\mu_k}{\mu_j}\varphi_j = \overline{\varphi}_k \sum_{j=1}^{N} \frac{\varphi_j}{\overline{\varphi}_j} = \overline{\varphi}_k \sum_{j=1}^{N} \frac{\frac{1}{\beta_j}\mu_j^2}{\overline{\varphi}_j \sum_{l=1}^{n} \mu_l^2 \left((1-r)\frac{1}{\alpha_l} + \frac{1}{\beta_l}\right)}$$

This expression then becomes

$$\overline{\varphi}_k \sum_{j=1}^{N} \frac{\frac{1}{\beta_j}\mu_j^2}{\overline{\varphi}_j \sum_{l=1}^{n} \left(\frac{\overline{\varphi}_l}{\overline{\varphi}_j}\right)^2 (\mu_j)^2 \left((1-r)\frac{1}{\alpha_l} + \frac{1}{\beta_l}\right)} = \overline{\varphi}_k \sum_{j=1}^{N} \frac{\frac{1}{\beta_j}\overline{\varphi}_j}{\sum_{l=1}^{n} (\overline{\varphi}_l)^2 \left((1-r)\frac{1}{\alpha_l} + \frac{1}{\beta_l}\right)}$$
$$= \overline{\varphi}_k \sum_{j=1}^{N} \frac{\frac{1}{\beta_j}\overline{\varphi}_j}{\sum_{l=1}^{n} \frac{\overline{\varphi}_l}{\beta_l}} = \overline{\varphi}_k.$$

[11] Indeed, an intermediate regime between partial and full transparency involves revealing R and $\tilde{\theta}_k$ for $k = 1, K$ while keeping confidential $\tilde{\theta}_k$ for $k = K+1, n$. In this case, R provides information about the (optimal) linear combination of signals $\tilde{\theta}_k$, $k = K+1, n$. A partially transparent central bank can always choose μ_k for $k = 0, K$ to mimic the corresponding full information and μ_k for $k = K+1, n$ to mimic optimal policy with partial release of the corresponding signals.

signaling instrument left to offset it. Under the partial transparency regime, revealing the interest rate is also the source of a common knowledge effect; here again, a shrewd central bank can minimize the distortion through its choice of the interest rate.

The case when $\beta_i = \beta_j \; \forall i, j$ further illustrates the role of the assumption that the interest rate does not play any macroeconomic role. We have seen that the optimal solution for the central bank is to set $\mu_k = \pm\infty$. In effect, the central bank creates maximum volatility to make the interest rate uniformative. Obviously, such a policy would be enormously costly if the interest rate had a macroeconomic effect and a partially transparent central bank most likely would trade-off the macroeconomic and communication effects.

3.5 The special case of full symmetry

As an illustration and for further reference, we consider the case where $\alpha_k = \alpha$ and $\beta_k = \beta \; \forall k$, i.e. signal precision is the same for each of the n fundamentals. Since we already assume that $\mathbf{A}\boldsymbol{\theta} = \sum_{k=1}^{n} \theta_k$, the full symmetry assumption makes the signals "equivalent", yet distinct. This simplification does not affect the opacity and full transparency regimes but it allows us to characterize optimal monetary policy in the partial transparency regime. This is why, in the rest of the paper, we will limit our study to the neighborhood of this full symmetry setup.

Under partial transparency, the price level is given by (6). Using the constraint $\sum_{i=1}^{n} \mu_i = 1$, we find:

$$p^i = \frac{\alpha R}{\left[\alpha + (1-r)\beta\right] \sum_{k=1}^{n} \mu_k^2} + \sum_{k=1}^{n} \left(1 - \frac{\alpha \mu_k x_k^i}{\left[\alpha + (1-r)\beta\right] \left(\sum_{k=1}^{n} \mu_k^2\right)}\right)$$

The Appendix shows that the central bank optimizes by setting $\mu_k^* = \frac{1}{n}$. $\forall k = 1, n$ if the following second order condition is satisfied:

$$(1 - k_1)\,\alpha + (1 - r)\,(1 - 2k_1)\beta > 0 \tag{11}$$

Then equilibrium prices are:

$$p^i = n \frac{\alpha}{\alpha + \beta\,(1-r)} R + \sum_{k=1}^{n} \frac{\beta\,(1-r)}{\alpha + \beta\,(1-r)} x_k^i \tag{12}$$

which are the same as under full transparency when $R = \frac{1}{n} \sum \tilde{\theta}_k$. It follows that $L^{PT}(\boldsymbol{\mu}^*) = L^{FT}$ under symmetry, where $\boldsymbol{\mu}^* = (\frac{1}{n}, ... \frac{1}{n})$.

To understand this result, recall that we have normalized the fundamentals so that $\mathbf{A}\boldsymbol{\theta} = \sum \theta_k$. The assumption $\alpha_k = \alpha$ and $\beta_k = \beta \; \forall k$ implies that, when they make their forecasts, both the central bank and the private sector attribute the same weight $\frac{1}{n}$ to all signals. It is natural therefore for the central bank to choose $R = \frac{1}{n} \sum \tilde{\theta}_k$. Using Bayes rule, the private sector then uses this information to infer that the central bank has received the signals $\tilde{\theta}_k = \frac{R}{n}\ \forall k$. This prevents the central bank from manipulating private sector expectations fundamental by fundamental. Put differently, when the central bank is fully transparent, the private agents use this information to set their prices p^i by combining the signals $\tilde{\theta}_k,\ k = 1, n$ revealed by the central bank as if (12) applies with $R = \frac{1}{n}\sum \tilde{\theta}_k$.

When the second order condition (11) is not satisfied, the loss function is minimized when the central bank sets $\mu_k = \pm\infty$ with signs such that $\sum \mu_k = 1$. Denote as $\boldsymbol{\mu}^{\infty}$ the corresponding vector of policy parameters. The partially transparent central bank creates maximum interest rate volatility to remove any information value from its policy decision. As a consequence, the partial transparency and opacity regimes are identical, as previously noted. The fact that optimized partial transparency delivers opacity also establishes that opacity welfare-dominates full transparency. Summarizing, we have established the following:

$$\begin{array}{lrcl} \text{When } (1-k_1)\,\alpha + (1-r)\,(1-2k_1)\beta & > & 0: & L^{PT}(\boldsymbol{\mu}^*) = L^{FT} < L^{op} \\ \text{When } (1-k_1)\,\alpha + (1-r)\,(1-2k_1)\beta & < & 0: & L^{PT}(\boldsymbol{\mu}^{\infty}) = L^{op} < L^{FT} \end{array}$$

The second order condition plays an important role. It involves all of the model's parameters and can be rewritten as $\frac{\alpha}{\beta} > -(1-r)\,\frac{1-2k_1}{1-k_1}$. Intuitively, it is satisfied when the relative precision of central bank signals $\frac{\alpha}{\beta}$ is high enough, when the common knowledge effect is moderate because private agents are not too reactive to each other's prices, and when price dispersion is perceived as a negative externality ($k_1 < 0$) or a relatively low positive externality ($k_1 > 0$ but not too large). It is always satisfied when is $k_1 < \frac{1}{2}$.

The combined role of the relative precision of central bank signals and of private sector reactivity is illustrated by previous results from in the literature. As noted in Section 2.1, the welfare function chosen by M&S corresponds to $k_1 = r$. In this case the second order condition is satisfied and full transparency welfare-dominates opacity when $\frac{\alpha}{\beta} > 2r - 1$, while opacity is the preferable regime in the opposite case. The welfare function advocated by Woodford (2005) corresponds to $k_1 = -r$ in which case the second order condition is always satisfied and opacity is never desirable.

The role of k_1 is further illustrated as follows. We have seen that, when it sets the interest rate under partial transparency, the central bank can reproduce the full transparency outcome, and that it can even do better for social welfare, which implies $L^{FT} \geq L^{PT}$. We can make a similar, symmetric argument regarding the private sector. Under full transparency, when the central bank releases all its information, the private sector can always choose the same prices (6) as under partial transparency, and it can do better by optimizing. This does not imply that $L^{FT} \leq L^{PT}$, however, because private agents cannot react to the aggregate price dispersion externality since they are atomistic. The best that they can individually do is not socially optimal, while the central bank internalizes the externality and delivers the social optimum. This is why, in the end, as long as the externality is not strongly welfare-increasing, i.e. when

$k_1 < \frac{1}{2}$, we have $L^{FT} > L^{PT}$, with $L^{FT} = L^{PT}$ when $k_1 = 0$. A conjecture, which is confirmed below, is that the difference in losses $L^{FT} - L^{PT}$, which is non-negative, is proportional to k_1^2.

4 Private Information Precision Unknown to the Central Bank

So far we have followed the existing literature in assuming that the variances of the signals received by individual private agents and by the central bank are known. We now allow for information precision to be imperfectly known. Specifically, we assume that the central bank information precision α_k about signal θ_k, for $k = 1, n$, is known to all but that the private sector information precision β_k is unknown to the central bank. Put differently, we assume that the private sector knows its own precision but has no way to reveal it to the central bank.

The justification for this assumption is that the central bank forecasts are closely monitored and evaluated by both the central bank itself and the private sector; presumably the central bank has the resources needed to evaluate its forecasting performance and has no reason to hide its results from its watchers. On the other hand, the central bank cannot observe the myriad of private sector forecasts well enough to infer their precision.[12] In the next section, we will consider the case when the private information precision is also unknown to the private sector itself.

To keep the analysis tractable, for all signals θ_k, $k = 1, n$, we will consider small deviations from the symmetric case studied in Section 3.5:

$$\begin{aligned} \alpha_k &= \alpha + u_k \\ \beta_k &= \beta + v_k \end{aligned} \tag{13}$$

where u_k and v_k are zero-mean random variables whose variances are unknown.[13] While α_k is public knowledge, we assume that private agents know β_k, which is the same for every agent. In contrast, the central bank erroneously believes that the private sector precision is:

$$\beta'_k = \beta_k + v'_k \tag{14}$$

[12] Why can't the private sector communicate its own precision to the central bank? Conceivably, it could, as it could reveal its signals; this would be welfare improving since it would eliminate the information heterogeneity problem. The assumption that private sector information is heterogeneous rests on the view that private sector information is inherently diffuse, presumably because of the multiplicity of agents, maybe also because of their limited resources.

[13] Otherwise we would have to formulate a hypothesis on the variances of u_k and v_k (the variances of the variances of signals), a somewhat far-fetched variable, and we could not assume Bayesian inference anymore.

where v'_k, $k = 1, n$, are independent random variables with zero mean and variances $F_k^2 v_k^2$. The proportionality term F_k represents a sort of "fog" under which the imperfectly informed central bank operates. Because of this fog, the central bank will be unable to choose the same optimal interest rate as was the case in the previous section. Instead of choosing the policy parameters $\boldsymbol{\mu} = (\mu_1, ..., \mu_N)$ it will set $\boldsymbol{\mu}' = (\mu'_1, ..., \mu'_N)$, which is socially suboptimal.

4.1 Transparency regimes

When the central bank does not know the precision of private signals, we can identify four transparency regimes: 1) full opacity; 2) interest rate (partial) transparency (RPT) when the central bank only reveals its interest rate decision R; 3) interest rate and precision (partial) transparency (RPPT) when the central bank reveals both the interest rate and its estimates $\boldsymbol{\beta}'$ of private sector precision; 4) full transparency (FT) when it also reveals its own signals $\widetilde{\boldsymbol{\theta}} = (\tilde{\theta}_1, ..., \tilde{\theta}_n)$. As before, in our setup, the interest rate decision is irrelevant in the polar regimes of opacity and full transparency. It follows that the situation under opacity and full transparency is the same irrespective of whether private sector precision is known or not.

In Section 3 partial transparency always welfare-dominates full transparency because the central bank can use the interest rate signal to partially offset the common knowledge effect. Does this result carry through to the case when the central bank does not know the precision of private signals? Not necessarily so. Indeed, because the interest rate decision will now rely upon erroneous knowledge, it may be that full transparency provides a better outcome than either partial transparency regime.

Informally, we know that when all precision is known, $L^{PT}(\boldsymbol{\mu}^*) < L^{FT}$. The only difference between partial tranparency when all precision is known and RPPT when private sector precision is not known to the central bank is that, in the latter case, the central bank uses incorrect precision estimates $\boldsymbol{\beta}' = (\beta_1, ..., \beta_N)$ to set the interest rate. Thus it is likely to choose a suboptimal $\boldsymbol{\mu}' = (\mu'_1, ..., \mu'_n)$ and $L^{RPPT}(\boldsymbol{\mu}') \geq L^{PT}(\boldsymbol{\mu}^*)$. Thus, we cannot directly compare $L^{RPPT}(\boldsymbol{\mu}')$ and L^{FT}. Yet, for the same reason as before, we know that there exists a $\hat{\boldsymbol{\mu}}$ such that, if chosen by the central bank, would replicate the full transparency regime outcome, i.e. that $L^{RPPT}(\hat{\boldsymbol{\mu}}) = L^{FT}$. There even exist optimal policy parameters $\boldsymbol{\mu}'^*$ such that $L^{RPPT}(\boldsymbol{\mu}'^*) < L^{FT}$. However, since the central bank does not know private sector precision, it can only choose $\boldsymbol{\mu}'^*$ by sheer luck. In fact, if the central bank is sufficiently off the mark - if the fog is thick - it will in fact choose $\boldsymbol{\mu}'$ such that $L^{RPPT}(\boldsymbol{\mu}') > L^{FT}$. We now prove this conjecture.

4.2 Welfare comparisons

4.2.1 Interest rate and precision partial transparency (RPPT) vs. full transparency (FT)

We know from Section 3.5 that when precision is known, under symmetry, in the partial transparency regime the central bank optimal policy is to set $\mu_k^* = \frac{1}{n}\ \forall k$ when the second order condition (11) is satisfied. In the neighborhood of the symmetric equilibrium, we assume that the optimal policy parameters will be close to μ_k^*:

$$\mu_k = \frac{1}{n} + m_k$$

where m_k is presumed to be small.

If it imperfectly estimates private sector precision, the central bank chooses instead $\mu_k' = \frac{1}{n} + m_k'$. The resulting unconditional expectation of the loss is $E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)\right]$. The Appendix shows that $E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)\right] > L^{FT}$ when:

$$F > \bar{F} = \frac{\frac{\alpha}{\beta}|k_1|}{\frac{\alpha}{\beta} + (1-r)(1-2k_1)}\sqrt{\frac{n}{2(n-1)}} \tag{15}$$

where $F^2 = E\left[\frac{\sum_{k=1}^{n}\left(\frac{v_k}{\beta}\right)^2 F_k^2}{\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{u_i-u_j}{\alpha} - \frac{v_i-v_j}{\beta}\right)^2}\right]$ is the relevant aggregate measure of the fog effect on central bank policy decisions. The Appendix also shows that $\frac{\alpha}{\beta} + (1-r)(1-2k_1) > 0$ when the second order condition (11) is satisfied.

Thus the presence of fog, the fact that the central bank is uncertain about private signal precision, may reverse the welfare ranking of the partial and full transparency regimes. When the central bank knows private information precision, it can optimally choose the interest rate to deal with the common knowledge effect. When it mistakenly appraises private sector information, the interest rate that it chooses is no longer socially optimal. Full transparency, which makes the interest rate signal useless, becomes more desirable when the fog is thick enough.

To interpret (15), note that when there is no price dispersion externality, i.e. when $k_1 = 0$, the threshold $\bar{F} = 0$ and the slightest degree of fog is enough to make FT the best communication regime. We have seen that, when the private sector signal precision is known, partial and full transparency deliver the same welfare when $k_1 = 0$. Obviously, the presence of fog, which leads the central bank to make a mistake when setting the interest rate, worsens the situation under partial transparency.

When the price dispersion externality is present so that $k_1 \neq 0$, partial transparency becomes desirable because, by manipulating the interest rate, the central bank partially internalizes the externality. The fog must be thick enough to make FT welfare-superior. The threshold $\bar{F}$ increases with $|k_1|$ when $k_1 > 0$ and declines with $|k_1|$ when $k_1 < 0$. When $k_1 > 0$, the price dispersion externality raises welfare; the common knowledge effect becomes increasingly undesirable as k_1 becomes larger and interest manipulation under partial transparency stands to raise welfare. Conversely, when $k_1 < 0$, the price dispersion externality reduces welfare; the common knowledge effect is good, as in Woodford (2005), and FT dominates even for low levels of fog.

The threshold $\bar{F}$ increases with $\frac{\alpha}{\beta}$, the relative precision of central bank signals. Quite intuitively, a better informed central bank is better able to use the interest rate to manipulate private expectations. The threshold also increases with the degree r of reactivity of private agents to each other expectations. Indeed, a higher degree of reactivity increases the common knowledge effect that the central bank can partially offset when it sets the interest rate.

The following proposition summarizes our results for the case when the second order condition is satisfied:

Proposition 2 *When the central bank does not know the precision of private sector signals and when the relative information precision of the central bank is large enough for the second order condition (11) to hold, full transparency is more desirable than interest rate and precision partial transparency when the fog effect is large enough. The threshold is lower, and full transparency is more desirable:*
- the less precise is relative central bank information,
- the less reactive are private agents to each other expectations
- the stronger is the price dispersion externality when it reduces welfare
- the weaker is the price dispersion externality when it increases welfare.

When the second order condition (11) is not satisfied, the best option for the central bank is to let the policy parameters μ_k become arbitrarily large in absolute value, i.e. to mimic the opacity regime. This is the same result as when precision is known, see Section 3.5. The only difference is that, when it is mistaken about private sector precision, the central bank does not achieve what it wishes, which makes RPPT less desirable. But this is a second order effect compared to the difference between opacity and full transparency.[14]

Thus we reach the following result:

$$\text{When } (1-k_1)\,\alpha + (1-r)\,(1-2k_1)\beta < 0: \qquad L^{op} \simeq E\left[L^{RPPT}\right] < L^{FT}$$

which can be summarized as follows:

Proposition 3 *When the central bank does not know the precision of private sector signals, full opacity is the most desirable communication strategy when the second order condition (11) does not hold.*

A comment is in order. The proposition favors opacity even though we stated that $L^{op} \simeq E\left[L^{RPPT}\right]$. In Section 3.5, under full symmetry when $\alpha_k = \alpha$ and $\beta_k = \beta$

[14]Formally, to a first order of approximation, we have $L^{op} - L^{FT} = n\alpha\frac{(1-k_1)\alpha+(1-r)(1-2k_1)\beta}{\beta(\alpha+(1-r)\beta)^2} < 0$. The fog effects are of second order.

$\forall k$, the optimal choice of the policy parameters is $\mu_k = \pm\infty$ and $L^{op} = E\left[L^{RPPT}\right]$. In the neighborhood of full symmetry, the optimal parameters become arbitrarily large in absolute values ($\mu_k \to \pm\infty$) but they remain finite. We can only state that $E\left[L^{RPPT}\right]$ is close to L^{op}. We do not examine further whether $E\left[L^{RPPT}\right]$ is larger or smaller than L^{op} because this solution depends on the unrealistic assumption that the interest rate plays no macroeconomic role.

4.2.2 Interest rate partial transparency (RPT) vs. interest rate and precision partial transparency (RPPT)

In both cases the central bank sets the interest rate optimally based on incorrect information about private sector precision. Under RPT, the private sector does not know the central bank's estimates of its precision. As a consequence its estimate of the optimally chosen policy parameters, denoted $\widetilde{\boldsymbol{\mu}} = (\tilde{\mu}_1, ...\tilde{\mu}_n)$, differs from the parameters $\boldsymbol{\mu}'$ actually chosen by the central bank. In order to set her price, each agent must therefore estimate both $\tilde{\mu}_k$ and the central bank signals $\tilde{\theta}_k$, $k = 1, n$ but she does not observe $\widetilde{\boldsymbol{\mu}}$. In order to estimate $\widetilde{\boldsymbol{\mu}}$, therefore, she combines her knowledge of the interest rate R with her guess of the central bank's belief about her own signal precision, given by (14). We assume that she makes the following guess:

$$\tilde{\beta}_k = \beta_k + v'_k + \tilde{v}_k$$

with $\tilde{v}_k$ centered around 0 and of variance $\tilde{F}_k^2 v_k^2$. This additive uncertainty captures the assumption that the central bank misestimates private sector precision and that the private sector observes this estimate with a noise. The central bank fog F_k generates a private sector fog $\tilde{F}_k$. [15]

The Appendix shows that, when the second order condition is satisfied, the unconditional expectation of the social loss under RPT is higher than the unconditional expectation of the social loss under RPPT:

$$E\left[L^{RPT}(\boldsymbol{\mu}', \widetilde{\boldsymbol{\mu}})\right] > E\left[L^{RPPT}(\boldsymbol{\mu}')\right] \tag{16}$$

This result naturally reflects the spreading of uncertainty under RPT, which does not occur under RPPT. In both regimes, the central bank optimally uses the interest rate to fashion private sector expectations but its ignorance of private sector precision leads it to choose a socially suboptimal set of policy parameters $\boldsymbol{\mu}'$. Under RPPT, the private sector can correctly estimate $\boldsymbol{\mu}'$ because the central bank has revealed its estimate $\boldsymbol{\beta}'$; under RPT, the private sector makes the imprecise inference $\widetilde{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}'$ which leads to socially suboptimal prices.

[15] All the results that follow generalize to the case where the central bank also ignores its own precision. While formally identical to the problem at hand, this generalization has little economic justification.

When the second order condition is not satisfied and the optimal parameters $\mu_k \to \pm\infty$, as before. We can show in the same way that (16) still holds, for the same reason.

Proposition 4 *When the central bank does not know the precision of private signals, if it publishes its interest rate, it is always preferable that it also reveals its assessment of private signal precision, even though it is erroneous.*

Finally, the analysis of the opacity regime is essentially the same as in Section 3. When the second order condition (11) holds, partial transparency - both RPT and RPPT - welfare-dominates opacity for the same reason. When (11) does not hold, it is possible for the central bank under either partial transparency regime to let $\mu_k \to \pm\infty$ which delivers an outcome close to that achieved under the opacity regime. And here again, an optimizing central bank can do better than that, unless the fog is thick and the central bank's optimal choice is badly flawed. We do not pursue this comparison further because the policy under partial transparency implies approximately mimicking opacity by making the interest rate highly volatile, which we view as an unrealistic implication of our assumption that the interest rate plays no macroeconomic role.

4.3 Discussion

The literature on monetary policy under perfect information has so far focused on uncertainty about the economic fundamentals. Section 3 essentially generalizes that literature to the case of an indefinite number of fundamentals to show that, indeed, information heterogeneity leads to a common knowledge effect. In the present section, we have added a second level of uncertainty, which concerns the precision of the signals.

"Central bank information" therefore is now multidimensional. While poor information about the signals creates the common knowledge effect, poor information about private signal precision generates a fog effect that reduces the effectiveness of the central bank. While the welfare effects of signal uncertainty are ambiguous (as reflected in the contrasted results of M&S and Woodford), the fog effect unambiguously makes full transparency more desirable. The intuition is clear. The central bank uses the interest rate to affect private sector expectations to deal with the common knowledge effect and to correct for the price dispersion externality. When its understanding of private sector pricing decision is flawed because it misestimates private sector precision, the central bank better contributes to welfare by not using the interest rate as a signal. This is achieved by revealing directly all the information rather a partial summary as with the interest rate.

A less obvious intuition is that a central bank that is mistaken about private sector signal precision should truthfully reveal its mistaken beliefs. The reason is that the the central bank uncertainty about private sector signal precision has two effects: it leads to a socially suboptimal interest rate decision, the fog effect, and it forces the private sector to take into account the central bank mistaken beliefs, which leads to another fog effect, which results in socially suboptimal pricing decisions. Removing this second fog effect through full transparency can be welfare enhancing.

Yet, it is not always the case that more transparency is always better than less. When its own signal precision is relatively low - when the second order condition (11) is not satisfied - it may make sense for the central bank to be fully opaque and not to reveal its interest rate. In that case, if the central bank cannot hide its interest rate decision, it becomes optimal to make the rate uninformative. This result, as previously mentioned, crucially depends on our assumption that the interest rate has only a signalling role, i.e. it has no macroeconomic effect.

5 Private Information Precision Unknown to Both the Central Bank and the Private Sector

We now extend the previous case to the situation where neither the central bank nor the private sector know the precision of private sector information $\boldsymbol{\beta}$. This may be an assumption more germane to the idea of information heterogeneity. The underlying view is that the central bank is very carefully monitored and devotes substantial resources to collecting and processing information. On the other hand, the private sector is composed of a large number of agents with limited resources and amongst which information collection and processing is a strategic instrument, hence rather secretive.

In line with the previous treatment of imperfect information, we consider the situation in the neighborhood of the symmetric case, see (13), and we assume that each private sector agent believes that her information precision for fundamental θ_k is:

$$\beta_k'' = \beta_k + v_k''$$

where the error terms are independently distributed with zero mean and variance $G_k^2 v_k^2$ for all $k = 1, n$. The assumptions about the central bank assessment of $\boldsymbol{\beta}$ is the same as in the previous section, see (14). The transparency regimes - publishing only the interest rate (RPT) or both the interest rate and the central bank beliefs about private sector precision (RPPT) - are also the same. As before, the polar regimes of opacity and full transparency are not affected by the uncertainty about signal precision because under either regime there is no (information) role for the interest rate. We assume Knightian uncertainty, i.e. that the central bank knows the existence of this fog but not the variances $G_k^2 v_k^2$. It follows that the central bank still chooses $\mu_k' = \frac{1}{n} + m_k'$ when the second order condition (11) is satisfied, otherwise it sets $\boldsymbol{\mu} \to \boldsymbol{\mu}^\infty$.

5.1 Interest rate and precision partial transparency (RPPT) vs. full transparency (FT)

We proceed by looking at a difference in differences: we compare the difference of social losses $E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{both}$ suffered under the RPPT and FT regimes when private signal precision is unknown to both the central bank and the private sector with the corresponding difference $E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{CB\ only}$ when it is only the central bank that is ill-informed.

When the second order condition (11) is satisfied, the Appendix shows that:

$$\begin{aligned} & E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{both} - E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{CB\ only} \\ = & -(1-r)^2 \alpha\beta \frac{\alpha + (1-r)\beta - 3k_1\alpha}{[\alpha + (1-r)\beta]^4} \frac{n-1}{n} G^2 \end{aligned} \quad (17)$$

where G is a measure of private fog, similar to the measure F of central bank fog. (17) shows that the impact of private sector uncertainty about its own precision depends on the sign of $\alpha + (1-r)\beta - 3k_1\alpha$.

Note first that the central bank fog does not affect this difference in differences: the two fog effects are additive. We exploit this result as follows. In the FT regime, the central bank does not make any useful decision, so the only optimizer is the price-setting private sector. In the RPPT transparency regime, both the central bank and the private sector optimize, but the additivity result allows us to interpret (17) by reasoning as if the only optimizer in this regime is the central bank.

A first intuition from (17) is that the fog effect reduces the effectiveness of the optimizer agent. We already saw in Section 4 that the central bank is less effective when it optimizes under uncertainty about private sector signal precision; full transparency, when because the interest rate becomes a useless signal, tends to be welfare-dominant. When private agents also suffer from their own fog effect, they are less good at setting prices and this effect tends to make full transparency less desirable. The effect is captured in (17) by the term $\alpha + (1-r)\beta > 0$.

In order to interpret the remaining term $-3k_1\alpha$, we need to remember the result from Section 3.5 that the price dispersion externality captured by k_1 favors partial transparency because the central bank can internalize this component of social welfare. When $k_1 = 0$ and there is no externality, the presence of a private fog effect unambiguously makes RPPT more socially desirable than FT. This conclusion is reinforced when $k_1 < 0$, i.e. when price dispersion is a social bad, because the central bank is the optimizer under RPPT (in the sense indicated above). When $k_1 > 0$, we face a trade-off. Now the price dispersion externality is a social good, which the central bank takes into account as it makes its decision under interest rate and precision partial transparency. But the private sector fog effect too leads to more price dispersion under both regimes.[16] Because it ignores G - a case of Knightian uncertainty -

[16] More precisely, the presence of a private fog raises the unconditional expectation of price dispersion.

the central bank cannot take this additional effect into account under RPPT, which favors the FT regime. When k_1 is large enough, this latter effect dominates. Note that the role of the price dispersion externality is stronger the more precise is the central bank - the larger is α - because a highly precise central bank has a stronger influence on private sector pricing decisions.

For completeness, we briefly mention the case when the second order condition (11) is not satisfied. As in Section 4, the central bank makes the interest rate uninformative by choosing $\boldsymbol{\mu}$ close to $\boldsymbol{\mu}^{\infty}$. Since the fog effects are of second order of magnitude, opacity remains the best regime:

$$L^{op} \simeq E\left[L^{RPPT}\right] < L^{FT}$$

5.2 Interest rate partial transparency vs. interest rate and precision partial transparency

The Appendix shows that, when the second order condition (11) is satisfied, the central bank optimally sets $\mu_i \simeq \frac{1}{n}$ and the result of Section 4 still holds: RPPT dominates RPT. Indeed, the existence of a private sector fog does not affect the central bank behavior. Facing Knightian uncertainty about private sector fog, it still chooses policy parameters $\boldsymbol{\mu}'$; under RPT, the private sector still infers that the central bank has chosen $\widetilde{\boldsymbol{\mu}}$, which leads to the welfare reducing bias previously described. When, in addition, it is subjected to its own fog, the private sector sets socially suboptimal prices. The resulting adverse effect on welfare is similar under RPT and RPPT; whatever difference exists, it is small relative to the bias due to the central bank fog.

The same reasoning applies when (11) is not satisfied.

5.3 Welfare implications

The previous analysis is summarized as follows for the case when the second order condition (11) holds:

Proposition 5 *Comparing the situation when the private sector knows its own signal precision and when it does not, and still assuming that the central bank does not know private sector signal precision:*
- interest rate transparency is always welfare-dominated by interest rate and precision partial transparency
- the welfare case for interest rate and precision partial transparency is enhanced when the price dispersion externality reduces welfare
- the welfare case for full transparency is enhanced when the price dispersion externality raises welfare, especially when the (actual) relative precision of central bank information is relatively large relative to private sector information.

In the end, private sector fog does not play as strong a qualitative role as central bank fog. The reason is that, through the interest rate, the central bank plays a signaling role, while the private sector only make pricing decisions. The central bank's signaling role implies a common knowledge effect, which is partly welfare reducing, because of too much attention, and partly welfare-increasing, because it reduces price dispersion. The resulting trade-off remains unchanged even in the presence of private sector fog.

Finally, for completeness, we note that the conclusions previously reached regarding the opacity regime remain valid. When (11) is verified, a partially transparent central bank can always do better than a fully opaque one. When (11) does not hold, opacity is optimal.

6 Conclusions

Information heterogeneity among private agents has emerged as a key consideration in the literature on central bank transparency. Information heterogeneity leads to the common knowledge effect whereby private agents attach a strong weight to central bank signals not necessarily because the central bank is well informed but because its signals are widely observed. Knowing that other agents will respond to central bank signals give these signals an importance that exceeds their precision. This effect can make transparency desirable or not, depending on the assumed social welfare function.

The present paper extends the literature in four directions. First, it allows for more than one economic fundamental. Second it adds the interest rate to the list of signals that the central bank can reveal. Third, it extends the range of uncertainties that matter. So far the literature has focused on uncertainty about the economic fundamentals, which are supposed to be estimated with known precision; here we also allow for uncertainty about precision. Fourth, it derives results that are general in the sense that they do not depend on any particular social welfare criterion. Each extension sheds new light on the role of central bank transparency.

Allowing for more than one economic fundamental shows that the central bank communication policy rests on exploiting the differences in the stochastic patterns of the corresponding signals. *A contrario*, when all signals are drawn from the same known distribution,[17] which may differ between the central bank and the private sector signals, any partial release by the central bank of its own signals, e.g. by announcing the optimally chose interest rate, amounts to releasing all signals. Then the central bank is left with a binary choice of transparency regimes, full opacity or full transparency. The multiplicity of independent signal fundamentals provides the central bank with a wider menu of transparency regimes.

The use of the interest rate as a communication tool has not been examined so

[17] When $\alpha_i = \alpha_j$ and $\beta_i = \beta_j \ \forall i, j$.

far, we believe. Here we go to the polar situation when the interest rate is only a communication tool. Full transparency occurs when the central bank faithfully reports all the signals that it has received. Full transparency, therefore, is a passive regime, since it deprives the interest rate from any additional information value. In contrast, with less than full transparency, the central bank can use the interest rate to shape private sector expectations. This makes the interest rate a strategic signal.[18] In particular, the central bank can optimally set the interest rate to mitigate the common knowledge effect when it is detrimental to social welfare or to exploit it when it enhances social welfare.

When signal precision is known, we essentially reproduce the results previously established in the literature, although we cast them more generally using the social welfare function proposed by Hellwig (2005), which encompasses the special cases proposed by M&S and by Woodford (2005). In fact, in this case, we establish results that are independent of the specification of the welfare function. Partial transparency, defined as the publication of the optimally set interest rate, dominates both opacity and full transparency. The reason is that opacity prevents the central bank from affecting private sector expectations while full transparency makes the interest rate uninformative since the central bank has revealed everything that it knows. We show that the result by M&S, that opacity can be optimal, depends on two assumptions: that all signals are drawn from the same distribution and, as shown by Woodford (2005), that the social welfare function ignores the negative externality associated with individual price dispersion. Similarly, we show that the result by Woodford (2005), that full transparency dominates, also depends on two assumptions: that all signals are drawn from the same distribution and that the negative externality associated with individual price dispersion is strong enough.

Allowing for uncertainty regarding the precision of private signals profoundly changes the picture. Because it misjudges private signal precision, the central bank operates in sort of fog. As a result, its optimally chosen interest rate is in fact socially suboptimal. This makes partial transparency regimes less desirable since the central bank's ability to optimally shape private expectations is impaired. In this case, there is no generally optimal transparency regime. When the fog is thick, i.e. when the misjudgment of private sector precision has a large variance, full transparency becomes the most desirable regime. Obviously, as the fog gets thinner, we move back toward the case when signal precision is known and partial transparency becomes optimal again.

This result holds whether the private sector knows or not its own signal precision. When the private sector also operates in a fog because it misjudges its own signal precision, nothing is qualitatively changed regarding the central bank interest rate decision. The main difference is that private agents caught in the fog make individually optimal decisions that are in fact socially suboptimal. Whether it makes one

[18] The release of some optimally chosen signals would provide the central bank with a similar tool. We have not examined this issue, which is left for further research.

transparency regime more preferable or not depends on the relative actual precision of central bank signals relative to private sector signals. Quite logically, the more precise is this actual relative precision, the better is the central apt at shaping private sector expectations and, *ceteris paribus*, the more desirable is the partial transparency regime.

Obviously, the present paper suffers from a number of limitations that should be kept in mind before drawing policy conclusions. To start with, the interest rate plays no direct macroeconomic role in our model. Its only function is to convey some information about the central bank signals. While unrealistic, this assumption allows us to isolate the information content of the interest rate. If the interest rate were to also play a macroeconomic role, the central bank would have to trade off the macroeconomic and signaling effects of its monetary policy decisions. This would reduce the attractiveness of the kind of interest rate manipulation that we focus upon and, probably, increase the attractiveness of the full transparency regime. Indeed, under full transparency, the interest rate loses its signal content, which makes it entirely available to play its macroeconomic role.

Another limitation is that we assume that the only source of uncertainty concerns the economic fundamentals. It can be argued that, in fact, this uncertainty is rather small, at least in comparison with our lack of understanding of the "true" economic model. In that view, the most challenging communication issue faced by central banks is to give a sense of the model that they have in mind.[19] In our framework, the economic model is subsumed by just one equation. It is captured in (1) by the term $\mathbf{A}\boldsymbol{\theta}$. We assume that vector $\mathbf{A}$, which captures the model's structure, is known while the fundamentals included in vector $\boldsymbol{\theta}$ are stochastic. Treating both $\mathbf{A}$ and $\boldsymbol{\theta}$ as stochastic would be a major complication; it is left for further research. One possibility is to invert things: let $\boldsymbol{\theta}$ be known and allow $\mathbf{A}$ to be stochastic. Obviously, then, this is a matter of rewriting the story and the results would qualitatively survive intact.[20]

Finally, partial information here means revealing some categories of information (the signals, the interest rate, signal precision). Another approach would be for the central bank to reveal a subset of each category, for example a subset $\tilde{\theta}_k$ of $k = 1, K$ with $K < n$ of its signals. Indeed, it could be argued that the set of relevant signals is too large for a central bank to ever be fully transparent. This issue is left for further research but the following remarks suggest the issues likely to emerge. Under partial transparency, the central bank reveals the interest rate, which is a linear combination of its signals. The only difference between revealing the interest rate and just one of the n signals is that the interest rate is an optimal combination of the signals,

[19] This is the point made by our discussant, Charles Bean.

[20] We have not chosen this route because it has proven convenient to normalize $\mathbf{A}$ by setting $A_k = 1\ \forall k$. We could normalize the fundamentals and set $\theta_k = 1\ \forall k$, but we find this approach unappealing. Indeed, it becomes unclear what are the fundamentals if they are all constant and equal.

which allows the central bank to fashion private expectations and thus deal with the common knowledge effect. For that reason, revealing the interest rate stands to welfare dominate revealing one signal. It may even dominate revealing two or more signals but, some way along the road, revealing a large number of signals may dominate. The larger is the fog effect, the more this form of partial transparency is likely to be socially optimal.

References

Amato, Jeffery D., Stephen Morris and Hyun Song Shin (2002) “Communication and Monetary Policy,” Oxford Review of Economic Policy 18: 495—503.

Amato, Jeffery D., and Hyun Song Shin (2003) “Public and Private Information in Monetary Policy Models,” BIS Working Paper No. 138, www.bis.org.

Bean, Charles (2005) "Monetary Policy in an Uncertain World", unpublished, Bank of England.

Blinder, Alan S. (1998) *Central Banking in Theory and Practice*, MIT Press, 1998.

Cukierman, Alex (2007) "The Limits of Transparency", forthcoming in Siklos P. et.al. (eds.), *Frontiers in Central Banking*, Elsevier.

Cukierman, Alex and Allan H. Meltzer (1986) "A Theory of Ambiguity, Credibility, and Inflation Under Discretion and Asymmetric Information", *Econometrica* 54(5): 1099–1128.

Geraats, Petra (2002) "Central Bank Transparency", *Economic Journal* 112 (483): F532-F565.

Geraats, Petra (2005) "Transparency and Reputation: The Publication of Central Bank Forecasts", *Topics in Macroeconomics* 5(1): 1-26.

Hellwig, Christian (2005) "Heterogeneous Information and the Welfare Effects of Public Information Disclosures", unpublished, UCLA

Morris, Stephen and Hyun Song Shin (2002) “The Social Value of Public Information”, American Economic Review 92: 1521—1534.

Kydland, F. E. and Prescott, E. C. (1977), "Rules Rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy* 85(3): 473–491.

McCallum, Bennett (1995) "Two Fallacies Concerning Central Bank Independence", *American Economic Review, Papers and Proceedings* 85(2): 207–211.

Morris, Stephen and Hyun Song Shin (2005) "Central Bank Transparency and the Signal Value of Prices", Brookings Papers on Economic Activity, forthcoming.

Roca, Mauro (2006) "Transparency and Monetary Policy with Imperfect Common Knowledge", unpublished paper, Columbia University.

Svensson, Lars E.O. (2005) "Social Value of Public Information: M&S Is Actually Pro Transparency, Not Con", American Economic Review, forthcoming.

Vickers, John (1986) ”Signalling in a Model of Monetary Policy with Incomplete Information,” *Oxford Economic Papers* 38(3): 443-55.

Walsh, Carl (2007) "Optimal Economic Transparency", *International Journal of Central Banking*, forthcoming.

Woodford, Michael (2003) *Interest and Prices*, Princeton, NJ: Princeton University Press.

Woodford, Michael (2005) "Central-Bank Communication and Policy Effectiveness", in: *The Greenspan Era: Lessons for the Future*, Kansas City: Federal Reserve Bank of Kansas City.

Appendix

Losses when private sector variances are known

$$L^{op} = (1-k_1)\sum_{k=1}^{n}\frac{1}{\beta_k}$$

$$L^{PT}(\boldsymbol{\mu}) = \sum_{k=1}^{n}(1-r)\left(\frac{\sum_{j=1}^{n}\frac{\mu_j}{\beta_j}}{\sum_{j=1}^{n}\mu_j^2\left(\frac{1-r}{\alpha_j}+\frac{1}{\beta_j}\right)}\right)^2\frac{\mu_k^2}{\alpha_k} + (1-k_1)\left(1-\frac{\sum_{j=1}^{n}\frac{\mu_j\mu_k}{\beta_j}}{\sum_{j=1}^{n}\mu_j^2\left(\frac{1-r}{\alpha_j}+\frac{1}{\beta_j}\right)}\right)^2\frac{1}{\beta_k}$$

$$L^{FT} = \sum_{k=1}^{n}(1-r)\left(\frac{\alpha_k}{\alpha_k+(1-r)\beta_k}\right)^2\frac{1}{\alpha_k} + (1-k_1)\left(\frac{(1-r)\beta_k}{\alpha_k+(1-r)\beta_k}\right)^2\frac{1}{\beta_k}$$

Proof of (6)

Given (5), 2 implies:

$$p^i = (1-r)\sum_{n=0}^{\infty} r^n E^i\left(\bar{E}^n(A\theta)\right) = AM\frac{1-r}{1-rM}\begin{bmatrix} R \\ x_1^i \\ \dots \\ x_n^i \end{bmatrix}$$

where M and A are the matrix given by:

$$\begin{aligned} M_{1j} &= \delta_{1j} \\ M_{i1} &= \frac{\gamma_{i-1}}{\mu_{i-1}} \text{ for } i > 1 \\ M_{ij} &= \delta_{ij} - \frac{\mu_{j-1}\gamma_{i-1}}{\mu_{i-1}} \text{for } i > 1, \text{ for } j > 1 \end{aligned}$$

$$A = (1, \dots, 1) \qquad 1 \times n$$

A straightforward computation of $AM\frac{1-r}{1-rM}$ leads to (6).

Proof of the results in Section 3.5

Under partial transparency, using (6), we have:

$$\begin{aligned} E\left(\sum_{j=1}^{n}\frac{\varphi_j}{\mu_j}\right)^2\left(R-\sum_{k=1}^{n}\mu_k\theta_k\right)^2 &= \frac{\alpha}{[\alpha+(1-r)\beta]^2\left(\sum_{k=1}^{N}\mu_k^2\right)} \\ E\left(\sum_{k=1}^{n}\left(1-\sum_{j=1}^{n}\frac{\mu_k}{\mu_j}\varphi_j\right)\left(x_k^i-\theta_k\right)\right)^2 &= \frac{n}{\beta} + \frac{\alpha^2 - 2(\alpha+(1-r)\beta)\alpha}{\beta[\alpha+(1-r)\beta]^2\left(\sum_{k=1}^{N}\mu_k^2\right)} \end{aligned}$$

so that the central bank will minimize:

$$E^{CB}L^{PT} = \int \left[(1-r)\left(p^i - A\theta\right)^2 + r\left(p^i - \bar{p}\right)^2 - k_1 \int_j \left(p^i - \bar{p}\right)^2 \right] di$$

$$= \quad (1-k_1)\left(\frac{n}{\beta} + \frac{\alpha^2 - 2\alpha\left[\alpha + (1-r)\beta\right]}{\beta\left[\alpha + (1-r)\beta\right]^2\left(\sum_{k=1}^n \mu_k^2\right)}\right) + (1-r)\frac{\alpha}{\left[\alpha + (1-r)\beta\right]^2\left(\sum_{k=1}^n \mu_k^2\right)}$$

Noting that this expression can be written $(1-k_1)\frac{n}{\beta} + \frac{K}{\left(\sum_{k=1}^n \mu_k^2\right)}$ and recalling the restriction $\sum_{k=1}^n \mu_k = 1$, it follows that the first order condition implies $\mu_k = \frac{1}{n}$.

Using Kronecker's δ_{ij}, the second order conditions requires that the $(n-1)\times(n-1)$ matrix with elements

$$\frac{\partial^2}{\partial\mu_i\partial\mu_j}L^{PT}(\boldsymbol{\mu}) = 2\frac{(1+\delta_{ij})\left[(1-k_1)\alpha + (1-r)(1-2k_1)\beta\right]}{\beta\left(\alpha + (1-r)\beta\right)^2\left(\sum_{k=1}^n \mu_k^2\right)^2}$$

be positive semi-definite. This condition is satisfied iff $(1-k_1)\alpha + (1-r)(1-2k_1)\beta > 0$. If not, the minimum is achieved for some $\mu_k = \pm\infty$ with signs such that $\sum \mu_k = 1$.

Proof of (15)

In what we follows, we use the constraint $\sum_{k=1}^n \mu_k = 1$ to eliminate μ_n. We first compute the optimal policy parameters $\mu_k = \mu_k^* + m_k$ around the symmetric equilibrium $\mu_k^* = \frac{1}{n}$ when the private sector signal precision is known. These parameters are are such that $\frac{\partial L^{PT}}{\partial\mu_k} = 0$ for all $k = 1, n-1$. A second order expansion of this condition around the symmetric equilibrium yields for all $k = 1, n-1$:

$$\sum_{j=1}^{n-1} m_j \frac{\partial^2 L^{PT}}{\partial\mu_k\partial\mu j} + \sum_{j=1}^{n}\left(u_j\frac{\partial^2 L^{PT}}{\partial\mu_k\partial\alpha_j} + v_j\frac{\partial^2 L^{PT}}{\partial\mu_k\partial\beta_j}\right) = 0 \tag{A1}$$

where the second order derivatives are evaluated at the symmetric equilibrium $\alpha_k = \alpha$, $\beta_k = \beta$, $\mu_k^* = \frac{1}{n}$., which assumes that the second order condition is satisfied. Defining as $\mathbf{m}$, $\mathbf{u}$ and $\mathbf{v}$ as the vectors of m_k, u_k and v_k, respectively, the first order conditions can be rewritten in matrix form:

$$\left[\partial^2_{\boldsymbol{\mu}} L^{PT}\right]\mathbf{m} + \left[\partial^2_{\boldsymbol{\alpha}} L^{PT}\right]\mathbf{u} + \left[\partial^2_{\boldsymbol{\beta}} L^{PT}\right]\mathbf{v} = 0$$

where we define $\left[\partial^2_{\boldsymbol{\mu}} L^{PT}\right]_{ij} = \frac{\partial^2 L^{PT}}{\partial\mu_i\partial\mu_j}$, $\left[\partial^2_{\boldsymbol{\alpha}} L^{PT}\right]_{ij} = \frac{\partial^2 L^{PT}}{\partial\mu_i\partial\alpha_j}$ and $\left[\partial^2_{\boldsymbol{\beta}} L^{PT}\right]_{ij} = \frac{\partial^2 L^{PT}}{\partial\mu_i\partial\beta_j}$. Denoting matrix $\mathbf{N}^{\boldsymbol{\alpha}} = -\left[\partial^2_{\boldsymbol{\mu}} L^{PT}\right]^{-1}\left[\partial^2_{\boldsymbol{\alpha}} L^{PT}\right]$ and $\mathbf{N}^{\boldsymbol{\beta}} = -\left[\partial^2_{\boldsymbol{\mu}} L^{PT}\right]^{-1}\left[\partial^2_{\boldsymbol{\beta}} L^{PT}\right]$, the optimal policy is therefore characterized as:

$$\mathbf{m} = \mathbf{N}^{\boldsymbol{\alpha}}\mathbf{u} + \mathbf{N}^{\boldsymbol{\beta}}\mathbf{v} \tag{A2}$$

i.e. $m_k = \sum_{j=1}^{n} (N_{kj}^{\alpha} u_j + N_{kj}^{\beta} v_j)$. Since we have eliminated μ_n the matrices $\mathbf{N}^{\boldsymbol{\alpha}}$ and $\mathbf{N}^{\boldsymbol{\beta}}$ have size $n-1 \times n$.

Close to the symmetric equilibrium, using the condition $\sum \mu_k = 1$, we have:

$$\frac{\partial L^{PT}}{\partial \alpha_i \partial \mu_j} = -2(1-r)\alpha \frac{\alpha + \beta(1-r)(1-2k_1)}{\alpha(\alpha+\beta(1-r))^3}(\delta_{ij} - \delta_{in}) n$$

$$\frac{\partial L^{PT}}{\partial \beta_i \partial \mu_j} = 2(1-r)\alpha \frac{\alpha + \beta(1-r)(1-2k_1)}{(\alpha+\beta(1-r))^3 \beta}(\delta_{ij} - \delta_{in}) n$$

$$\frac{\partial^2}{\partial \mu_i \partial \mu_j} L^{PT}(\boldsymbol{\mu}) = -2 \frac{(1+\delta_{ij})\left[(1-k_1)\left(\alpha^2 - 2(\alpha + (1-r)\beta)\alpha\right) + (1-r)\alpha\beta\right]}{\beta(\alpha+(1-r)\beta)^2} n^2$$

where we use Kronecker's δ_{ij}. This allows us to compute:

$$N_{ij}^{\alpha} = -2\beta \frac{\alpha + \beta(1-r)(1-2k_1)}{(\alpha+\beta(1-r))} \frac{(1-r)\left(\delta_{ij} - \frac{1}{n}\right)}{2\left((1-k_1)\left(\alpha^2 - 2(\alpha+(1-r)\beta)\alpha\right) + (1-r)\alpha\beta\right) n}$$

$$N_{ij}^{\beta} = 2\alpha \frac{\alpha + \beta(1-r)(1-2k_1)}{(\alpha+\beta(1-r))} \frac{(1-r)\left(\delta_{ij} - \frac{1}{n}\right)}{2\left((1-k_1)\left(\alpha^2 - 2(\alpha+(1-r)\beta)\alpha\right) + (1-r)\alpha\beta\right) n}$$

Using (A2), we get:

$$\begin{aligned} m_k &= \frac{2(1-r)\alpha\beta}{2\left((1-k_1)\left(\alpha^2 - 2(\alpha+(1-r)\beta)\alpha\right) + (1-r)\alpha\beta\right) n} \\ &\quad \times \frac{\alpha + \beta(1-r)(1-2k_1)}{\alpha + \beta(1-r)} \sum_{j=1}^{n} \left(\delta_{jk} - \frac{1}{n}\right)\left(-\frac{u_j}{\alpha} + \frac{v_j}{\beta}\right) \end{aligned}$$

which applies to $k = 1, ...n$ since the constraint $\sum \mu_k = 1$ can be used to compute m_n.

We use these results to compute $E\left[L^{RPPT}(\boldsymbol{\mu}')\right] - L^{FT}$. When the central bank believes that private sector precision is $\boldsymbol{\beta}' = \boldsymbol{\beta} + \mathbf{v}'$, under interest rate and precision partial tranparency it optimally chooses $\boldsymbol{\mu}' = \boldsymbol{\mu} + \mathbf{m}'$, which delivers social loss $L^{RPPT}(\boldsymbol{\mu}')$. The second-order development of $E\left[L^{RPPT}(\boldsymbol{\mu}')\right]$ around $L^{PT}(\boldsymbol{\mu})$ is:

$$\begin{aligned} E\left[L^{RPPT}(\boldsymbol{\mu}')\right] &= L^{PT}(\boldsymbol{\mu}) + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial \mu_i \partial \mu_j} E\left[(\mu_i' - \mu_i)\left(\mu_j' - \mu_j\right)\right] \\ &= L^{PT}(\boldsymbol{\mu}) + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial \mu_i \partial \mu_j} E\left[(m_i' - m_i)\left(m_j' - m_j\right)\right] \end{aligned}$$

which implies:

$$E\left[L^{RPPT}(\boldsymbol{\mu}')\right] - L^{FT} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial \mu_i \partial \mu_j} E\left[(m_i' - m_i)\left(m_j' - m_j\right)\right] + \left(L^{PT}(\boldsymbol{\mu}) - L^{FT}\right)$$

Using (A2), we get:

$$E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)-L^{FT}\right]=\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sum_{k=1}^{n}\frac{\partial^2 L^{PT}\left(\boldsymbol{\mu}\right)}{\partial\mu_i\partial\mu_j}N_{ik}^{\beta}N_{jk}^{\beta}F_k^2v_k^2+\left[L^{PT}\left(\boldsymbol{\mu}\right)-L^{FT}\right] \quad \text{(A3)}$$

Moreover a second order expansion, using (A1), shows that :

$$L^{PT}\left(\boldsymbol{\mu}\right)-L^{FT}=-\frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial^2}{\partial\mu_i\partial\mu_j}L^{PT}\left(\boldsymbol{\mu}\right)m_im_j$$
$$+\frac{1}{2}\left(\sum_{j=1}^{n}\sum_{i=1}^{n}v_iv_j\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\beta_i\partial\beta_j}+u_iu_j\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\alpha_i\partial\alpha_j}+2v_iu_j\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\beta_i\partial\alpha_j}\right)$$

Now, at the second order, we can show the following identities around the symmetric equilibrium:

$$\frac{1}{2}\left(\sum_{j=1}^{n}\sum_{i=1}^{n}v_iv_j\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\beta_i\partial\beta_j}+u_iu_j\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\alpha_i\partial\alpha_j}+2v_iu_j\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\beta_i\partial\alpha_j}\right)$$
$$=\frac{1}{2n}\frac{\alpha\beta\left(1-r\right)^2\left(\alpha+\left(1-r\right)\beta+k_1\left(\alpha-2\left(1-r\right)\beta\right)\right)}{\left(\alpha+\left(1-r\right)\beta\right)^4}$$
$$\times\sum_{j=1}^{n}\sum_{i=1}^{n}\left(\left(\frac{u_i-u_j}{\alpha}-\frac{v_i-v_j}{\beta}\right)^2\right)$$

$$-\frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial^2}{\partial\mu_i\partial\mu_j}L^{PT}\left(\boldsymbol{\mu}\right)m_im_j$$
$$=-\frac{\beta\alpha\left(1-r\right)^2}{2n\left[\left(1-k_1\right)\alpha+\left(1-2k_1\right)\left(1-r\right)\beta\right]}\left(\frac{\alpha+\beta\left(1-r\right)\left(1-2k_1\right)}{\left(\alpha+\left(1-r\right)\beta\right)^2}\right)^2$$
$$\times\sum_{i=1}^{n}\sum_{k=1}^{n}\left(\left(\frac{u_i-u_j}{\alpha}-\frac{v_i-v_j}{\beta}\right)^2\right)$$

$$\frac{1}{2}\sum_{k=1}^{n}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial^2}{\partial\mu_i\partial\mu_j}L^{PT}\left(\boldsymbol{\mu}\right)N_{ik}^{\boldsymbol{\beta}}N_{jk}^{\boldsymbol{\beta}}F_k^2v_k^2$$
$$=\frac{\beta\alpha\left(1-r\right)^2}{\left[\left(1-k_1\right)\alpha+\left(1-2k_1\right)\left(1-r\right)\beta\right]}\left(\frac{\alpha+\beta\left(1-r\right)\left(1-2k_1\right)}{\left(\alpha+\left(1-r\right)\beta\right)^2}\right)^2\frac{n-1}{n}\sum_{k=1}^{n}\left(\frac{\delta_k}{\beta}\right)^2F_k^2$$

We use these relationships in (A3) to obtain:

$$
\begin{aligned}
&E\left[L^{RPPT}(\boldsymbol{\mu}')\right]-L^{FT}=\frac{1}{2}E\left[\sum_{k=1}^{n}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial\mu_i\partial\mu_j}N_{ik}N_{jk}F_k^2 v_k^2\right]\\
&-\frac{1}{2}E\left[\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial\mu_i\partial\mu_j}x_i x_j\right]\\
&+\frac{1}{2}E\left[\sum_{j=1}^{n-1}\sum_{i=1}^{n-1}v_i v_j\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\beta_i\partial\beta_j}+u_i u_j\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\alpha_i\partial\alpha_j}+2u_j v_i\frac{\partial^2\left(L^{PT}-L^{FT}\right)}{\partial\beta_i\partial\alpha_j}\right]\\
=\;&\frac{2\beta\alpha(1-r)^2}{2\left[(1-k_1)\alpha+(1-2k_1)(1-r)\beta\right]}\left(\frac{\alpha+\beta(1-r)(1-2k_1)}{(\alpha+(1-r)\beta)^2}\right)^2\frac{(n-1)}{n}E\left[\sum_{k=1}^{n}F_k^2 v_k^2\right]\\
&-\frac{(1-r)^2\alpha^2 k_1^2}{2\left[(1-k_1)\alpha+(1-2k_1)(1-r)\beta\right](\alpha+(1-r)\beta)^4 n}\alpha\beta E\left[\sum_{i=1}^{n}\sum_{k=1}^{n}\left(\frac{u_i-u_j}{\alpha}-\frac{v_i-v_j}{\beta}\right)^2\right]
\end{aligned}
$$

Now define the fog effect as F with $F^2=E\left[\frac{\sum_{k=1}^{n}\left(\frac{v_k}{\beta}\right)^2F_k^2}{\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{u_i-u_j}{\alpha}-\frac{v_i-v_j}{\beta}\right)^2}\right]$. With this definition, we have:

$$
\begin{aligned}
&E\left[L^{RPPT}(\boldsymbol{\mu}')\right]-L^{FT}=\\
&\frac{1}{n}\frac{\alpha\beta(1-r)^2}{(\alpha+(1-r)\beta)^4}\frac{2\left[\alpha+\beta(1-r)(1-2k_1)\right]^2\frac{(n-1)}{n}F^2-\alpha^2k_1^2}{2\left[(1-k_1)\alpha+(1-2k_1)(1-r)\beta\right]}E\left[\sum_{i=1}^{n}\sum_{k=1}^{n}\left(\frac{u_i-u_j}{\alpha}-\frac{v_i-v_j}{\beta}\right)^2\right]
\end{aligned}
$$

It follows that $E\left[L^{RPPT}(\boldsymbol{\mu}')\right]-L^{FT}>0$ when $F^2>\left(\frac{k_1\frac{\alpha}{\beta}}{\frac{\alpha}{\beta}+(1-r)(1-2k_1)}\right)^2\frac{n}{2(n-1)}$. To obtain (15), we note that $\frac{\alpha}{\beta}+(1-r)(1-2k_1)>0$ when (11) is satisfied. Indeed, since $0<r<1$, this is the case when $k_1<0$. When $k_1>0$, (11) also ensures that this is the case.

Proof of (16)

When precision is known, the loss under the partial transparency regime is:

$$
\begin{aligned}
L^{PT}(\boldsymbol{\mu}^*)\;=\;&(1-r)E\left[\left(\sum_{j=1}^{n}\frac{\varphi_j}{\mu_j}\right)\left(R-\sum_{k=1}^{n}\mu_k\theta_k\right)\right]^2\\
&+(1-k_1)E\left[\sum_{k=1}^{n}\left(1-\frac{\sum_{j=1}^{N}\frac{1}{\beta_j}\mu_j\mu_k}{\sum_{j=1}^{N}\mu_j^2\left(\frac{1}{\alpha_j}(1-r)+\frac{1}{\beta_j}\right)}\right)^2\left(x_i^k-\theta_k\right)^2\right]
\end{aligned}
$$

The first term corresponds to the deviation of prices from their fundamentals. Denoting $\bar{L}(\boldsymbol{\mu})$ the second term, which corresponds to signal heterogeneity, we rewrite the loss as:

$$L^{PT}(\mu) = cE\left[\left(\sum_{j=1}^{n}\frac{\varphi_j}{\mu_j}\right)\left(R-\sum_{k=1}^{n}\mu_k\theta_k\right)\right]^2 + \bar{L}(\mu)$$

where $c = 1 - r$.

The same decomposition applies to the interest rate partial transparency regime when private sector precision is unknown to the central bank:

Similarly, when private sector precision is unknown to the central bank, the loss corresponding to the interest rate partial transparency regime:

$$E\left[L^{RPT}(\boldsymbol{\mu}', \tilde{\boldsymbol{\mu}})\right] = cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R-\sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 + E\left[\bar{L}\left(\tilde{\boldsymbol{\mu}}\right)\right]$$

where $E\left[\bar{L}\left(\widetilde{\boldsymbol{\mu}}\right)\right] = (1-k_1)\,E\left[\sum_{k=1}^{N}\left(1-\frac{\sum_{j=1}^{N}\frac{1}{\beta_j}\tilde{\mu}_j\tilde{\mu}_k}{\sum_{j=1}^{N}\tilde{\mu}_j^2\left(\frac{1}{\alpha_j}(1-r)+\frac{1}{\beta_j}\right)}\right)^2\left(x_i^k-\theta_k\right)^2\right]$.

Note that $\bar{L}\left(\widetilde{\boldsymbol{\mu}}\right)$ now is a stochastic variable because, since the private section precision is stochastic, the central bank's choice of the interest rate itself is stochastic.

Recall that $R = \sum_{k=1}^{n}\mu_k'\tilde{\theta}_k$ is the interest rate actually chosen by the central bank based on its estimate $\boldsymbol{\beta}'$ of private sector precision. We define $\tilde{R} = \sum_{k=1}^{n}\tilde{\mu}_k\tilde{\theta}_k$ as the notional interest rate that the private sector would expect if it could observe the central bank signals $\tilde{\theta}_k$ based on its own guess $\widetilde{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}'$. We make this mismatch explicit by rewriting the previous equation as:

$$\begin{aligned}
E\left[L^{RPT}(\boldsymbol{\mu}', \widetilde{\boldsymbol{\mu}})\right] &= cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R}-\sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 + E\left[\bar{L}\left(\widetilde{\boldsymbol{\mu}}\right)\right] \\
&\quad + cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R-\sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \\
&\quad - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R}-\sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \\
&= E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right] + cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R-\sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \\
&\quad - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R}-\sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2
\end{aligned}$$

where we define $E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right]$ as the unconditional expectation of the loss that would have occurred if the central would have chosen, and announced the notional interest rate $\tilde{R}$ and $\boldsymbol{\beta}'$. Collecting the expressions above, we have:

$$\begin{aligned} E\left[L^{RPT}(\boldsymbol{\mu}',\widetilde{\boldsymbol{\mu}}) - L^{RPPT}(\boldsymbol{\mu}')\right] &= E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right] - E\left[L^{RPPT}(\boldsymbol{\mu}')\right] \\ &\quad + cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \\ &\quad - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R} - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \qquad \text{(A4)} \\ &= \sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sum_{k=1}^{n}\frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial\mu_i\partial\mu_j}N_{ik}^{\boldsymbol{\beta}}N_{jk}^{\boldsymbol{\beta}}\tilde{F}_k^2 v_k^2 \\ &\quad + cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \\ &\quad - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R} - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \end{aligned}$$

The first term has already been shown to be positive when (11) is verified. It remains positive even when (11) is not met, at least when evaluated around the correspond optimum. The last two terms can be evaluated as follows:

$$\begin{aligned} & cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R} - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \\ = \; & cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R - \sum_{k=1}^{n}\mu_k'\theta_k - \sum_{k=1}^{n}\left(\tilde{\mu}_k - \mu_k'\right)\theta_k\right)\right]^2 \\ & - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R} - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \\ = \; & cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R - \sum_{k=1}^{n}\mu_k'\theta_k\right)\right]^2 + cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\sum_{k=1}^{n}\left(\tilde{\mu}_k - \mu_k'\right)\theta_k\right)\right]^2 \\ & - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R} - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \end{aligned}$$

we can evaluate $cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R-\sum_{k=1}^{n}\mu_k'\theta_k\right)\right]^2 - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R}-\sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2$ at the second order approximation as follows :

$$\begin{aligned} & cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R-\sum_{k=1}^{n}\mu_k'\theta_k\right)\right]^2 - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R}-\sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]^2 \\ \simeq\ & c\sum_{k=1}^{n}E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\mu_k'^2-\tilde{\mu}_k^2\right)\right]\frac{1}{\alpha} \end{aligned}$$

We can thus use this expression to rewrite :

$$\begin{aligned} E\left[L^{RPT}(\boldsymbol{\mu}',\widetilde{\boldsymbol{\mu}})-L^{RPPT}(\boldsymbol{\mu}')\right] &= cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\sum_{k=1}^{n}\left(\tilde{\mu}_k-\mu_k'\right)\theta_k\right)\right]^2 \\ &\quad +\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sum_{k=1}^{n}\frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial\mu_i\partial\mu_j}N_{ik}^{\boldsymbol{\beta}}N_{jk}^{\boldsymbol{\beta}}\tilde{F}_k^2 v_k^2 \\ &\quad +c\sum_{k=1}^{n}E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\mu_k'^2-\tilde{\mu}_k^2\right)\right]\frac{1}{\alpha} \qquad \text{(A5)} \\ &= c\sum_{k=1}^{n}E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\tilde{\mu}_k-\mu_k'\right)^2\right]E\left[\theta_k^2\right] \\ &\quad +\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sum_{k=1}^{n}\frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial\mu_i\partial\mu_j}N_{ik}^{\boldsymbol{\beta}}N_{jk}^{\boldsymbol{\beta}}\tilde{F}_k^2 v_k^2 \\ &\quad +c\sum_{k=1}^{n}E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\mu_k'^2-\tilde{\mu}_k^2\right)\right]\frac{1}{\alpha} \end{aligned}$$

The first term is positive and infinitely large with respect to the two last ones. As a consequence $E\left[L^{RPT}(\boldsymbol{\mu}',\widetilde{\boldsymbol{\mu}})-L^{RPPT}(\boldsymbol{\mu}')\right]>0$ as claimed in the text. This result evidences the role of the bias $\tilde{\mu}_k-\mu_k'$ between the parameters chosen by the central bank μ_k' and those $\tilde{\mu}_k$ guessed by the private sector. Note that the proof is independent of whether (11) is satisfied or not. (The result remains valid for $c=0$, i.e. $r=1$, since $\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sum_{k=1}^{n}\frac{\partial^2 L^{PT}(\boldsymbol{\mu})}{\partial\mu_i\partial\mu_j}N_{ik}^{\boldsymbol{\beta}}N_{jk}^{\boldsymbol{\beta}}\tilde{F}_k^2 v_k^2>0$).

Proof of (17)

When the second order condition (11) is satisfied, the central bank still chooses $\mu_k'=\frac{1}{n}+m_k'$. Then (A3) formally holds but the last term becomes $E\left[L^{PT}(\boldsymbol{\mu})-L^{FT}\right]$. Indeed $L^{PT}(\boldsymbol{\mu})-L^{FT}$ is now stochastic because the private sector mistakenly believes that the central bank estimate of private sector signals is $\boldsymbol{\beta}''$ whereas it really is $\boldsymbol{\beta}$. Using the expressions given above for the losses when private sector precision is known to both the central bank and the private sector, we expand $E\left[L^{RPPT}(\boldsymbol{\mu}) - L^{FT}\right]$ using the results from Proof of (15) to get:

$$
\begin{aligned}
& E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{both} \\
= \;& E\left[\sum_{k=1}^{n}(1-r)\left(\frac{\sum_{j=1}^{n}\frac{\mu'_j}{\beta_j+v''_j}}{\sum_{j=1}^{n}\mu_j'^2\left(\frac{1-r}{\alpha_j}+\frac{1}{\beta_j+v''_j}\right)}\right)^2\frac{\mu_k'^2}{\alpha_k} + (1-k_1)\left(1-\frac{\sum_{j=1}^{n}\frac{\mu'_j\mu'_k}{\beta_j+v''_j}}{\sum_{j=1}^{n}\mu_j'^2\left(\frac{1-r}{\alpha_j}+\frac{1}{\beta_j+v''_j}\right)}\right)^2\frac{1}{\beta_k}\right] \\
& -E\left[\sum_{k=1}^{n}(1-r)\left(\frac{\alpha_k}{\alpha_k+(1-r)\left(\beta_k+v''_k\right)}\right)^2\frac{1}{\alpha_k} + (1-k_1)\left(\frac{(1-r)\left(\beta_k+v''_k\right)}{\alpha_k+(1-r)\left(\beta_k+v''_k\right)}\right)^2\frac{1}{\beta_k}\right]
\end{aligned}
$$

Developing this expression to the second order around $v''_k = 0$ and computing the expectations yields:

$$
\begin{aligned}
& E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{both} \\
= \;& E\left[\sum_{k=1}^{n}(1-r)\left(\frac{\sum_{j=1}^{n}\frac{\mu'_j}{\beta_j}}{\sum_{j=1}^{n}\mu_j'^2\left(\frac{1-r}{\alpha_j}+\frac{1}{\beta_j}\right)}\right)^2\frac{\mu_k'^2}{\alpha_k} + (1-k_1)\left(1-\frac{\sum_{j=1}^{n}\frac{\mu'_j\mu'_k}{\beta_j}}{\sum_{j=1}^{n}\mu_j'^2\left(\frac{1-r}{\alpha_j}+\frac{1}{\beta_j}\right)}\right)^2\frac{1}{\beta_k}\right] \\
& -E\left[\sum_{k=1}^{n}(1-r)\left(\frac{\alpha_k}{\alpha_k+(1-r)\beta_k}\right)^2\frac{1}{\alpha_k} + (1-k_1)\left(\frac{(1-r)\beta_k}{\alpha_k+(1-r)\beta_k}\right)^2\frac{1}{\beta_k}\right] \\
& -(1-r)^2\alpha\beta\frac{(1-3k_1)\alpha+(1-r)\beta}{(\alpha+(1-r)\beta)^4}\frac{n-1}{n}E\left[\sum_{k=1}^{n}(G_k)^2\left(\frac{v_k}{\beta}\right)^2\right]
\end{aligned}
$$

where we have replaced μ'_j with $\frac{1}{n}$ and α_k, β_k with α, β in the last term. Noting that:

$$
\begin{aligned}
& E\left[\sum_{k=1}^{n}(1-r)\left(\frac{\sum_{j=1}^{n}\frac{\mu'_j}{\beta_j}}{\sum_{j=1}^{n}\mu_j'^2\left(\frac{1-r}{\alpha_j}+\frac{1}{\beta_j}\right)}\right)^2\frac{\mu_k'^2}{\alpha_k} + (1-k_1)\left(1-\frac{\sum_{j=1}^{n}\frac{\mu'_j\mu'_k}{\beta_j}}{\sum_{j=1}^{n}\mu_j'^2\left(\frac{1-r}{\alpha_j}+\frac{1}{\beta_j}\right)}\right)^2\frac{1}{\beta_k}\right] \\
& -E\left[\sum_{k=1}^{n}(1-r)\left(\frac{\alpha_k}{\alpha_k+(1-r)\beta_k}\right)^2\frac{1}{\alpha_k} + (1-k_1)\left(\frac{(1-r)\beta_k}{\alpha_k+(1-r)\beta_k}\right)^2\frac{1}{\beta_k}\right] \\
= \;& E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{CB\ only}
\end{aligned}
$$

Defining the private sector fog G such that $G^2 = E\left[\sum_{k=1}^{n}G_k^2\left(\frac{v_k}{\beta}\right)^2\right]$, we finally find:

$$
\begin{aligned}
& E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{both} = E\left[L^{RPPT}(\boldsymbol{\mu}') - L^{FT}\right]_{CB\ only} \\
& -(1-r)^2\alpha\beta\frac{(1-3k_1)\alpha+(1-r)\beta}{(\alpha+(1-r)\beta)^4}G^2\frac{n-1}{n}
\end{aligned} \tag{A6}
$$

as asserted in the text. Adding private sector imperfect knowledge of its own precision introduces a new source of uncertainty captured by the terms v_k''.

The additional effect created by the assumption that the private sector believes that its own signal precision is $\boldsymbol{\beta}''$ is captured by the second term. If this term is positive, resp. negative, interest rate and precision partial transparency becomes more, resp. less, desirable than when the private sector knows its own signal precision. This establishes (17).

Proof of the result in Section 5.2

We show that the result $E\left[L^{RPT}(\boldsymbol{\mu}', \widetilde{\boldsymbol{\mu}})\right] > E\left[L^{RPPT}(\boldsymbol{\mu}')\right]$ obtained when private sector precision is unknown to the central bank only also holds when it is also unknown to the private sector. In brief, the additional source of uncertainty affects $E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right]$ and $E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)\right]$ in nearly the same way so that the sign of the difference between these two terms is unaffected.

Formally, following the same approach as for (A4), we have:

$$\begin{aligned} E\left[L^{RPT}(\boldsymbol{\mu}', \widetilde{\boldsymbol{\mu}}) - L^{RPPT}(\boldsymbol{\mu}')\right]_{both} \quad = \quad & \left[E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right] - E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)\right]\right]_{both} \\ & + cE\left[\left(\sum_{j=1}^{n} \frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R - \sum_{k=1}^{n} \tilde{\mu}_k \theta_k\right)\right]_{both}^2 \\ & - cE\left[\left(\sum_{j=1}^{n} \frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R} - \sum_{k=1}^{n} \tilde{\mu}_k \theta_k\right)\right]_{both}^2 \end{aligned}$$

Next, we note that (A6) holds at the second order of approximation when replacing $\boldsymbol{\mu}'$ with $\widetilde{\boldsymbol{\mu}}$:

$$\begin{aligned} E\left[L^{RPPT}\left(\widetilde{\boldsymbol{\mu}}\right) - L^{FT}\right]_{both} \quad = \quad & E\left[L^{RPPT}\left(\widetilde{\boldsymbol{\mu}}\right) - L^{FT}\right]_{CB\ only} \\ & - (1-r)^2 \alpha\beta \frac{(1-3k_1)\alpha + (1-r)\beta}{(\alpha + (1-r)\beta)^4} \frac{n-1}{n} E\left[\sum_{k=1}^{n} G_k^2 \left(\frac{v_k}{\beta}\right)^2\right] \end{aligned}$$

Subtracting these two equations, we find:

$$E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right] - E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)\right]_{both} = E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right] - E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)\right]_{CB\ only}$$

and:

$$\begin{aligned} E\left[L^{RPT}(\boldsymbol{\mu}', \widetilde{\boldsymbol{\mu}}) - L^{RPPT}(\boldsymbol{\mu}')\right]_{both} \quad = \quad & E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right] - E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)\right]_{CB\ only} \\ & + cE\left[\left(\sum_{j=1}^{n} \frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R - \sum_{k=1}^{n} \tilde{\mu}_k \theta_k\right)\right]_{both}^2 \\ & - cE\left[\left(\sum_{j=1}^{n} \frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R} - \sum_{k=1}^{n} \tilde{\mu}_k \theta_k\right)\right]_{both}^2 \end{aligned}$$

Using (A5), we evaluate the terms in this expression:

$$E\left[L^{RPPT}(\widetilde{\boldsymbol{\mu}})\right] - E\left[L^{RPPT}\left(\boldsymbol{\mu}'\right)\right]_{CB\ only} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sum_{k=1}^{n} \frac{\partial^2 L^{PT}\left(\boldsymbol{\mu}\right)}{\partial\mu_i\partial\mu_j} N_{ik}^{\boldsymbol{\beta}} N_{jk}^{\boldsymbol{\beta}} \tilde{F}_k^2 v_k^2$$

$$cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(R - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]_{both}^2 - cE\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)\left(\tilde{R} - \sum_{k=1}^{n}\tilde{\mu}_k\theta_k\right)\right]_{both}^2$$

$$= c\sum_{k=1}^{n} E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\mu_k'^2 - \tilde{\mu}_k^2\right)\right]_{both}\frac{1}{\alpha} + c\sum_{k=1}^{n} E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\tilde{\mu}_k - \mu_k'\right)^2\right]_{both} E\left[\theta_k^2\right]$$

so that :

$$\begin{aligned} E\left[L^{RPT}(\boldsymbol{\mu}', \widetilde{\boldsymbol{\mu}}) - L^{RPPT}(\boldsymbol{\mu}')\right]_{both} \quad = \quad & \sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sum_{k=1}^{n} \frac{\partial^2 L^{PT}\left(\boldsymbol{\mu}\right)}{\partial\mu_i\partial\mu_j} N_{ik}^{\boldsymbol{\beta}} N_{jk}^{\boldsymbol{\beta}} \tilde{F}_k^2 v_k^2 \\ & + \frac{c}{\alpha}\sum_{k=1}^{n} E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\mu_k'^2 - \tilde{\mu}_k^2\right)\right]_{both} \\ & + c\sum_{k=1}^{n} E\left[\theta_k^2\right] E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\tilde{\mu}_k - \mu_k'\right)^2\right]_{both} \end{aligned}$$

As before, the bias term $c\sum_{k=1}^{n} E\left[\theta_k^2\right] E\left[\left(\sum_{j=1}^{n}\frac{\tilde{\varphi}_j}{\tilde{\mu}_j}\right)^2\left(\tilde{\mu}_k - \mu_k'\right)^2\right]$ is infinitely larger because θ is uniformly distributed. As a consequence $E\left[L^{RPT}(\boldsymbol{\mu}', \widetilde{\boldsymbol{\mu}}) - L^{RPPT}(\boldsymbol{\mu}')\right]_{both} > 0$, which proves our assertion. This conclusion has been established irrespective of whether condition (11) is satisfied or not.