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## A Time-Varying Parameter Model of A Monetary Policy Rule for Switzerland. The Case of the Lucas and Friedman Hypothesis.

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### Abstract

This paper is an empirical research of a monetary policy rule for a small open economy model, taking Switzerland as a case-study. A time-varying parameter model of a monetary policy reaction function is proposed to integrate various trade-offs to be made about various macroeconomic variables -- inflation, the output gap and the real exchange rate gap. The Kalman filter estimations of the time-varying parameters shows how rational economic agents combine past and new information to make new expectations about the state variables. The uncertainty created by the time-varying parameter model, and estimated by the conditional forecast error and conditional variance, is decomposed into two components, the uncertainty related to the time-varying parameters and the uncertainty related to the purely monetary shock. Most of the monetary shock uncertainty comes from the time-varying parameters and not from the pure monetary shock.

The Lucas and Friedman hypotheses about the impact of uncertainty on output are revisited, using a conditional variance to test them. Both hypothesis are confirmed, using the one-step ahead conditional variance of the monetary shock. An inverse relation between the magnitude of the response on output to the nominal shock and the variance of this shock is found, as Lucas

had predicted. Moreover, there is a direct negative impact of uncertainty which reduces output in the long-term.

JEL Classification: C32, E52

Keywords: time-varying parameter model, Taylor rule, Kalman Filter

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# A Time-Varying Parameter Model of A Monetary Policy Rule for Switzerland. The Case of the Lucas and Friedman Hypothesis.\*

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## Abstract

This paper is an empirical research of a monetary policy rule for a small open economy model, taking Switzerland as a case-study. A time-varying parameter model of a monetary policy reaction function is proposed to integrate various trade-offs to be made about various macroeconomic variables – inflation, the output gap and the real exchange rate gap. The Kalman filter estimations of the time-varying parameters shows how rational economic agents combine past and new information to make new expectations about the state variables. The uncertainty created by the time-varying parameter model, and esti-

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# 1 Introduction

Central Bank's reaction functions are very fragile objects due to changing policies and uncertainty about the future. Over time, the importance attached to conflicting objectives – inflation, the output gap, the rate of unemployment, the exchange rate – may change and policy makers' views on the structure of the economy may change. That creates uncertainty for the private sector about the central bank's monetary policy rule. This uncertainty is exacerbated by the central bank's changing policies that is represented by changing weights on the deviations of key variables from their targets<sup>1</sup>.

This paper is an empirical estimation of a simple theoretical monetary policy rule with time-varying coefficients for the Swiss Central Bank (SNB). This paper does not assert that the SNB formally uses such a rule to conduct its monetary policy. Instead, what this paper claims, is that, an augmented Taylor rule that adjusts the short-term interest rate in response to an output gap, deviation of expected inflation from a time-varying target correctly reflects the SNB's monetary objectives. In addition, the augmented rule includes a real exchange rate gap. This rule should provides us some insights about the SNB's sensitivities towards stabilising key variables such as inflation and the output gap.

The first goal in this paper is to estimate the changing coefficients of a monetary policy rules, using a Kalman filter estimation of the time-varying coefficients. The SNB strategies and instruments change over the sample period, rules change, judgment or discretion is used at time,

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<sup>1</sup>Harrison and Stevens (1976) : "... a person's uncertainty about the future arises not simply because of future random terms but also of uncertainty about current parameter values of the model's ability to link the present to the future".

overruling the prescribed rules. This is reflected by regressor coefficients that would also vary accordingly. A Kalman filter algorithm will be used to estimate the time-varying coefficients.

As a by-product of the Kalman filter approach, recursive forecast errors of the policy instrument, and their conditional variance are obtained.

The second goal of the paper is a variance analysis to decompose uncertainty that is caused by time-varying parameters and other kinds of heteroskedasticity, such as a Switching Markov regime heteroskedasticity.

The third goal of the paper is to assess the impact of uncertainty on the economy, by testing two hypothesis: the Lucas (1973) hypothesis and the Friedman (1977) hypothesis. The Lucas hypothesis predicts that unanticipated demand shifts do have positive output effects. Lucas (1973) tested the hypothesis, using the unconditional constant variance of nominal shocks. But what really matters for the behavior of economic agents, is the conditional variance, not the unconditional variance. On the other hand, Friedman(1977) tests if increased volatility of inflation may raise the rate of unemployment and consequently lower GDP. To test the Friedman hypothesis, I use the conditional variance of monetary forecast errors as a proxy for the variability of inflation rate. I test whether the time-varying conditional variance of the monetary shock has a negative impact on the cyclical and potential output. Friedman (1977) predicted that increased variability of inflation rate causes a reduction in the allocative efficiency of the price system, causing a reduction in the national level of output. Both Lucas and Friedman hypothesis were confirmed.

My main findings are as follows: (i) a time-varying parameter model of a monetary policy reaction function is proposed to integrate changing policies and strategies, reactions and decisions over various trade-offs to be made about various macroeconomic variables – inflation, the output gap and the real exchange rate gap. The Kalman filter estimations of the time-varying parameters shows how rational economic agents combine past and new information to make



new expectations about the state variables; (ii) the uncertainty created by the time-varying parameter model, evaluated by the conditional forecast error and conditional variance, is decomposed into two components, the uncertainty related to the time-varying parameters and the uncertainty related to the purely monetary shock. Different assumptions are made about the monetary shock: homoskedastic errors are assumed and Markov switching regime heteroskedasticity are evaluated in turn. The model does not show any GARCH heteroskedasticity as such. Eighty percent of the uncertainty in a Markov switching regime heteroskedastic monetary shock is estimated to come from expected regime switching and not from time-varying parameters but this result is not significant; (iii) the Lucas and Friedman hypotheses about the impact of uncertainty on output are revisited, using a time-varying conditional variance to test them. Both hypothesis are confirmed, using the one-step ahead conditional variance of the monetary shock. An inverse relation between the magnitude of the response on output to the nominal shock and the variance of this shock is found, as Lucas had predicted. Moreover, there is a direct negative impact of uncertainty which reduces both output in the long-term and the long-term growth of potential output.

The rest of the paper runs as follows. Section 2 gives a brief outline of the post-Bretton Woods Swiss monetary policy rules and strategies. Section 3 presents the Kalman filter state-space for the monetary policy rule model with time-varying parameters. Section 4 presents and interprets the results of time-varying regressor coefficients. Section 5 combines time-varying coefficients with Markov switching regime heteroskedasticity. Moreover, as uncertainty for the public is increased by changing regressor weights, the impact of time-varying uncertainty on the economy would be assessed in section 6. Section 7 concludes.

## 2 A Brief History of the post-Bretton Woods Swiss Monetary Rules and Strategies

In his 1997 JME article, Rich (1997) provides a clear and precise history of the Swiss monetary policy targets from which I briefly extract the main points relevant to this paper. Prior to the end of 1999, the Swiss National Bank (SNB) never announced a formal inflation target. However, it is widely recognised that price stability has been a primary objective of the Swiss monetary policy.

Rich (1997) outlines three important policy principles of the SNB, of which the 'main objective' was price stability. Inflation is largely a monetary phenomenon. Therefore, a strict control of the money supply was key to fighting inflation. The way to achieve it was through the control of the growth of the money stock, a monetarist approach of the Swiss monetary policy. The three principles were: (i) the SNB regards price stability as the main objective of monetary policy, (ii) if the SNB is to achieve and maintain price stability, it must keep a tight control on the growth in the money stock and (iii) the SNB believes that it should "**precommit itself, as far as possible**", to a policy rule.

On balance, as Rich (1997) notes, the SNB's experience with monetary targets has been positive. Inflation was lowered to acceptable levels from the peaks of over 10 percent in the mid-1970's to less than 1 percent in 2000, with an average of 3.3 percent over the period 1975-1994. The SNB's experience with monetary targets has been positive. A shift to a temporary target for the CHF/DM exchange rate in 1978/79, held substantially above .80 CHF/DM to stave off the continued appreciation of the Swiss franc, which risked endangering the Swiss competitiveness and growth. There was a return to annual monetary targets at the end of 1979 and a shift to a medium term strategy at the end of 1990. As annual targets were no longer credible, the SNB adopted a five-year growth target for the monetary base.

Broadly speaking, one can observe four main regimes over the sample period:

- 1973-1978: annual monetary growth targets
- 1979: exchange rate targeting
- 1980s: annual monetary growth targeting combined with increasing concern over the currency overappreciation
- 1993-1999: interest rate strategy
- end of 1999 onwards: medium-term inflation targets

In the first half of the 1970s, the SNB modified fundamentally its approach to monetary policy. After the collapse of the Bretton Woods system, the Swiss authorities decided to float the currency in January 1973. The shift to a floating exchange rate enabled the SNB to pursue an autonomous monetary policy and to decouple the Swiss economy from world inflation. At the end of 1974, the SNB publicly announced specific targets for the yearly average growth targets in the money stock M1, until 1978. In 1979, the SNB did not announce any monetary target to temporarily target the nominal exchange rate to prevent an over-appreciation of the currency that would have reduced the Swiss competitiveness for its goods<sup>2</sup>. At the end of 1979, the SNB returned to the monetary growth targets, but altered its target variable by substituting the monetary base to M1. It considered monetary base to be more stable than aggregate M1. However, the adoption of the monetary base target did not entail a fundamental change in the SNB's policy approach. As in the case of M1, the SNB continued to target a yearly average monetary growth target of 2%, compatible with price stability in the long run.

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<sup>2</sup>Rich (1997) writes: "The upvaluation of the Swiss franc started to concern seriously the Swiss public. It undermined the competitive position of domestic industry and raised the specter of a slump in domestic economic activity".

The fight against inflation from the end of the 1980s to early 1990s was achieved at the cost of the "longest recession in the post-war period and a substantial rise in unemployment, reaching a level of over 5% at the beginning of 1994". Three consecutive monetary target misses from 1988 to 1990 prompted the SNB "to reconsider the wisdom of setting annual monetary targets". At the end of 1990, the annual growth target for the monetary base was replaced by a five-year medium-term objective.

Even though the medium-term strategy was more sensible than the annual targets, it did not stop occasional peaks in inflation, especially after the German reunification in 1989-1990. This led the SNB in the mid-1990's to reappraise its monetary strategy and base its policy decisions on inflation forecasts rather than monetary targets. And at the end of 1999, the SNB opted for an approach akin to inflation targeting, although it did not treat its new approach as inflation targeting: (i) it set a quantitative objective for price stability: the rate of increase in CPI of less than 2 percent. (ii) each quarter, the SNB established and published a forecast of inflation for the following three years, a period long enough to cover the lags in the effects of monetary policy. (iii) if the inflation forecast deviated from its objective, the SNB would consider changing its main policy instrument. (iv) its main policy instrument is the 3-month Libor rate of interest.

The changing monetary strategies could be translated by Markov switching regime models. But it can also be reflected by time-varying regressor coefficients, reflecting changing emphasis on targets and control variables. The two types of models will be examined and compared in this paper.

### 3 An Augmented Time-Varying Taylor Rule for A Small Open Economy

Are rules used by central bankers or do they use discretion ? I would not like to enter in the "rules versus discretion" debate which has been first introduced at the end of the seventies by Kydland and Prescott (1977) and Barro and Gordon (1983) and extensively developed thereafter. In this paper, I assume that the augmented Taylor rule with time-varying regressor coefficients used in this paper, given the information set, should reflect how the SNB' sensitivities towards key endogenous variables such as expected inflation, an output gap and an exchange rate variable, would have evolved in the way described thereafter. One thing that seems to be true for the SNB, as for many central banks, is that (a) central banks use rules, much more complex than the simple rule presented in this paper, but (b) they do not systematically or mechanically stick to any of them. They use rules as an indicator to evaluate in which direction is the economy heading, and complement this information with judgment and, at times, discretionary manoeuvring. Although central banks vary widely in degrees of transparency, what is quite explicit is the consensus over stabilising in the medium term, prices or its growth rate, inflation, and output growth around certain targets or equilibrium levels.

There is also a growing consensus that monetary behaviour, be it through a rule or even in a discretionary manner, could be appropriately represented by some fairly simple rules. The use of a Taylor-type of rule can, in a very simple manner, highlight those concerns by estimating and looking at the weights that are attached to each term. As all variables, except for the interest rate, are logarithmic transformations, these weights would reflect the various elasticities attached to each term.

Taylor (1993) suggested a very simple rule for monetary policy:

$$i_t = \pi_t + r^* + 0.5(\pi_t - \pi^*) + 0.5\tilde{y}_t \tag{1}$$

where  $\pi^*$  is the constant target inflation rate and  $r^*$  is the constant equilibrium real funds rate, a "natural" or 'NAIRU' rate, consistent with full employment. The Taylor rule suggests that the central bank reacts to the deviation of inflation from its target and output from its potential. It is fairly easy to estimate a Taylor-type of rule, as in equation (1) by simply adding a residual term to capture deviations from the rule and estimate the weights as coefficients. However, the main difficulty in estimating coefficients of the Taylor rule is that the constant (and even worse) time-varying target inflation rate, the natural rate of interest and the potential output are not known as they are neither observed nor are they specified by the monetary policy, nor is the term period clearly announced, rendering such rules inappropriate at best or simply inoperative at worse.

Several other problems can be noted with the original Taylor (1993) rule. One problem is that central banks seem to adjust interest rates in a **gradual manner**, by **smoothing** interest rates, by taking discrete jumps, 25 or 50 basis points, step by step. Another problem is that a Taylor type of monetary policy rule as the above equation (1) is neither optimal nor appealing for a small open economy highly sensitive to external shocks as is the Swiss economy. In general, small economies in a floating regime are balancing internal with external considerations in implementing their monetary policy. Thus, an appropriate specification of an exchange rate term could address concerns of the private sector about loss of competitiveness due to overappreciating nominal and real exchange rate. As explained in section 2, the SNB had, at times, overruled its monetary growth targets to stem an overappreciation of the currency. All these issues have been the object of a previous paper, Elkhoury(2005), which estimate an augmented Taylor rule with fixed coefficients and analyse the role for an exchange rate term, if any.

Including time-varying coefficients in the Taylor rule would address the issue of changing reactions of the SNB towards various macroeconomic variables. However, structural breaks or

changing policies would not be captured by fixed regressor coefficients. To take into account abrupt occasional but recurring changes in variables first or second moments, one needs to include into the rule switching regimes. Section 3 looks at a time-varying monetary policy rule. Section 5 combines time-varying coefficients with Markov switching regime heteroskedasticity. Moreover, as uncertainty for the public is increased by changing regressor weights, the impact of time-varying uncertainty on the economy would be assessed in section 6.

### **3.1 The Structural Time-Varying Parameter Model (TVP)**

The first goal of this paper is to estimate time-varying regression coefficients to reflect central bankers changing policies over time. As central bankers change their preferences for certain goals due to changing of personalities at the helm of the central bank, changing emphasis, improved monetary theories, improved financial technologies, the question arises for the econometrician, policy analysts and central banker watchers to look for as much flexibility in the model as it is possible, based on tractability and simplicity, in-as-much as possible, supposing the model under study was the 'true' one, in the sense that it would 'plausibly' track stylised facts. Asymmetries can also arise from within the rule. In the vein of Krugman's(1991) exchange rate target zone model, one can suppose that non-linearities and asymmetries do arise in a rule, whereby as one comes closer to a limiting zone or bound, of what is an acceptable, desirable or appropriate upper bound or lower bound, central bankers would react differently than if they were in the middle of the zone or close to an equilibrium value. Moreover, central bankers' reactions at the lower bound would be different than on the upper limit of the bound, i.e. rules do not have to be symmetric near to the bounds. Ranges of a bound could also change. Central bankers rarely or never make these issues clear to the public. The structural TVP model would then provide enough flexibility to the regression model to integrate these various unknowns and uncertainties in a state-space form that can be estimated by the Kalman filter algorithm. Therefore, given

an information set given by past data, one could extract the best possible signal in the sense of a minimising the mean squared error of the variance-covariance matrix of the state variables, in this case, the time-varying coefficients.

I assume in this paper that the preferred instrument for the SNB is the 3-month Swiss Libor rate of interest. The recommended instrument rate,  $i_t^*$ , is equal to the following rule:

$$i_t^* = \pi_t^e + r_t^* + (\beta - 1)(\pi_t^e - \pi^*) + \gamma_1 \tilde{y}_{t-1} + \gamma_2 \tilde{y}_{t-2} + \gamma_3 \tilde{e}_{t-1} + \gamma_4 dmb_t^e \quad (2)$$

In fact, the central bank prefers to smooth the rate over time with the following smoothing dynamics for the actual instrument:

$$i_t = (1 - \rho) i_t^* + \rho i_{t-1} + \delta \Delta i_{t-1} + \varepsilon_t \quad (3)$$

where  $i_t^*$ , the recommended rate, and  $i_t$ , the actual 3-month Libor rate,  $\tilde{y}_{t-1}, \tilde{y}_{t-2}$ , is the 1st and 2nd lagged output gaps,  $\tilde{e}_t$ , the real exchange rate gap,  $dmb_t^e$  is the expected deviation of money base growth from a target growth rate;  $\pi_t^e$ . The target inflation,  $\pi^*$ , is assumed constant and equal to 2 percent;  $r_t^*$  is the time-varying natural rate of interest. There are many reasons to believe that the natural rate of interest is time-varying. Looking at some of the determinants of the natural rate given by the theory, the natural rate of interest varies over time in response to changes in technological progress, the trend growth rate and shifts in preferences. All the equilibrium variables:  $r_t^*, y_t^*, e_t^*$ , the natural rate of interest, the potential output, the equilibrium real exchange rate are smoothed by a Hodrick-Prescott filter with smoothing coefficient,  $\lambda = 1600$ . There is no reason to believe that the smoothing coefficient should be  $\lambda = 1600$ , except that this is the standard measure for the quarterly US Real GDP. It would be of major interest to properly estimate the smoothing coefficient on a case-by-case study for each unobserved variable. Meanwhile, I am using the standard smoothing



measure of  $\lambda = 1600$ .  $\rho$  is the smoothing coefficient in the 'smoothed' Taylor rule equation. A first difference of the lagged instrument is added to the smoothing equation to stress that it is not just the level of the instrument that is smoothed but also the changes in the instrument.

In this paper, I integrate all the Hodrick-Prescott filter estimations of equilibrium target variables into my augmented Taylor rule equation, assuming time-varying regressor coefficients. The equation (2) is replaced with the following two equations, (4) and (5), respectively.

$$i_t^* = \pi_t^e + r_t^* + (\beta_t - 1)(\pi_t^e - \pi^*) + \gamma_{1t}\tilde{y}_{t-1} + \gamma_{2t}\tilde{y}_{t-2} + \gamma_{3t}\tilde{e}_{t-1} + \gamma_{4t}dmb_t^e \quad (4)$$

Equation (4) includes an additional lagged gap term along with the first lagged output gap, to allow for the possibility for the central bank to react not just to current variables but also past ones, considering the **long and variable lags** in which the monetary policy affects the economy. As explained earlier, the real exchange rate gap term,  $\tilde{e}_t$ , is added to take into account foreign demand for the small open economy model.  $\pi_t^e$  is the expected inflation rate.

The dynamics of adjustment of the actual level of the funds rate to  $i_t^*$  are given by:

$$i_t = (1 - \rho_t)i_t^* + \rho_t i_{t-1} + \delta_t \Delta i_{t-1} + \varepsilon_t \quad (5)$$

The instrument takes a convex combination of the recommended rate,  $i_t^*$ , and the one period lagged nominal interest rate. At time  $t$ , the monetary authorities set the level of the instrument to a weighted average of the current recommended level (the first term) as well as partially correcting the error between last period's setting and the current recommended level (the second term).

The Kalman filter is applied to the state-space form to make inferences on the changing regression coefficients. What is new here is that the weights attached to each regressor are also assumed to be time-varying. As the monetary policy rule changes over time, the weights attached to each variable will also change accordingly. Assuming time-varying coefficients allows

one to capture time-varying monetary policies that would reflect changing relative emphasis on the different policy trade-offs that are at stake; trade-offs between inflation and output and deviation from a target exchange rate. This could be interpreted as insight on how rational economic agents revise their estimates of the coefficients in a Bayesian fashion when new information is available in a world of uncertainty and under changing policy regimes.

Substituting equation (4) into (5), we obtain the following TVP model to be estimated:

$$\begin{aligned}
i_t = & (1 - \rho_t) r_t^* - (1 - \rho_t) (\beta_t - 1) \pi^* + (1 - \rho_t) \beta_t \pi_t^e + \\
& + (1 - \rho_t) \gamma_{1t} \tilde{y}_{t-1} + (1 - \rho_t) \gamma_{2t} \tilde{y}_{t-2} + (1 - \rho_t) \gamma_{3t} \tilde{e}_{t-1} + \rho_t i_{t-1} + \delta_t \Delta i_{t-1} + \\
& + (1 - \rho_t) \gamma_4 dm b_t^e + \varepsilon_t
\end{aligned} \tag{6}$$

Equation (6) is the key equation to be estimated in this model. In this equation, all the regression coefficients are time-varying and are to be estimated by a Kalman filter approach estimation.

### 3.2 Dynamics of the Regression Coefficients

The random walk structural time-varying parameter model is a special case of a Kalman filter model approach. The model is a time-series as all regressors vary through time. In addition, this model is also structural, due to the structural form relating dependent and independent variables. The key to handling structural time series models is the state-space form (SSF). Once in SSF, the Kalman filter provides the means of updating the state as new observations become available. Various filtering procedures exist, using only past information or the whole sample length. Updating and smoothing can only be carried out, once the hyperparameters governing the stochastic movements of the state variables have been estimated. These hyperparameters are themselves estimated by the Kalman filter by maximising a likelihood function.

In this paper, I use a filtering and smoothing algorithm originally developed by Kalman in the 1960s for engineers and then adapted for economists chiefly by Anderson and Moore(1979), Harvey (1992), Hamilton (1994) and Kim and Nelson (1999).

Expressed in generic terms, the TVP state-space model analysed in this paper is the following, with **the measurement equation** as follows:

$$i_t = H_t \beta_t + \varepsilon_t \quad (7)$$

where  $i_t$  is the 3-month Libor interest rate,  $H_t$  is the  $1 \times 9$  time-varying matrix of regressors,  $\beta_t$  is the  $9 \times 1$  time-varying vector of coefficients and  $\varepsilon_t$  is the financial innovation,  $\varepsilon_t \sim NIID(0, \sigma_\varepsilon^2)$ .

and **the transition equation** is:

$$\beta_{jt} = \beta_{jt-1} + \nu_{jt}, j = 0, 1, \dots, 6 \quad (8)$$

where  $\beta_{jt}$  is the  $j$ th time-varying coefficient and  $\nu_{jt}$  is the  $j$ th innovation,  $\nu_{jt} \sim NIID(0, \sigma_{\nu_j}^2)$ .

Or, in a generic form,

$$\beta_t = F \beta_{t-1} + \nu_t \quad (9)$$

where  $F$  is the  $9 \times 9$  identity matrix  $I_9$  and  $\nu_t \sim NIID(0, Q)$ .

I assume a random walk process without drift for the time-varying coefficients, as it is most commonly assumed in the literature, c.f Kim and Nelson (1999). One could as easily assume some AR(p), ARMA(p,q) process or even some non-parametric distribution process instead. That would complicate the model in many ways so as to have to keep track of several lags of state variables and to assume non-Gaussian distributions, and their impact on the overall heteroskedasticity of the variance-covariance matrix. The difference between the two models would be in the temporariness or long-lasting impact of shocks on the dynamic path of the state variables which would indeed have different implications to the overall model. There is no a priory reason to choose one model over the other. 'Practicality' of the state-space form

makes the random walk approach more 'appealing' as it requires less hyperparameters and state variables to assume, thus less errors in the model estimation. Furthermore, in the Random Walk model, because the regressor coefficients are non-stationary, the model can accommodate fundamental changes in the structure that are long-lasting. But other processes could be assumed. That could be the subject of future empirical research.

Putting the model in state-space form, the multivariate system that we estimate, is the following:

(i) Measurement Equation:

$$\begin{aligned}
i_t = & (1 - \rho_t) r_t^* - (1 - \rho_t) (\beta_t - 1) \pi^* + (1 - \rho_t) \beta_t \pi_t^e + \\
& + (1 - \rho_t) \gamma_{1t} \tilde{y}_{t-1} + (1 - \rho_t) \gamma_{2t} \tilde{y}_{t-2} + (1 - \rho_t) \gamma_{3t} \tilde{e}_{t-1} + \rho_t i_{t-1} + \delta_t \Delta i_{t-1} + \\
& + (1 - \rho_t) \gamma_{4t} dmb_t^e + \varepsilon_t
\end{aligned} \tag{10}$$

where the output gap,  $\tilde{y}_t$ , and the real exchange rate gap,  $\tilde{e}_t$ , are the gaps relative to the Hodrick-Prescott filtered potential output and the equilibrium real exchange rate respectively.

Equations (9) – (10) constitute the state space system. Estimated confidence intervals and corresponding standard errors for the estimates of the states, use Hamilton's(1986, 13.7) Monte Carlo procedure, which accounts for both filter and parameter uncertainty.

## 4 Estimation Results and Interpretation

### 4.1 The Data

The macro variables that matter for the augmented Taylor rule are the output gap and the lagged output gap, the real exchange rate gap, the short-term expected inflation, the lagged

nominal interest rate and the natural rate of interest. The data is quarterly, ranging from 1972:1 to 2003:3.

The monetary policy instrument is the 3-month Libor rate. For the Taylor rule, I construct a nominal interest rate to reflect, in as much as possible, the 3-month Libor money market rate. From 1962/01/01 - 1973/12/01, the rate is the Euromarket 3 month big banks lending rate; from 1974/01/01-1989/01/01, the rate is the 3 month Euromarket rate; and from 1989/02/01 - 2005/05/01, the rate is the SNB's 3 month Libor rate lagged one month. I constructed this interest rate due to two reasons. First, the Libor rate provided by the IFS database was not correct in the early 1980s, a remark kindly pointed out to me by Georg Rich in one of my discussions with him. Moreover, the SNB does not provide the Libor rates prior to 1989.

Expected inflation,  $\pi_t^e$ , is the CPI expected inflation, is the expectation of average CPI inflation over the four quarters ahead. The expectations are based on an out-of-sample forecast, using an AR(3) process with a 40-quarter rolling regression window. The target inflation,  $\pi^*$ , is assumed constant and equal to 2 percent, based on Rich(1999).

The output gap and the lagged output gap are the deviations of the actual log real GDP from potential output. Potential output is estimated by a Hodrick-Prescott filter estimation based on a signal extraction problem using a Hodrick-Prescott smoother  $\lambda = 1600$ . The time-varying natural rate of interest and the implicit inflation target are Hodrick-Prescott filter estimates with a smoothing coefficient of  $\lambda = 1600$ .

The base money growth target is constructed based on Rich(1999), table 1, p. 116.

## 4.2 Estimation Results

Rich(1997) explains that in the late 1980s, the SNB entered a turbulent period as it was confronted with several unexpected disturbances. First, the real exchange rate showed an appreciating trend which occurred right after the 1987 stock market crash and a perceived slowdown

in real growth. In the first half of 1987, the SNB allowed the monetary base to **grow slightly above the target rate** of 2 percent. Another substantial expansion of the monetary base in the aftermath of the stock market crash magnified the deviation from target. In 1988, the conduct of monetary policy was further complicated by the introduction of the SIC, a new electronic interbank payment system which would alter the demand for base money. Up till the end of 1988, the monetary policy was expansionary. From the end of 1988, the SNB **gradually tightened its policy** in view of a more vigorous expansion in economic activity, a weakening of the Swiss franc and raising concerns about rising inflation. As inflation remained high until the summer of 1992, the SNB continued to pursue a tight monetary policy, **tighter than it had anticipated**, causing the beginning of a recession that was going to be the longest of the post-war era coupled with a substantial rise in unemployment.

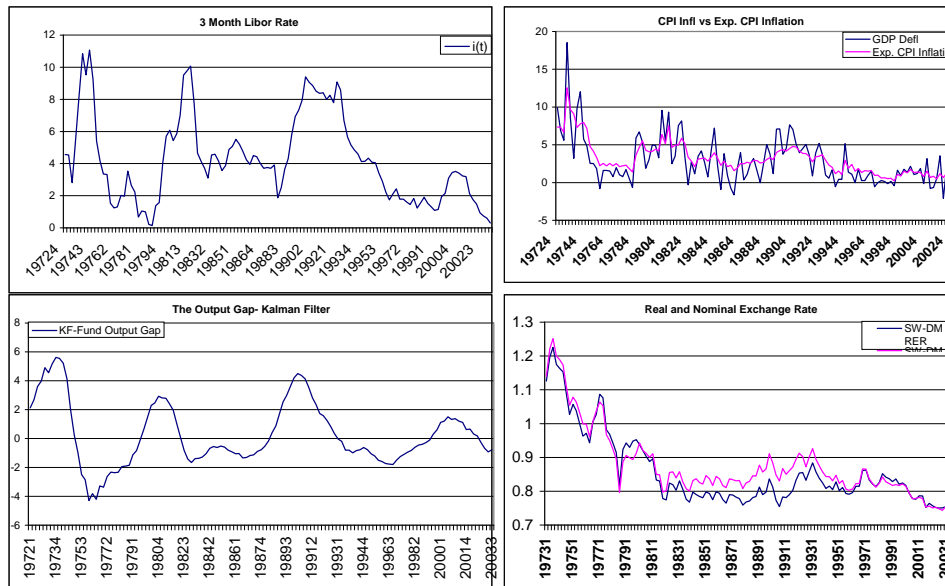


Figure 1: Macroeconomic Variables

Looking at the Figure(1), we see that the output gap is positive around 1974, between 1980

to 1982 and between 1988 to nearly 1993, and from 1999 to 2002 and is negative between 1975 to 1980, 1982 to 1988 and end of 1993 to 1998, and turns negative again after 2002. Short-term expected inflation is above 2 percent and high in and around 1974, between 1978 to 1986 and 1986 to end of 1993, early 1994. It then falls to below 2 percent.

The CHF/DM real exchange rate shows a continuous appreciating trend from 1974 to the end of 1981, then it stabilises around the 0.8 'target' level, which could indicate that the SNB intervened or threatened to intervene when the nominal CHF/DM reached that level; and if we assume that inflation in both Switzerland and Germany are very similar, than that is also true for the real CHF/DM real exchange rate.

Given these very stylised facts, how did the monetary authorities react to the various macro-economic variables and trade-offs, according to my model of the TVP monetary policy rule ?

Table(1) reports the parameter estimates of the TVP model for the period 1975:1-2003:3. Figure(2) presents the filtered and smoothed dynamics for the regressor coefficients. These are estimates on how the SNB reacted on a period by period basis to the output gaps, the real exchange rate gap, expected inflation and the lagged nominal interest rate. Smoothed estimates would be long-term estimates while filtered variables are the 1-step ahead forecasts. Deviations of the filtered time series from the smoothed one could be construed as deviations from equilibrium. A level for the 1-step-ahead coefficient variable above the equilibrium may signal a more aggressive reaction compared to the smoothed equilibrium while choosing a level below equilibrium is a less aggressive reaction relative to the equilibrium.

The smoothed regressor coefficients for the output gap varies between 0 and 2 with an average coefficient value of 0.84. At a time of high expected inflation, at the beginning of the sample period, the smooth coefficient increases rapidly from a near zero value to nearly 2 in 1978:1, from which it then decreases steadily to 0.5 in the 1980s and stabilises around 1 in

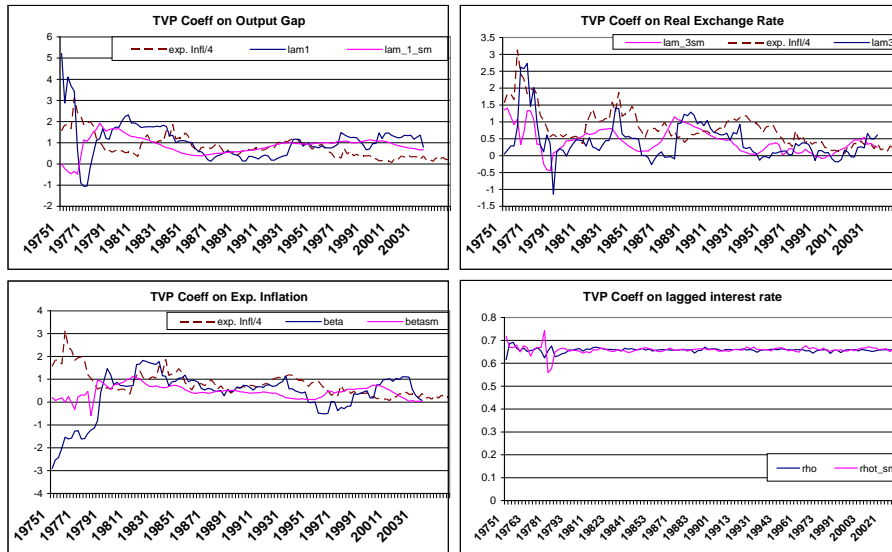


Figure 2: TVP Regressor Coefficient: Filtered vs Smoothed Estimates

the 1990s, decreasing slightly to 0.65 at the end of the sample period, in the early 2000s. The 1-step ahead filtered estimation remains above the equilibrium from the beginning of the sample period, although it falls below the smoothed coefficient value until 1979:2 and stays above the smooth coefficient up until 1986:3. From 1987 until 1993:2, the filtered coefficient on the output gap crosses again below the smoothed coefficient value, then onwards, it fluctuates around the smoothed coefficient values. The 1994-1999 long recession could be associated with a sharp reduction in the weight over the output gap, signalling increased concern over falling output and a reduction in the weight attached to the output gap.

For the coefficient attached to the real exchange rate gap, the filtered and smoothed coefficients are above 1 at the beginning of the sample and sharply drop to negative values in 1978:4. This corresponds to a period when the SNB was worried about the over-appreciation of the Swiss franc and intervened officially on the exchange rate market, setting a target of 0.8 CHF/DM in 1979. Ironically, this target has never been officially receded ever after. In 1979, there was real



concern over the appreciation of the currency, thus, the SNB reduces the impact of exchange rate gap on the monetary instrument. After 1979, it reverses again the trend, increasing its impact again, though, to a level that is below the 1977 peak levels. We see another increase for the smoothed and filtered series to values just above 1 in the second and fourth quarter of 1988, when the SNB is now fearing a too sharp depreciation of the Swiss real exchange rate. In the 1990s onwards, the weight on the exchange rate variable tends to decrease to fluctuate between 0 and 0.5. These changing weights on the real exchange rate could explain changing policy of the SNB towards the real exchange rate gap variable. The gradual decrease of both weights, smoothed and filtered, starting from 1990 onwards could signal a decreasing emphasis on the exchange rate compared to the other variables.

The smoothed regressor coefficient attached to the expected inflation is above 1 at the beginning of the sample period and decreases steadily from 1978 to 1998:1 to fluctuate between 0 and 0.5. It increases above 0.5 in the last period. The filtered coefficient is above the smooth coefficient level at the beginning of the sample period until 1983:4 and remains below the smoothed coefficient until 2000. The time-varying coefficient on expected inflation is, for most of the time, lower than 1, contrary to the Taylor principle. Taylor's principle calls for a coefficient on expected inflation above 1 to stabilise inflation. It is possible that the one-period ahead short-term forecast inflation does not accurately reflect the path to the medium-term and long-term inflation target. This could be better captured by the output gap which is a good predictor of long-term to medium term inflation. That would explain the fact that the rule puts a higher weight on the output gap and less on the short-term expected inflation. But, referring to the Benigno and Benigno (2000) determinacy condition, the higher the smoothing coefficient on the lagged instrument variable, the lower the coefficient on expected inflation, given that the coefficient on the exchange rate variable is positive.

The coefficient on the lagged nominal interest rate, both for the smoothed and the 1-step-

ahead Kalman estimate, remains fairly constant around a value of 0.65. This suggests a gradual adjustment of the funds rate to the rule. The interest rate rule puts more weight, each quarter, on the lagged instrument, represented by  $\rho_t$ , than on the natural rate, which is determined by  $(1 - \rho_t)$ .

## 5 Uncertainty and the Time-Varying Parameter Model

### 5.1 Modelling Uncertainty

After analysing the impact of changing regressor coefficients, this section analyses the volatility created by these changing coefficients and error heteroskedasticity. Tsay (1987) and Bera and Lee (1993) show that parameter heterogeneity in the random coefficient autoregressive models is a source of changing conditional variance such as Engel's ARCH models.

The Kalman filter is a recursive procedure for calculating the optimal estimator of the state vector, given all the information which is currently available. On the other hand, smoothing is a backward recursion, which enables optimal estimators of the state vector to be calculated at all possible points in time, using the full sample. As a side-product, the Kalman filter algorithm produces forecast errors and conditional variances for the state vector and the measurement equation. (For a brief outline of the Kalman filter estimation, see Elkhoury (2004) Appendix 1, and for a more elaborate exposition, see Hamilton (1994) or Kim and Nelson (1999)).

The Kalman filter is obtained in two steps:

- **Step 1: the Prediction equations:** At the beginning of time  $t$ , we want to form an optimal predictor of  $y_t$ , based on all available information up to time  $t - 1$ :  $y_{t|t-1}$ . This is done by estimating the expected state variable  $\beta_{t|t-1} = E(\beta_t | \psi_{t-1})$ .
- **Step 2: the Updating equations:** Once  $y_t$  is realised at the end of time  $t$ , the predictor

error,

$\eta_{t|t-1} = y_t - y_{t|t-1} = y_t - T_t\beta_{t|t-1}$  can be calculated, where  $T_t$  is the Transition matrix. This prediction error contains new information about  $\beta_t$  beyond that contained in  $\beta_{t|t-1}$ . Thus, after observing  $y_t$ , we can update our inference with a more accurate estimate that can be made of  $\beta_t$ ,  $\beta_{t|t}$ .

$\beta_{t|t} = \beta_{t|t-1} + K_t\eta_{t|t-1}$ , where  $K_t = P_{t|t-1}T_t'f_{t|t-1}^{-1}$ , the '**Kalman gain**', is the weight assigned to the new information about  $\beta_t$  contained in the prediction error,  $P_{t|t-1}$  is the variance-covariance matrix of the state  $\beta_t$ , conditional on information up to  $t-1$ ,  $T_t$  is the Transition matrix, and  $f_{t|t-1}$ , the conditional variance of the error forecast. The updated state vector  $\beta_{t|t}$  can be viewed as a 'weighted average' of the predicted state vector,  $\beta_{t|t-1}$ , and the new information contained in the prediction error,  $\eta_{t|t-1}$ .

The prediction error,  $\eta_{t|t-1}$ , plays a key role in updating the state vector  $\beta_{t|t}$ . The greater is  $\eta_{t|t-1}$ , the greater the 'correction' in the updated state vector. Moreover, the larger is the uncertainty associated with the predicted state vector  $\beta_{t|t-1}$ , the more weight is given to new information in the prediction error  $\eta_{t|t-1}$ . Kim and Nelson(1999) show that  $\frac{\partial K_t}{\partial(P_{t|t-1}x_t^2)} > 0$ , where  $(P_{t|t-1}x_t^2)$  is the portion of the prediction error variance due to uncertainty in the state vector  $\beta_{t|t-1}$ .

The conditional variance of the error forecast,  $f_{t|t-1} = E(\eta_{t|t-1}^2) = T_tP_{t|t-1}T_t' + R$ , consists of two parts: the uncertainty due to the error in making inference about the state  $\beta_t$ ,  $(\beta_t - \beta_{t|t-1})$ , and the prediction error due to the random shock to  $y_t$ . Thus the conditional variance of the prediction error,  $f_{t|t-1}$ , is a function of the uncertainty associated with  $\beta_{t|t-1}$ ,  $P_{t|t-1}$ , and with  $R$ , the variance of the measurement equation.

Thus, there are two types of uncertainty that one has to account for when analysing the impact of monetary policy shocks which may affect differently the economy: the uncertainty

related to changing policies, that is represented in the first term of the forecast error variance and the uncertainty related to the purely monetary shock, which is represented by the second part of the forecast error variance, which will be assumed in the next sections to be heteroskedastic.

## 5.2 Conditional Heteroskedasticity

There are several ways of capturing a changing conditional variance that depends on past observations. In this paper, I investigate three types of conditional heteroskedasticity. The first model being the TVP model already discussed in the previous section. Another way is the ARCH (Autoregressive Conditional Heteroskedasticity) model introduced by Engle's (1982) and further generalised by Bollerslev's GARCH model (Generalised Autoregressive Conditional Heteroskedasticity). A third option is Hamilton's Markov switching regime (thereafter denoted by MSR) distributions for the variance.

The three types of heteroskedasticity are fundamentally different. The TVP model does not assume any a priori distribution for the conditional variance. It simply estimates it as the conditional variance of the forecast error of the measurement equation in the Kalman filter algorithm. The unconditional variance, under ARCH is constant but subject to abrupt changes under MSR models. Moreover, and as suggested by Hamilton and Susmel (1994) and Kim and Nelson(1999), whereas long-run variance dynamics may be governed by regime shifts in unconditional variance, short-run variance dynamics within a regime may be governed by an ARCH-type process. Thus, MSR heteroskedasticity may be more appropriate for low frequency data over a long period of time, whereas the ARCH type heteroskedasticity more appropriate for high frequency data over a short period of time. Kim and Nelson remark, that, for a given time series, it may be difficult to distinguish statistically between the two types of heteroskedasticity because the two models are non-nested.

In the TVP model, I decompose the uncertainty associated to monetary policy in two parts:

the uncertainty related to time-varying regressor coefficients, captured by  $P_{t|t-1}$ , conditional on information up to  $t - 1$ , and  $\sigma_\varepsilon^2$ , the uncertainty associated to the purely monetary shock . It is captured in the variance of the conditional forecast error in the Kalman filter:

$$f_{t|t-1} = x_t P_{t|t-1} x_t' + \sigma_\varepsilon^2 \quad (11)$$

Table (1)<sup>3</sup> report the estimates of the TVP hyperparameters of the model. Estimates of zero or non-significant standard deviations signal a Null hypothesis of constant coefficients for the parameters. Looking at table (1), we see that the regressor coefficients attached to inflation and the first difference in the interest rate are time-varying, with the rest of coefficients not significantly varying.

<b>TVP</b>	<b>MLE Prmtr</b>	<b>Std. Dev.</b>	<b>t-stats</b>
sig_e	0.2542	0.1230	2.0674
sig_π <sub>t</sub> *	0.1372	0.0320	4.2848
sig_ỹ <sub>t-1</sub>	0.0974	0.0125	7.7795
sig_ỹ <sub>t-2</sub>	0.0096	0.0281	0.3415
sig_ẽ <sub>t</sub>	0.0001	0.0063	0.0210
sig_π <sub>t</sub> <sup>e</sup>	0.1372	0.0320	4.2848
sig_r <sub>t-1</sub> *	0.0006	0.0206	0.0297
sig_ı <sub>t-1</sub>	0.0006	0.0206	0.0297
sig_Δı <sub>t-1</sub>	0.0509	0.0294	1.7310
sig_Δm <sub>t</sub>	0.0049	0.0085	0.5808

Table 1: Time-Varying Hyper-parameters

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<sup>3</sup> The variance of the regressor coefficient,  $\rho_t$ , attached to the natural rate is assumed to be the same as the variance of the regressor coefficient,  $(1 - \rho_t)$ , attached to the lagged instrument regressor coefficient.

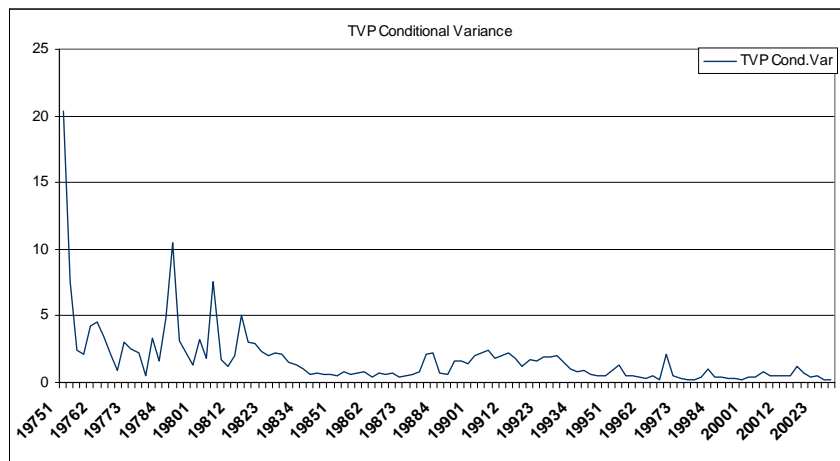


Figure 3: Time-Varying Heteroskedasticity

Figure (3) shows a graph of the changing conditional variance or uncertainty underlying monetary policy. There is a first peak in 1974, at the time of the implementation of the new post-Bretton Woods monetary policy. Post-Bretton Woods, the largest uncertainty occurs in 1979:2 with a peak of 12.6 percent. A shift to a temporary exchange rate target occurred during this period. This uncertainty prevailed until at least 1982, with two other peaks in 1980:4 and 1981:3 with 8 and 6 percent respectively. As Rich(1997) explains, the SNB temporarily relaxed the monetary targeting and imposed an exchange rate targeting. At the end of 1979, the SNB reinstated the monetary targets, using the monetary base instead of M1, as was previously used, considering the onetary base to be more stable than the aggregate M1. Smaller increases in conditional variance occur between 1988 and 1994, a period of uncertainty related to the German reunification period, the 1987 stock market crash and the SNB reappraisal of its monetary policy targets to medium term monetary targets. It is clear from the figure (3) that much of the

uncertainty present in the early post-Bretton Woods era until the early 1980s, is reduced to less than 2 percent from 1983 onwards. A new increase to 1.3 percent occurs in the early 2000, an increased uncertainty that may be linked to the introduction of the euro.

### **5.3 A Time-Varying Parameter Model assuming the Existence of Markov Switching Regime Heteroskedasticity(TVPMSR)**

The Markov switching models are useful tools for capturing occasional but recurrent discrete changes and endogenous regime shifts in time series. While the probability of switching from one regime to the other could be constant, as in Hamilton (1989), Diebold, Lee and Weinbach (1994) assume a time-varying transition probability that might depend on some underlying economic fundamentals. In a Markov Switching Regime (MSR) model, turning points are treated as structural events that are function of the data-generating process. One important feature of such models is that they can capture a particular form of non-linear dynamics or asymmetry in the policy rule, allowing for instance different means and variances for different regimes as in Hamilton (1989).

As an alternative to the TVP model, I now consider a TVPMSR model, a time-varying parameter model with Markov switching regimes heteroskedasticity. Kim and Nelson (1999) remark that the TVP model fails to incorporate changing uncertainties due to future random shocks while the MSR heteroskedasticity model with constant regression coefficients fail to incorporate the learning process of economic agents. These considerations suggest a general model that combines changing conditional variance due to time-varying coefficients and Markov switching heteroskedasticity.

### 5.3.1 Estimation Issues with TVPMSR

There are technical difficulties associated with Kalman filter inference on the state vector and the mean squared error associated to it to which Kim and Nelson(1999) propose efficient approximations to solve it. In the state-space MSR model, the goal is to form a forecast of the state (here the regressor coefficient) vector  $\beta_t$  based not just on the information set up to time  $(t-1)$ ,  $\psi_{t-1}$ , but also conditional on the random regime state  $S_t = j$  and  $S_{t-1} = i$  prevailing at time  $t$  and  $(t-1)$  respectively,  $\beta_{t|t-1}^{(i,j)} = E[\beta_t | \psi_{t-1}, S_t = j, S_{t-1} = i]$ ,

$$\text{and } P_{t|t-1}^{(i,j)} = E\left[\left(\beta_t - \beta_{t|t-1}\right)\left(\beta_t - \beta_{t|t-1}\right)' | \psi_{t-1}, S_t = j, S_{t-1} = i\right];$$

The KF algorithm calculates  $M^2$  such forecasts for each date  $t$ , corresponding to each possible value for  $i, j$ . Each iteration of the KF produces an M-fold increase in the number of cases to consider. If M=2, i.e two regimes, by the time  $t = 10$ , there would be 1,000 cases to consider. Kim and Nelson(1999) introduce some approximations to collapse the  $M^2$  posteriors at each iteration to M posterior distributions, taking weighted averages over the states at time  $t$ , to make the filter operable.

### 5.3.2 Model Specification of the TVPMSR Heteroskedasticity

The TVPMSR model which I consider is a 2-state switching regime heteroskedasticity with the following state-space form:

$$i_t = H_t \beta_t + \varepsilon_{it} \tag{12}$$

$$\beta_t = F \beta_{t-1} + \nu_t \tag{13}$$

where  $H_t = (\pi_t^*, \tilde{y}_{t-1}, \tilde{y}_{t-2}, \tilde{e}_{t-1}, \pi_t^e, r_t^*, i_{t-1}, \Delta i_{t-1}, dm_t^e)$  is a  $1 \times 9$  time-varying matrix of regressors,  $\beta_t$  is a  $9 \times 1$  vector of time-varying regressor coefficients.

The monetary shock,  $\varepsilon_{it} \sim N(0, \sigma_{i,S_t}^2)$ , is assumed to follow a 2-state Markov switching regime



with variance as follows:

$$\sigma_{i,S_t}^2 = \sigma_0^2 + (\sigma_1^2 - \sigma_0^2) S_t \quad (14)$$

where transition probabilities  $Prob[S_t = 1|S_{t-1} = 1] = p_{11}$  and  $Prob[S_t = 0|S_{t-1} = 0] = p_{00}$ ; and  $\nu_t \sim iidN(0, Q)$ .

The TVPMSR can be viewed as an alternative to the TVPGARCH heteroskedastic model. A fundamental difference with the latter is that part of the changes in conditional variance of the forecast error can result from endogenous regime changes. For the TVPMSR model, the conditional variance of the monetary forecast error, given the states  $S_t = j, S_{t-1} = i$ , can be represented as follows:

$$f_{t|t-1}^{(i,j)} = x_t P_{t|t-1}^i x_t' + \sigma_j^2, i, j = 0, 1; \quad (15)$$

$$= f_{1t}^i + f_{2t}^j, i, j = 0, 1; \quad (16)$$

where  $x_t$  is the  $9 \times 1$  regressor vector,  $P_{t|t-1}^i$  is the mean squared error matrix of  $\beta_{t|t-1}^i$ , an inference of  $\beta_t$  based on information up to time  $(t-1)$ , given  $S_{t-1} = i$ .  $\sigma_j^2$  is the variance of the shock, given  $S_t = j$ .  $f_{1t}^i = x_t P_{t|t-1}^i x_t', i = 0, 1$ ;  $f_{2t}^j = \sigma_j^2 = \sigma_0^2 + (\sigma_1^2 - \sigma_0^2) j, j = 0, 1$ ;

The uncertainty term would be calculated as follows:

$$\tilde{f}_t = \tilde{f}_{1t} + \tilde{f}_{2t} \quad (17)$$

where  $\tilde{f}_{1t}, \tilde{f}_{2t}$  are a complicated probability weighted average of  $f_{1t}^i, f_{2t}^j$  respectively. (See Kim and Nelson(1999), ch.5, p.119-120 for more details).

Again, the above equation states that the conditional variance of the forecast errors consists of two terms: the conditional variance related to the uncertainty due to changing coefficients,  $f_{1t}$ , and the conditional variance due to the regime change heteroskedasticity. The former is dependent on  $S_{t-1} = i$ , the regime at date  $(t-1)$  and the latter is dependent upon  $S_t = j$ , the regime at time  $t$ .

### 5.3.3 Results

Figure (4,5) shows the decomposition of the monetary shock uncertainty between the time-varying parameter uncertainty and the uncertainty due to switching regime heteroskedasticity. Around 80 percent of the variation comes from the time-varying parameter heteroskedasticity and only 20 percent from the regime switching heteroskedasticity. Moreover, both the variances for the low inflation regime and high inflation regime,  $\text{sig\_e\_0}$  and  $\text{sig\_e\_1}$  respectively, are significantly different from zero. The degree of uncertainty about the type of regime, whether a regime 0 with expected low inflation and low interest rate or a regime 1 of expected high inflation and high interest rate, is drastically reduced by more than two-third after 1976.

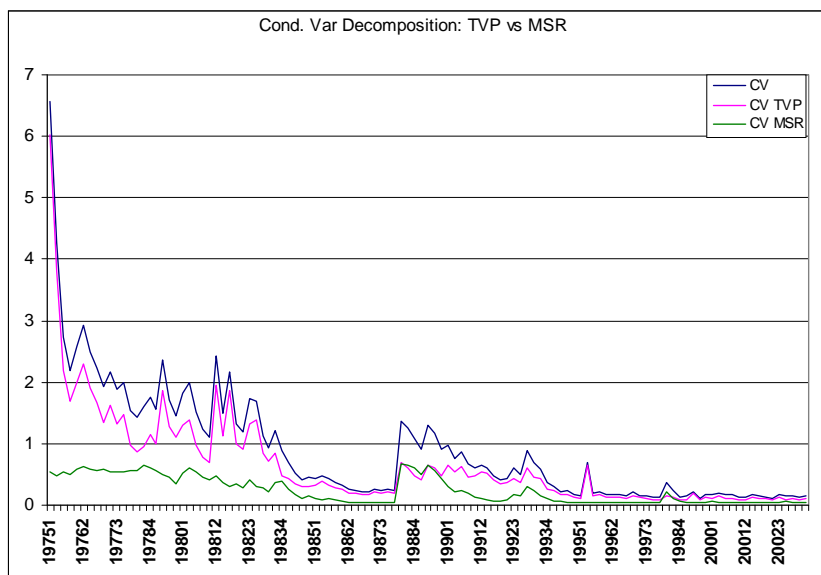


Figure 4: TVP-Markov Switching Regime Heteroskedasticity

The next Figure (6) confirms the findings. The filtered and smoothed probability of being in regime 0, a regime with low inflation target, is approximately 30 percent at the beginning of the period, 1975 and gradually increases to 100 percent at the end of the 1980s, 1988 to be more precise. However, this probability falls abruptly to 2 percent in 1988:1 and hovers below 30 percent until the beginning of the 1990s where it again climbs towards 90 percent, fluctuating in the 1990s between 60 percent and 95 percent to reaching nearly 100 percent at the end of the 1990s. 1988:1 was a time when the SIC system became operative and close to the German reunification. Rich(1997) observes that the a new electronic interbank payment system was introduced in 1987 the Swiss Interbank Clearing (SIC) system which became fully operative

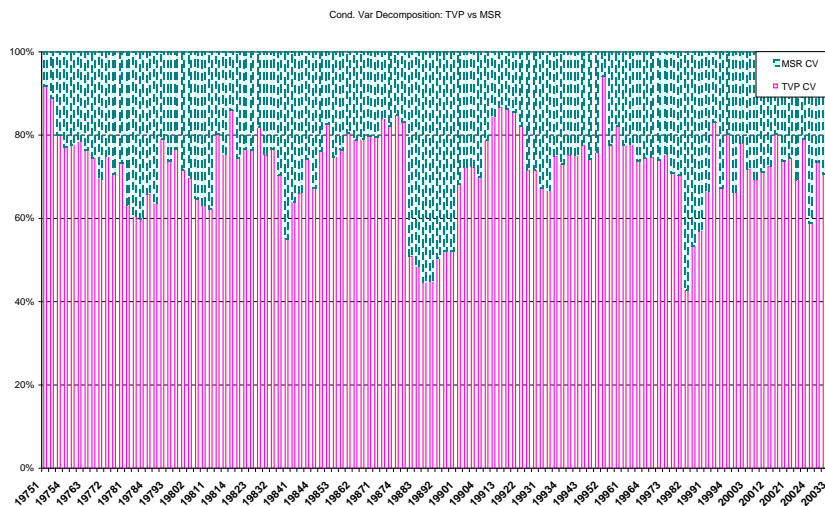


Figure 5: Shares of Cond.Var. Decomposition: TVP vs MSR

in 1988 and reduced drastically the balances of base money the banks required to settle their payments. However, this new system put the SNB "in the dark" as forecast of money base demand became unreliable. Meanwhile, the impact of the German reunification in October 1990 was also unpredictable for the Swiss economy. However, at the end of 1990, the SNB decided to shift to a medium-term strategy for the growth target of the money base. Inflation increased between 1988 to 1993 and price stability was restored in 1994. Inflation remained below the 2 percent level thereafter, achieving to reducing structurally the level of expected inflation to below the 2 percent level after 1993. The expected duration of the high inflation regime was estimated to be about 6 years and that of the low inflation regime to be about 15.6 years, highly persistent level of low inflation regime, larger than the US standard business cycle of 8-10 years.

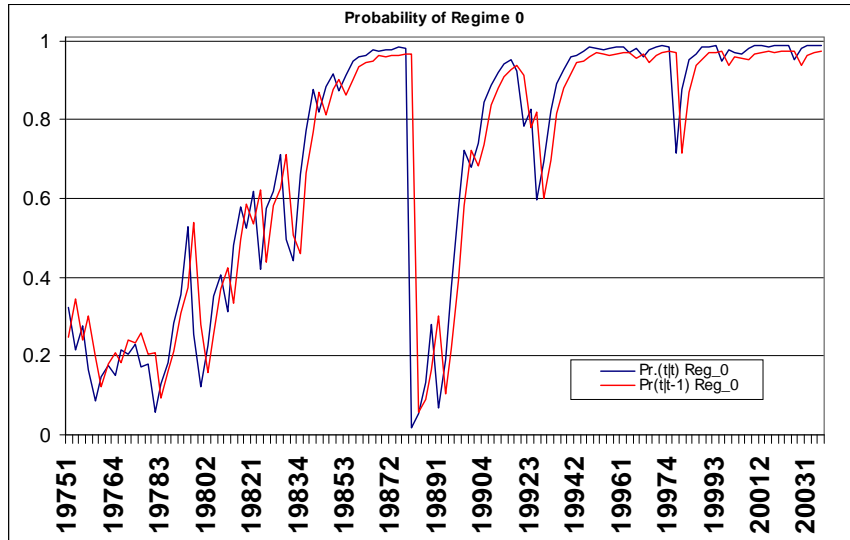


Figure 6: Markov Probability of Low Inflation Regime (Regime 0)

## 6 An Empirical Test for the Lucas and Friedman Hypothesis

As a by-product of the Kalman filter approach, recursive forecast errors of the policy instrument, and their conditional variance are obtained. The third goal of this paper is to test two hypothesis: the Lucas (1973) hypothesis and the Friedman (1977) hypothesis. The Lucas hypothesis predicts a negative relationship between the variance of nominal shocks and the magnitude of the output response to nominal shocks. Lucas (1973) has examined the hypothesis, using the unconditional constant variance of nominal shocks. But what really matters for the behavior of economic agents, is the conditional variance, given the information set up to time  $t-1$ , not the unconditional variance. In this paper, I use a time-varying conditional variance to test the Lucas hypothesis. Friedman (1977) predicted that increased variability of inflation rate causes a reduction in the allocative efficiency of the price system, causing a reduction in the national

level of output. The conditional variance of monetary forecast errors is used as a proxy for the variability of inflation rate and the Friedman hypothesis is also tested in this paper.

Solving the Lucas(1973)'s island model for output and price inflation, Lucas gets the following two equations:

$$y_{c,t} = -\pi\delta + \pi shock_t + \lambda y_{c,t-1} \quad (18)$$

$$\Delta P_t = -\beta + (1 - \pi) shock_t + \pi shock_{t-1} - \lambda \Delta y_{c,t-1} \quad (19)$$

The object of Lucas(1973) paper was not to explain output and price level movements within a given country but to see whether the terms of the output-inflation 'trade-off' vary across countries in an inverse manner. Lucas predicts an inverse relationship between the magnitude of the response to the nominal shock and the variance of this shock. To test his hypothesis, Lucas assumes that reaction to the shock and the variance of the shock are steady within a country but vary across countries. He takes a panel of 18 countries. His output gap is a deviation from a time trend.

To test the Lucas hypothesis for the case of Switzerland, I assume time-varying shocks and time-varying conditional variance. Instead of the Lucas(1973) defined output gap as the deviation from a time trend, I use for the output gap, the deviation of output from a smoothed Kalman estimated variable trend, the potential output. I test the Lucas hypothesis in two steps. Step 1 estimates equation (18) with time-varying coefficients:

$$\tilde{y}_{c,t} = -\alpha_t + \pi_t shock_t + \lambda_t \tilde{y}_{c,t-1} + \varepsilon_t \quad (20)$$

where  $\tilde{y}_{c,t}$  is the output gap,  $shock_t$  is the forecast error for the monetary shock; the three time-varying regressor coefficients are significant and of the correct sign.

The mean values in Lucas(1973) for  $\pi, \lambda$  have to be positive and below 1. Lucas's estimate for  $\pi$  are [.287,.910] and for  $\lambda$  are [.178,.937].

In step 2, I regress the estimate of the time-varying parameter  $\pi_t, \hat{\pi}_t$ , on the conditional variance of the nominal shock,  $CV_t = f_{t|t-1}$ ,

$$\hat{\pi}_t = \underset{(0.0022)}{0.65^{***}} - \underset{(0.0011)}{0.0016^*} CV_t + \kappa_t \quad (21)$$

$$R^2 = 0.02 \quad (22)$$

where standard deviations are in brackets. I find a **negative and significant** relation between the parameter  $\hat{\pi}_t$  and the conditional variance  $CV_t, \beta_1 = -0.0016$ . Thus, there is an output-inflation trade-off. The higher the uncertainty on inflation, the lower the nominal impact on output. However, this impact is quite small, of the order of  $1.6 \times 10^{-3}$ : for a 1 percent increase in the uncertainty, there is a  $1.6 \times 10^{-3}$  decrease in the nominal impact on the output gap.

Next, to test for the Friedman hypothesis, I regress the dependent variable on a constant, the shock, the conditional variance and the product of the shock by the log of the conditional variance:

$$\boxed{y_t} = \alpha_0 + \gamma_1 shock_t + \gamma_2 shock_t \cdot \ln(CV_t) + \gamma_3 CV_t + \gamma_4 CV_{t-1} + \varepsilon_t \quad (23)$$

I run two regressions, changing the dependent variable such as:

(i) the first difference of output gap. Running the regression in levels, the coefficient on the lagged output gap is close to 1. To avoid a unit root case, I turn into first differences.

(ii) the first difference of potential output

Standard deviations are in brackets. I get:

$$\begin{aligned}
d\tilde{y}_{c,t} &= \frac{0.000}{(0.12337)} + \frac{0.0126}{(0.0801)} shock_t + \frac{0.0221}{(0.054)} shock_t \cdot \ln(CV_t) + \\
&\quad - \frac{0.0985}{(0.0761)} CV_t - \frac{0.0887^{**}}{(0.0493)} CV_{t-1} + \varepsilon_t
\end{aligned} \tag{24}$$

$$R^2 = 0.03$$

$$\begin{aligned}
dy_t^* &= \frac{0.42361^{***}}{(0.024)} + \frac{0.0187}{(0.0156)} shock_t - \frac{0.0097}{(0.0105)} shock_t \cdot \ln(CV_t) + \\
&\quad - \frac{0.0139}{(0.0148)} CV_t - \frac{0.0239^{***}}{(0.0096)} CV_{t-1} + \varepsilon_t
\end{aligned} \tag{25}$$

$$R^2 = 0.15$$

The direct impact of the nominal shock on the dependent variable is positive in both cases but not always significant. Controlling for the shock and cross-terms, I find, again, in both cases, there is a direct negative impact of the uncertainty on the output gap and the potential output gap, the effect of the uncertainty, lagged one period, being significant to 5 percent and 1 percent respectively. Moreover, in equation(25), nearly 15 percent of the drop in growth output is explained by the effects of uncertainty and the forecast error over the growth rate. The rest would be explained by other variables, like productivity growth, investment strategies, labour reform policies and other variables.

## 7 Conclusion

A conventional fixed-coefficient model of a Taylor-type monetary policy rule does not integrate the learning process by economic agents about the economy. In this paper, a time-varying parameter model of a monetary policy reaction function was proposed to integrate such changing policies and strategies, reactions and decisions over various trade-offs to be made about various macroeconomic variables – inflation, the output gap and the real exchange rate gap. The



Kalman filter estimations of the time-varying parameters shows how rational economic agents combine past and new information to make new expectations about the state variables.

Secondly, a time-varying parameter model creates more uncertainty in the rule than a fixed-coefficient model. This monetary uncertainty, is estimated by the conditional forecast error and conditional variance; it can be decomposed into two components, the uncertainty related to the time-varying parameters and the uncertainty related to the purely monetary shock. Almost all of the uncertainty comes from time-varying parameters compared to the pure monetary shock. In the Markov switching regime, 80 percent of the monetary shock uncertainty is estimated to come from the time-varying parameter uncertainty and 20 percent from the expected regime change.

Thirdly, the Lucas and Friedman hypothesis about the impact of uncertainty on output are revisited, using a conditional variance to test them. I find an inverse relationship between the magnitude of the response on output to the nominal shock and the variance of this shock. Moreover, there is a direct negative impact of uncertainty which reduces both output in the long-term and the long-term growth of potential output. Unanticipated nominal shocks and the increased volatility of expected inflation, respectively proxied by the forecast error and the conditional variance of this forecast error, can explain more than 27 percent of the reduction in the long-term of growth output.

The Lucas test shows that increased volatility of nominal shocks reduce the impact of the unanticipated shock on the economy. The Friedman test shows that, controlling for the shock, increased volatility has a direct negative impact on the economy. This paper shows that most of the uncertainty in the Taylor rule comes from time-varying coefficients. One way to improve the economy is by keeping coefficients constant. As in the Kydland-Prescott and Barro-Gordon model, central bankers should stick to the rule and not change coefficients. Coefficients can change because, either policies change, which is under the control of the central bank, or there

are structural changes in the economy, such as the introduction of the SIC system in 1988, which may change structurally the relation between the dependent and the independent variables. In that case, coefficients change. To reduce the impact on the economy, central bankers can signal to the public the changes to be made. In fact, we see that central bankers do smooth changes in the interest rate as the weight on the lagged interest rate is significant and fairly stable throughout this thirty years period, estimated at around 0.65 percent. This gradual manner in reaching the recommended rate does attenuate fairly the changes and the uncertainty linked to these changes.

To conclude, this paper suggests that (a) regressor coefficients are time-varying and (b) most of the uncertainty comes from time-varying coefficients rather than the pure monetary shock. This could suggest a role for central bankers to reduce in as much as possible the uncertainty linked to time-varying coefficients, either, simply by sticking, if at all possible, to constant regressor coefficients, or if policy changes due to structural changes in the economy, smoothing the changes through gradual changes or by being transparent in communicating to the public, as clearly as possible, the changes that need to be done or the new weights attached to the various key variables and the timing of the changes to be made to reduce the uncertainty related to time-varying coefficients.

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<b>TVP</b>	<b>MLE Prmtr</b>	<b>Std. Dev.</b>	<b>t-stats</b>
sig_e_0	0.1286	0.0837	1.5371
sig_e_1	0.8443	0.2098	4.0248
sig_π <sub>t</sub> <sup>*</sup>	0.0600	0.0211	2.8489
sig_ỹ <sub>t-1</sub>	0.0000	0.0000	0.0000
sig_ỹ <sub>t-2</sub>	0.0129	0.0188	0.6855
sig_ẽ <sub>t</sub>	0.0000	0.0007	0.0103
sig_π <sub>t</sub> <sup>e</sup>	0.1290	0.0221	5.8308
sig_r <sub>t</sub> <sup>*</sup>	0.0000	0.0011	0.0193
sig_i <sub>t-1</sub>	0.0000	0.0011	0.0193
sig_Δi <sub>t-1</sub>	0.0344	0.0287	1.1987
sig_m <sub>t</sub>	0.0000	0.0000	0.0431
Prob_00	0.9840	0.0150	65.4697
Prob_11	0.9602	0.0285	33.7507
<b>Expected (Yearly) Duration for:</b>			
<b>Regime 0</b>	15.6245		
<b>Regime 1</b>	6.2842		

Table 2: Markov Switching Regime TVP

<b>TVP Lucas</b>	<b>MLE Prmtr</b>	<b>Std. Dev.</b>	<b>t-stat.</b>	
sig_ε	0.2154	0.1503	1.4329	
sig_α <sub>t</sub>	0.2579	0.1105	2.3335	
sig_π <sub>t</sub>	0.0000	0.0119	0.0042	
sig_λ <sub>t</sub>	0.0823	0.1041	0.7902	
<b>TVP Mean Values</b>		<b>Filtered</b>	<b>Smoothed</b>	<b>Lucas(1973)</b>
-α <sub>t</sub>		-0.1682	-0.1735	
π <sub>t</sub>		0.6481	0.6476	[.287,.910]
λ <sub>t</sub>		0.6551	0.6687	[.178,.937]

Table 3: Time-Varying Coefficients in the Lucas Equation