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Abstract

We characterize the degree of price discretion that competing principals award their agents in a framework where agents are informed about demand and seek to pass on their unverifiable distribution costs to consumers at the principals' expense. Principals learn demand probabilistically and may exchange this information on a reciprocal basis. While equilibria with full price delegation never exist, partial delegation equilibria exist with and without information sharing and feature binding price caps (list prices) that prevent agents from passing on their distribution costs to consumers. Yet, these equilibria are more likely to occur with than without information sharing. Moreover, while principals exchange information when products are sufficiently differentiated and downstream distribution costs are not too low, expected prices are unambiguously lower with than without information sharing. These results have potential implications for recent and ongoing antitrust investigations and damage claims in prominent sectors both in the US and the EU.

KEYWORDS: Competing Principals, Delegates Sales, Discretion, Information Sharing, List Prices

JEL CODES: L42, L50, L81.

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1 Introduction

Information sharing agreements are common in many prominent sectors and have been under close antitrust scrutiny on both sides of the Atlantic for many years.[1] An established literature has extensively studied the determinants and the competitive effects of communication in oligopoly games where firms first decide whether to share their private information (about demand or costs) and then compete à la Cournot or Bertrand (see, e.g., Khün and Vives, 1995, and Vives, 2006, for surveys of this literature). These models reach their conclusions under the traditional hypothesis that firms are profit-maximizing black boxes and are thus silent on the interplay between the incentives to exchange information and firms' internal delegation decisions.

Yet, when firms are viewed as an organization or collective of different agents with dispersed information, responsibilities, and non-congruent interests, the allocation of decision rights becomes an integral component of their organizational architecture and competitive strategies.[2] Therefore, to fill this gap, in this paper we study the link between firms' incentives to enter an information-sharing agreement, wherewith they exchange demand information, and their price delegation strategies, with these decisions being jointly determined at equilibrium. We consider a competitive environment where principals cannot internalize agents' incentives with monetary transfers but they can only decide what their representatives are entitled to do by designing permission sets from which these agents must select their pricing choices. Hence, principals face the so-called 'delegation dilemma': giving up control to gain flexibility, or imposing rigid rules unresponsive to changes of the environment?

We consider a stylized two-stage game, with linear demand functions, in which two upstream principals (e.g., manufacturers) compete by producing differentiated products and choose simultaneously, in the first stage, how much price authority to grant their downstream agents (dealers, sales managers, retailers, etc.) who are privately informed about an aggregate, additive and binary demand shock — i.e., being closer to the final market, these agents are better informed than principals about consumers' willingness to pay. In the second stage, after demand has been realized, agents simultaneously set actual prices given the constraints (if any) imposed by the principals in the first stage. We assume that agents incur an observable but not verifiable distribution cost to introduce a simple wedge between upstream and downstream objectives.[3] The presence of this cost, bundled with agents' private information, creates a

[1] https://www.ftc.gov/enforcement/competition-matters/2014/12/information-exchange-be-reasonable https://www.arnoldporter.com/en/perspectives/publications/2018/12/exchange-of-competitively-sensitive-information

[2] Recent evidence shows that price delegation is a profitable business strategy in several industries. Exploiting data from the auto-lending market, Phillips et al. (2015) estimate that sales forces' freedom to adjust prices to local market fluctuations generates an average profit increase of approximately 11%. Similarly, by comparing a sample of 181 companies from Germany's industrial machinery and electrical engineering industry, Frezen et al. (2010) document a positive effect of price delegation on firm performance, which amplifies under high market uncertainty and information asymmetry. Instead, Homburg et al. (2012) consider various B2B industries in Germany and find an inverted U-shaped relationship between delegation and profitability.

[3] For example, the opportunity cost of the time required to convince a buyer into purchasing the product, the cost of the effort that must be invested to build a relationship with the buyer (which may depend, among other things, on its unverifiable propensity to acquire the product), the cost of targeting prospective customers with informative and/or promotional activities, etc. In a recent paper, Quérou et al. (2020) argue that there are important contracting environments where agents' tasks

natural misalignment of preferences. Since principals cannot internalize the non-verifiable cost through monetary incentives, agents will pass on this cost to consumers at the expense of sale volumes, profits and consumer surplus. Therefore, although agents are better informed than principals on demand and can tailor prices to this information, they tend to charge excessive prices compared to what competition between principals mandates. Yet, unlike in other delegation models, principals are not totally uninformed in our framework: with some probability, they observe the state of demand (i.e., high or low willingness to pay) and can, therefore, condition the degree of price authority granted to their agents on this information. We study their incentives to exchange information and examine the competitive and welfare effects of these agreements. We show the following results.

First, we establish that full delegation never occurs in equilibrium regardless of whether principals share information. That is, as long as the distribution cost is positive (even negligible), principals will never allow agents to set prices in all demand states freely. Specifically, principals always have an incentive to constrain their agents' pricing decisions (in at least one state of nature) to prevent them from charging excessive prices compared to what upstream profit maximization mandates.

Second, given the impossibility of full delegation, we characterize equilibria with partial delegation in both information-sharing regimes. In these equilibria, agents are entitled to choose their preferred price only when demand is low and are, instead, constrained to charge a price lower than what they would like in the high-demand state. Hence, partial delegation equilibria feature a price cap or, equivalently, a list price: a result broadly consistent with customized pricing with discretion (see, e.g., Phillips et al., 2021).[4] As intuition suggests, partial delegation equilibria exist when distribution costs are not too high, meaning that the conflict of interest between principals and agents is not too pronounced. The region of parameters where partial delegation occurs in equilibrium shrinks when products become closer substitutes because agents are more incentivized to pass on their distribution costs to consumers to shield against increased competition. On the contrary, this region of parameters expands as principals' information accuracy and the significance of demand uncertainty increase. The higher the probability that principals are informed, the easier for them to sustain an equilibrium with partial delegation. Specifically, a principal that expects its rival to be informed with a higher probability will be keener to delegate because it expects a lower opponent's price, which, by strategic complementarity, also makes his agent willing to charge a lower price, endogenously reducing its bias towards an excessive price. The opponent will, in turn, expect a lower price and accordingly reduce its price too indirectly. Moreover, the greater the significance of demand uncertainty, the higher the cost for the principals to give up flexibility and implement a rigid pricing rule (pooling) that makes prices unresponsive to demand shocks — i.e., the greater is the cost of not allowing the agents to engage in price tailoring.

Third, we show that principals have a reciprocal incentive to enter an information-sharing agreement only when products are sufficiently differentiated, when their information accuracy is high, and distribu-

involve non-monetary and monetary costs that are unverifiable due to imperfect monitoring or lack of expert knowledge.

[4]This practice is widespread in many B2B and B2C markets where prices are inherently customized due to the additional costs needed to satisfy buyers (e.g., quality customization, delivery requirements, service provision such as loan and insurance application, etc.).

tion costs are not too low. The following effects determine this result. When principals share information, each learns the demand shock with greater probability than without information sharing (because an uninformed principal learns the state of demand when the rival is informed and has committed to share this information). Hence, other things being equal, the information exchange benefits principals because it mitigates the conflict of interest with their agents. Yet, when both principals are uninformed and know this is the case because they agreed to share information, agents charge higher prices in the low-demand state. In this state of nature, each agent is sure that the opponent will pass on its cost to consumers and will be keener to raise its price, as opposed to the no information sharing regime where there is uncertainty on whether the rival principal is informed or not. This price-increasing effect has an ambiguous impact on the principals' profits: it benefits principals since it softens competition; but, when distribution costs are excessively high, the pass-through is too high, reducing sales and thus profits. On the net, information sharing benefits principals if agents' distribution costs are not too small and products are sufficiently differentiated. In this case, solving internal agency conflicts is relatively more important than softening competition. From a consumer welfare point of view, instead, we show that expected prices are unambiguously lower with than without information sharing. Essentially, by aligning incentives within organizations, information sharing reduces the pass-through rate according to which distribution costs are passed on to consumers. Hence, from an ex ante point of view, the exchange of information benefits consumers.

Besides emphasizing the pros and cons of information sharing on demand in contexts where these agreements impact firms' delegation strategies, our results may also have broader implications. In particular, in the online Appendix, we also demonstrate that, within our delegation framework, an exchange of demand information can be implemented through disclosure of list price intentions. This observation may help understanding recent patterns of information sharing on list prices that are under investigation both in the US and EU — i.e., cases involving the exchange of information on prices that are different from those that customers eventually pay.[5] Although derived in a stylized framework, our analysis shows that, from a static point of view, the welfare consequences of these exchanges of information are unclear and depend on the purpose of the exchange of information and its impact on the firms' organizational design.

Finally, to discuss the robustness and the limitations of these results, in the online Appendix, we also examine (pooling) equilibria in which, when uninformed, principals retain full price authority — i.e., they force a singleton price irrespective of the demand state. In these equilibria, results align with the existing literature (see, e.g., Khün and Vives, 1995, and Vives, 2006) since principals de facto behave as uninformed, vertically integrated oligopolists. When a pooling equilibrium arises with and without information sharing, the exchange of information has a neutral impact on expected prices and consumer

[5] The relevance of examining this issue is demonstrated by the ongoing EU trucks cartel case — the largest-ever EU cartel infringement by fine size that projected a wave of damage claims across the EU. In this case, the European Commission has concluded that executives of major truck manufacturers regularly met and exchanged gross list price information and that this conduct constitutes an infringement by object of Article 101 TFEU. Harrington (2021) also discusses other cases including high fructose corn syrup, urethane, cement, air freight, air passengers, and railroads.

surplus. However, sharing information unambiguously benefits principals because it allows them to tailor prices to demand, while increasing the extent of vertical control. By contrast, when under information sharing the equilibrium features partial delegation and pooling without information sharing, we find that consumers are hurt by information sharing. Yet, in this hybrid scenario, principals will not share information when products are sufficiently differentiated and the significance of demand uncertainty is not too high. We also argue why our results remain qualitatively valid under alternative information structures, demand specifications, and organizational structures (i.e., inter-brand competition), drawing new avenues for future research whenever possible.

The rest of the paper is organized as follows. Section 2 lays down the baseline model. In Section 3, we develop two useful benchmarks and show that full delegation cannot occur in equilibrium irrespective of the information sharing regime. In Section 4, we characterize and show the existence of partial delegation equilibria with and without information sharing and then study the effect of these agreements on equilibrium prices and profits. In Section 5, we discuss the policy implications of the model, its robustness/limitations and relate our results to existing work. Section 6 concludes. Proofs are in the Appendix. In the online Appendix, we provide additional material and robustness checks.

2 The baseline model

Environment. Consider two principals (e.g., manufacturers), each denoted by P_i with $i = 1, 2$, distributing their products through exclusive agents (dealers, retailers, etc.), each denoted by A_i with $i = 1, 2$, to whom they can grant price authority. Demand functions are derived from the preferences of a representative consumer with a Shubick-Levitan utility function (see, e.g., Motta, 2004, Ch. 8.4.2.) — i.e.,

$$D_i(\theta, p_i, p_{-i}) \triangleq a + \theta - p_i + d(p_{-i} - p_i), \quad \forall i = 1, 2,$$

where $a > 0$ is the exogenous demand intercept, while θ is an additive shock distributed with equal probability on the support $\Theta \triangleq \{-\epsilon, \epsilon\}$.[6] The parameter $d \geq 0$ represents the degree of product differentiation: the larger d, the less differentiated (more homogenous) products are.

Agents are privately informed about the demand shock θ and condition their pricing decision (if they are entitled to do so) on its realization. Each principal is informed about θ with probability $\alpha \in [0, 1]$ and uninformed otherwise (an all-or-nothing information structure). For expositional purposes, we introduce a signal $s_i \in \{\varnothing, 1\}$ (for each $i = 1, 2$) describing the principals' information structure: $s_i = 1$ means that P_i is informed about θ, while $s_i = \varnothing$ means that it is uninformed and bases its decisions on the (common) prior. In the following we shall refer to $\alpha \triangleq \Pr[s_i = 1]$ as to principals' information accuracy (in Section 5 we discuss alternative information structures).

Payoffs, conflict of interest and delegation. Following the delegation literature[7], we rule out mon-

[6] See the online Appendix for a discussion of the continuum of types.

[7] For example, Amador and Bagwell (2013), Dessein (2002), Holmstrom (1977-1984), Martimort and Semenov (2006),

etary incentives — i.e., there are frictions (e.g., limited enforcement and/or incomplete contracts) that prevent principals to fully internalize the downstream profits through monetary incentives (e.g., fixed fees). Specifically, while P_i maximizes sale profit

$$\pi_i(\cdot) \triangleq D_i(\theta, p_i, p_{-i})\, p_i, \quad \forall i = 1, 2,$$

with technologies being linear and marginal costs normalized to zero, A_i's objective function is

$$u_i(\cdot) \triangleq \pi_i(\cdot) - cD_i(\theta, p_i, p_{-i}) = D_i(\theta, p_i, p_{-i})(p_i - c), \quad \forall i = 1, 2. \tag{1}$$

The parameter $c \geq 0$ plays a key role in the analysis: it can be interpreted as the distribution cost that an agent incurs to finalize a sale — e.g., the opportunity cost of the time required to convince a buyer into purchasing the product, the cost of effort that must be invested in this negotiation (which may depend, among other things, on the buyers' unverifiable propensity to acquire the product), the cost of targeting perspective buyers through informative and/or promotional activities, etc. Hence, de facto, c represents A_i's bias vis-à-vis P_i. If $c = 0$ their preferences are fully aligned; otherwise, A_i has an incentive to set a price higher than P_i's ideal price as we explain below.

We assume that c is not verifiable. Therefore, monetary transfers contingent on this parameter cannot be enforced — see, e.g., also Green and Laffont (1994) and Aghion and Tirole (1994).[8] P_i can only limit A_i's discretion by determining the permission set $\mathcal{P}_i$ within which A_i must choose its price p_i. We restrict attention to equilibria in pure strategies and focus, without loss of generality, on interval delegation equilibria. That is, we look for pure strategy equilibria in which every principal P_i offers a (connected) permission set $\mathcal{P}_i \triangleq [\underline{p}_i, \overline{p}_i]$, with $\overline{p}_i \geq \underline{p}_i \geq 0$.[9] The width of this segment represents a measure of the price authority delegated to A_i. Notice that, depending on the shape of $\mathcal{P}_i$, three scenarios can emerge:

- P_i fully delegates price authority to A_i — i.e., $\mathcal{P}_i$ is wide enough not to constrain A_i's decision (say, for example $\mathcal{P}_i = [0, +\infty)$);

- P_i can retain full price control by setting a singleton (pooling) price that A_i must charge — i.e.,

Melumad and Shibano (1991), among many others.

[8] Green and Laffont (1994) study a general model with pure non-verifiability where two players would ideally like to contract contingent on the state of nature that will become known to both of them in the future before any payoff-relevant actions must be taken. They assume that the state of nature is not verifiable by any third party. Thus, although it is assumed that there is a third party present to enforce the contract, this third party has less information than either of the contracting players, and this fact may limit how the agreement can function in the mutual interest of the players. They identify conditions under which the first-best is achievable and show that, when these conditions are not satisfied, the only feasible solution to the problem is a delegation scheme where only one of the parties gets control of the payoff relevant actions.

[9] Restricting principals to offer connected permission sets is without loss of generality because the demand state is binary. Agents will always tailor prices to the demand state. Hence, there will always be two ideal prices, one for each realization of θ. These two prices will be ordered — i.e., the higher θ, the larger the price chosen by an agent (for given expectations on the rival's price). Therefore, any allocation a principal can achieve by offering a disconnected permission set can also be achieved with a connected set. Specifically, a partial-delegation equilibrium in which agents are restricted to set a price given price in only one state of nature can be implemented either with a simple cap or with a simple floor. As a result, with pure strategies, more complex delegation schemes than a connected permission set are redundant.

$\overline{p}_i = \underline{p}_i = p_i$;

- P_i can restrict A_i's authority within a given binding interval (partial delegation) — i.e., the set $\mathcal{P}_i$ restricts A_i's choice in one state of nature (more below).

Information sharing. We examine and compare two alternative information-sharing (IS) regimes:

a A regime in which principals do not share their private information;

b A regime in which principals share their private information.

Following the bulk of literature on information sharing in oligopoly, we assume that principals commit to share information ex ante (before observing signals) and that only if both agree to do so, the information exchange takes place.[10] As standard, we assume that principals cannot falsify the information they share.[11]

Timing. The timing of the game is as follows:

1. Principals choose whether to share information;
2. The demand shock realizes. Each agent learns this realization with probability 1, while each principal learns it with probability α;
3. Principals share their private information if they committed to do so;
4. Principals simultaneously and non-cooperatively choose their permission;
5. Agents choose prices simultaneously within the permission sets designed by their principals;
6. Demand is allocated between the two products. Profits are made.

Equilibrium concept. Since principals offer permission sets secretly to their agents — i.e., A_i is unable to observe the permission set offered to A_{-i} and vice-versa — the equilibrium concept will be Perfect Bayesian Nash Equilibrium. We impose the refinement of 'passive beliefs' widely used in the vertical contracting literature (Hart and Tirole, 1990; McAfee and Schwartz, 1994; Rey and Tirole, 2007). With passive beliefs, an agent's conjecture about the permission sets offered to its rivals is not influenced by an out-of-equilibrium offer (i.e., an unexpected permission set) he receives: the so-called 'no signaling what you don't know' condition (see, e.g., Fudenberg and Tirole, 1988, Ch. 8). We shall describe the equilibrium

[10] See, e.g., Calzolari and Pavan (2006), Gal-Or (1985-1986), Khün and Vives (1994), Piccolo and Pagnozzi (2013), Raith (1996), Shapiro (1986), Vives (1984-2006), among many others.

[11] Alternatively, the truthfulness of the shared data may be guaranteed by (un-modeled) reputation costs incurred by firms that falsify or misreport their private information — e.g., rivals may no longer trust them in the future — implying that the potential benefits of information sharing will be foregone in the future.

structure in every IS regime as we go along the paper, after having established the impossibility of full delegation.

Parametric restrictions. In developing the analysis, we assume that

$$a > \epsilon, \tag{\textbf{A1}}$$

which guarantees that demand functions are always positive, and that

$$c \leq \frac{a - \epsilon}{2 + d}, \tag{\textbf{A2}}$$

which guarantees that downstream margins are positive irrespective of the demand shock and the information sharing regime.[12]

Discussion and interpretation. The delegation framework laid down above hinges on a few assumptions worth explaining before describing the analysis.

First, we assumed that the distribution cost c is not verifiable by a third party in charge of enforcing contracts (e.g., a Court of Law). As argued by Quérou et al. (2020), there are important contracting environments where agents' tasks involve non-monetary and monetary costs, which are unverifiable due to high costs of monitoring or lack of expert knowledge — e.g., subcontractors operating at arm's length face this type of costs, as in the automobile industry (Kawasaki & McMillan, 1987) or for construction projects.[13]

Second, as in the delegation literature, the non-verifiability of the agents' bias makes monetary incentives contingent on this parameter impossible to enforce. However, the above payoff formulation can also be seen as a reduced form of a simple sharing agreement according to which principals and agents split the sales profit $\pi_i(\cdot)$ with fixed proportions (say with weights $\beta \in (0,1)$, and $1-\beta$, respectively). Neither of these shares affect our analysis. While the scalar β does not alter principals' decisions, we can redefine the agents' utility to embed the share $1-\beta$ into their distribution cost. To fix ideas, denote by $\hat{c}$ the true distribution cost, the utility of each agent would then be

$$u_i(\cdot) = (1-\beta) D_i(\theta, p_i, p_{-i}) p_i - c D_i(\theta, p_i, p_{-i}), \quad \forall i = 1, 2,$$

where c in equation (1) is equal to $\frac{\hat{c}}{1-\beta}$. Then, the coefficient $1-\beta$ is irrelevant for our analysis, and positive changes in the principals' bargaining position β can be just interpreted as increases of the cost parameter c.

[12] As it will be clear, for a sufficiently large, this restriction does not affect the equilibrium characterization.

[13] Since we focus on the delegation scheme that emerges in these instances, we purposefully rule out the possibility that principals reimburse in full or partly these costs. Yet, it is intuitive that agents will always have an incentive to overstate their costs. Therefore the reimbursement will only be partial, thereby generating a payoff structure like the one stated in (1).

Third, the hypothesis that principals can truthfully share their information hinges on the idea that the information exchange is coordinated by specialized intermediaries that certify the truthfulness of the data shared among rivals — e.g., auditors, data analytic companies, marketing information services firms, trade associations, etc. — who own the technology to discover the private information of the participants to the agreement and can commit to disclosure rules that disseminate this knowledge among them (see, e.g., Lizzeri, 1999). In Section 5, we discuss informally how these agreements can be implemented in practice and provide a formal argument in the online Appendix.

Finally, it is worth noting that we have opted for a simultaneous pricing game to capture the idea that, in practice, price tailoring and discounts are often secretly negotiated with buyers. A model with a sequential pricing timing — e.g., such that there is a price leader — would implicitly implement some degree of information sharing because it would enable the follower to learn demand from the leader's moves.

3 Benchmarks and impossibility of full delegation

To gain insights on the key forces driving the results, it is useful to first examine the case in which principals are fully informed ($\alpha = 1$) about the state of demand. In this scenario, they can force the (unique) equilibrium outcome of the standard pricing game with differentiated products — i.e.,

$$p^P(\theta) \triangleq \frac{a+\theta}{2+d}, \quad \forall \theta \in \Theta.$$

This price determines the principals' ideal point in the equilibrium of the game with complete information.

Suppose now, to the other extreme, that principals are uninformed ($\alpha = 0$) but grant full price authority to the agents — i.e., $\mathcal{P}_i = [0, +\infty)$ for every $i = 1, 2$. Agents set prices by maximizing their profit functions, which include the distribution cost c. The (unique) equilibrium of this (sub)game yields

$$p^A(\theta) \triangleq \frac{a+\theta}{2+d} + \underbrace{\frac{1+d}{2+d}}_{\text{Pass-through}} c, \quad \forall \theta \in \Theta.$$

This expression determines the agents' ideal point which features a standard pass-through rate. Agents have an incentive to pass on their distribution cost to consumers. Comparing $p^A(\theta)$ with $p^P(\theta)$ we obtain the agents' bias compared to the principals' ideal price — i.e.,

$$\Delta p(\theta) \triangleq p^A(\theta) - p^P(\theta) = \frac{1+d}{2+d} c \geq 0, \quad \forall \theta \in \Theta. \tag{2}$$

This expression is increasing in d: the stronger competition, the higher the incentive of the agents to shift the burden of their distribution cost to consumers — i.e., the higher the pass-through rate is.

We can thus state the following lemma, which rules out equilibria with full delegation.

Lemma 1. *If $c > 0$, full delegation never occurs in equilibrium irrespective of the IS regime.*

The reason why an equilibrium with full delegation never exists when agents have positive (even arbitrarily small) distribution costs is straightforward. Since $\Delta p(\theta) > 0$ for $c > 0$, principals always have an incentive to cap their agents' pricing choices because such constraint aligns upstream and downstream incentives — i.e., it mitigates the negative effect of the pass-through on the sales volume in the high-demand state where profit margins are relatively large.

Hence, throughout the analysis, in each information sharing regime, we will focus on the most interesting class of pure strategy equilibria in which principals partially delegate price authority to their agents. The reason why these equilibria are more interesting than equilibria in which agents are not granted price authority at all (pooling or singleton-price equilibria) is that in the latter class of equilibria principals de facto behave as integrated duopolists, exactly as in the traditional literature studying information sharing in oligopoly (see Section 5 for a discussion of this class of equilibria).

4 Partial delegation equilibria

This section characterizes equilibria with partial delegation in each IS regime, establishes the conditions under which these equilibria exist, and examines the effects of the exchange of information on principals' expected profits and expected prices.

4.1 The regime without information sharing

Suppose that principals do not exchange information. Consider a symmetric (candidate) equilibrium with the following properties:

N1 When P_i $(i = 1, 2)$ is informed $(s_i = 1)$, in every demand state θ it forces the singleton price $p^N(\theta, s_i = 1)$ that maximizes its profit (given the rival's equilibrium strategy);

N2 When P_i $(i = 1, 2)$ is uninformed $(s_i = \varnothing)$, it chooses a permission set that requires a binding price cap (list price) — i.e., $\mathcal{P}^N \triangleq [0, \overline{p}^N]$. The cap $\overline{p}^N$ only binds in state $\theta = \epsilon$, while A_i sets a price $p^N(-\epsilon, s_i = \varnothing) < \overline{p}^N$ in state $\theta = -\epsilon$.

N1 is intuitive since it simply imposes that, when informed, a principal forces the state-dependent price that maximizes its profit. **N2**, instead, implies that the candidate equilibrium features partial delegation because every agent is free to pick its preferred price when its principal is uninformed and demand is low. We first characterize the candidate equilibrium described above and then find the conditions under which it exists and is unique.

Suppose that P_i is informed $(s_i = 1)$. Given properties **N1-N2**, for every demand state θ, principal P_i solves

$$\max_{p_i \geq 0} \mathbb{E}_{s_{-i}} \left[D_i \left(\theta, p_i, p^N(\theta, s_{-i}) \right) p_i \right].$$

The above expectation is taken with respect to the information status of P_i's rival, which modifies its pricing behavior according to the rules stated in **N1-N2**. For given $p^N(\theta, s_i = \varnothing)$, which will be characterized below, optimality (see the Appendix) requires

$$p^N(\theta, s_i = 1) = \frac{a+\theta}{2(1+d) - \alpha d} + \frac{d}{2(1+d) - \alpha d}(1-\alpha)\, p^N(\theta, s_{-i} = \varnothing), \quad \forall \theta \in \Theta, \tag{3}$$

which, as intuition suggests, converges to the principals' ideal point when $\alpha = 1$ — i.e., when principals are fully informed.[14]

Next, consider the case in which P_i is uninformed ($s_i = \varnothing$) and assume that it grants A_i price authority within the set $\mathcal{P}_i = [0, \overline{p}_i]$, with $\overline{p}_i$ being binding in state $\theta = \epsilon$ only. We will check ex post that these properties are indeed optimal from P_i's standpoint. Hence, in state $\theta = -\epsilon$, A_i solves the following maximization problem

$$\max_{p_i \geq 0} \mathbb{E}_{s_{-i}} \left[D_i \left(-\epsilon, p_i, p^N(-\epsilon, s_{-i}) \right) (p_i - c) \right],$$

whose solution, in equilibrium, yields

$$p^N(-\epsilon, s_i = \varnothing) = p^P(-\epsilon) + \frac{2 + d(2-\alpha)}{2(2+d)} c.$$

To complete the characterization of equilibrium prices we must determine the cap $\overline{p}^N$. We assume, and check in the Appendix, that $\overline{p}^N$ exceeds $p^N(-\epsilon, s_i = \varnothing)$ and falls short of A_i's ideal price in state $\theta = \epsilon$, so that the cap binds in the high demand state and there is neither full delegation nor a singleton (pooling) price. Since P_i's profit in state $\theta = -\epsilon$ does not depend on the cap imposed in the high-demand state, this value solves the following maximization problem

$$\max_{\overline{p}_i \geq 0} \mathbb{E}_{s_{-i}} \left[D_i \left(\epsilon, \overline{p}_i, p^N(\epsilon, s_{-i}) \right) \overline{p}_i \right],$$

whose solution, in equilibrium, yields $\overline{p}^N = p^P(\epsilon)$ — i.e., under partial delegation principals force their ideal price in state $\theta = \epsilon$ (see the Appendix). Then, it is easy to show that A's unconstrained price choice in state $\theta = \epsilon$ would violate the cap $\overline{p}^N$ since this choice must factor in the distribution cost c.

Summing up, the equilibrium prices charged by A_i when P_i is uninformed are

$$p^N(\theta, s_i = \varnothing) \triangleq \begin{cases} p^P(\epsilon) & \text{if } \theta = \epsilon \\ p^P(-\epsilon) + \frac{2+d(2-\alpha)}{2(2+d)} c & \text{if } \theta = -\epsilon \end{cases}, \tag{4}$$

with $p^N(\theta, s_i = \varnothing)$ being increasing in θ if and only if c is not too large (see the Appendix).

Under partial delegation, uninformed principals always force their ideal price when demand is high: in this state, agents would appropriate high margins if they could choose prices according to their own biased

[14] This price positively depends on the price chosen by an uninformed principal because, without IS, each principal is uncertain about whether the other principal is informed or not.

objectives. By contrast, to gain flexibility, principals let agents distort prices upward in the low demand state — i.e., where such a distortion impacts relatively less sales volumes. Clearly, partial delegation can occur in equilibrium only if the distribution cost c is not too high, otherwise the gain in flexibility of partial delegation would be offset by the loss of control owing to the agent's need to pass on their costs to consumers.

Finally, substituting $p^N(\theta, s_i = \varnothing)$ into (3) we also obtain the prices of an informed principal — i.e.,

$$p^N(\theta, s_i = 1) \triangleq \begin{cases} p^P(\epsilon) & \text{if } \theta = \epsilon \\ p^P(-\epsilon) + \frac{d}{2+d}(1-\alpha)c & \text{if } \theta = -\epsilon \end{cases}. \tag{5}$$

Notice that when demand is high, an informed principal will set the full information benchmark, even if it does not know whether the rival is informed. The reason is that an informed principal anticipates that even if the rival is uninformed, in equilibrium, the rival's agent will face a price cap only in the high demand state. By contrast, since principals are uninformed about the rival's signal, the price in the low-demand state features an upward distortion because, in this state, agents can pass on their cost to consumers. Clearly, this distortion vanishes when products are independent ($d = 0$) and principals are fully informed ($\alpha = 1$)

The expected equilibrium price charged by each principal is

$$p^N \triangleq \mathbb{E}\left[p^P(\theta)\right] + \underbrace{\frac{2 + d(2+\alpha)}{4(2+d)}(1-\alpha)c}_{\text{Price distortion without IS}},$$

which, as intuition suggests, is higher than the expected ideal price that principals would set when they are fully informed. Notice that α has a potentially ambiguous effect on p^N. First, the higher α, the larger the probability that a principal is informed and thus able to force the singleton price that maximizes its expected profit. Second, other things being equal, the higher the probability that a principal is informed, the higher the likelihood that the singleton price that this principal charges in the low demand state is upward distorted. The reason is that, without information sharing, an informed principal still faces uncertainty about the rival's signal and, thus, about its ability to discipline its agent, which is expected to charge an excessive price with probability $1 - \alpha$. On the net, it can be shown that the first effect always dominates, and that the price distortion without information sharing is decreasing in α. Moreover, as expected, this price distortion is increasing in d because the pass-through rate rises with the degree of product substitutability — i.e., the closer substitutes the products are, the stronger the agents' incentive to pass on their costs to consumers.

We can now turn to study the conditions under which the candidate equilibrium characterized above exists. Clearly, an informed principal has no incentive to deviate because the price $p^N(\theta, s_i = 1)$ maximizes its objective function in every demand state (given the rival's equilibrium behavior). Hence, the only deviation that may disrupt the candidate equilibrium characterized above is that of an uninformed principal. Given the restriction to pure strategies, there are two possible deviations that such a principal may consider: (*i*) a deviation to full delegation, and (*ii*) a deviation to pooling — i.e., such that the deviating principal retains full price control. The first deviation is clearly unprofitable: as argued above, an uninformed principal will always cap its agent's choice in the high-demand state to prevent the agent form passing the distribution cost to consumers. Hence, the main conceivable deviation for an uninformed principal is to set a singleton (pooling) price. The following holds.

Proposition 1. *Without IS, there exists a threshold $c_N > 0$ such that a symmetric equilibrium with partial delegation satisfying conditions* **N1-N2** *exists if and only if $c \leq c_N$ and is unique within the class of partial-delegation equilibria in pure strategies. The threshold c_N is decreasing in d and increasing in α and ϵ.*

When the misalignment of preferences between principals and agents is sufficiently strong, as reflected by a high distribution cost, principals do not find it profitable to leave price authority to the agents because they would pass on this high cost to consumers, reducing sales and profits. The region of parameters where partial delegation occurs in equilibrium shrinks when d increases. Since the pass-through increases when products become closer substitutes, the conflict of interest between principals and agents worsens, which makes a pooling equilibrium relatively more likely since agents would make choices too distant from what the principals would like. Yet, as α and ϵ increase, the region of parameters where partial delegation occurs in equilibrium expands. The higher the probability that principals are informed, the easier it is for them to sustain an equilibrium with partial delegation: since prices are strategic complements, a principal that expects its rival to be informed with a higher probability will be keener to delegate because it expects the rival's price to be more aligned to what upstream competition mandates, which in turn lowers the price charged by its agent, mitigating the agency conflict. Instead, as the significance of demand uncertainty increases, the cost for the principals to implement a rigid pricing rule (pooling) that makes prices unresponsive to demand fluctuations increases. Hence, they will be more willing to delegate for higher levels of ϵ.

4.2 The regime with information sharing

Suppose now that principals share information. There are two relevant aggregate states of nature in this regime: (*i*) the state in which none of the principals is informed (hereafter denoted $\sigma = \varnothing$) whose probability is $(1-\alpha)^2$, and (*ii*) the state in which at least one of them is informed (hereafter denoted $\sigma = 1$) whose probability is $1-(1-\alpha)^2$. Consider a symmetric (candidate) equilibrium with the following properties:

- **S1** When at least one principal is informed ($\sigma = 1$), each learns the state of nature θ and charges a singleton price $p^S(\theta, \sigma = 1)$ that maximizes its profit (given the rival's equilibrium strategy);

- **S2** When no principal is informed ($\sigma = \varnothing$), each chooses a permission set that requires a binding price cap — i.e., $\mathcal{P}^S \triangleq [0, \overline{p}^S]$.[15] The cap $\overline{p}^S$ only binds in state $\theta = \epsilon$, while A_i $(i = 1, 2)$ sets a price $p^S(-\epsilon, \sigma = \varnothing) < \overline{p}^S$ in state $\theta = -\epsilon$.

The logic behind the structure of this candidate equilibrium is the same as in the regime without IS, the difference being that prices now are contingent on the aggregate state $\sigma \in \{1, \varnothing\}$ — i.e., when one principal is informed, also the rival is informed. As before, we first characterize the above candidate equilibrium, and then find the conditions under which it exists and is unique.

When $\sigma = 1$, it is easy to show that principals force a singleton price equal to their 'ideal point' — i.e.,

$$p^S(\theta, \sigma = 1) = p^P(\theta), \quad \forall \theta \in \Theta.$$

One difference with the case of no information sharing is that there is no upward price pressure under information sharing when principals are informed and demand is low. When at least one principal is informed, each knows exactly what price the rival will set under information sharing because demand is symmetric.

Next, consider the state $\sigma = \varnothing$. Suppose that P_i offers a permission set $\mathcal{P}_i = [0, \overline{p}_i]$, with $\overline{p}_i$ being binding in state $\theta = \epsilon$ only. Given property **S2,** in state $\theta = -\epsilon$, A_i solves the following maximization problem

$$\max_{p_i \geq 0} D_i\left(-\epsilon, p_i, p^S(-\epsilon, \sigma = \varnothing)\right)(p_i - c).$$

In a symmetric equilibrium, the solution of this problem yields

$$p^S(-\epsilon, \sigma = \varnothing) = p^P(-\epsilon) + \frac{1+d}{2+d}c.$$

In contrast to the no IS regime, this expression is independent of α because every principal knows that the rival is uninformed in the state of nature under consideration.

To complete the characterization we must find the price cap $\overline{p}^S$. Again, since P_i's profit in the low-demand state does not depend on the cap $\overline{p}_i$ imposed in the high-demand state, the maximization problem that determines this value is

$$\max_{\overline{p}_i \geq 0} D_i\left(\epsilon, \overline{p}_i, \overline{p}^S\right)\overline{p}_i,$$

whose first-order condition, in equilibrium, yields again $\overline{p}^S = p^P(\epsilon)$ (see the Appendix).

Summing up, the prevailing prices when both principals are uninformed are

$$p^S(\theta, \sigma = \varnothing) = \begin{cases} p^P(\epsilon) & \text{if } \theta = \epsilon \\ p^P(-\epsilon) + \frac{1+d}{2+d}c & \text{if } \theta = -\epsilon \end{cases},$$

with $p^S(\theta, \sigma = \varnothing)$ being increasing in θ if and only if c is not too large (see the Appendix). Hence, even with information sharing, partial delegation requires principals to force their ideal price in the high

[15] We show in the Appendix that this is indeed the unique equilibrium.

demand state, while agents can distort prices upward in the low demand state.

The expected equilibrium price charged by each principal is

$$p^S \triangleq \mathbb{E}\left[p^P(\theta)\right] + \underbrace{\frac{1+d}{2(2+d)}(1-\alpha)^2 c}_{\text{Price distortion with IS}},$$

which, once again, is higher than the expected ideal price that principals would set when they are fully informed. As intuition suggests, the price distortion with information sharing is decreasing in d and α and vanishes when principals are fully informed ($\alpha \to 1$).

We can then turn to characterize the conditions under which the above candidate equilibrium is immune to deviations by the principals. As before, the only deviation that may disrupt the above candidate equilibrium is that of an uninformed principal that deviates to set a singleton (pooling) price. The following holds.

Proposition 2. *With IS, there exists a threshold c_S such that the symmetric equilibrium with partial delegation satisfying conditions* **S1-S2** *exists if and only if $c \leq c_S$ and is unique within the class of partial-delegation equilibria in pure strategies. The threshold c_S is independent of α, decreasing in d and increasing in ϵ.*

The intuition for this result is as before, and will thus be omitted for brevity.

4.3 Sharing information or not?

We now compare principals' equilibrium profits and consumer surplus with and without IS. To gain insights on how information sharing affects the equilibrium outcome of the game, we start with the following useful result.

Lemma 2. *Suppose that $\alpha \in (0,1)$, then if an equilibrium with partial delegation exists with no information sharing, it also exists with information sharing, but not vice-versa — i.e., $c_N < c_S$.*[16]

The intuition for this result follows from the comparative statics discussed after Propositions 1 and 2. The higher the probability that principals are informed, the easier it is for them to sustain an equilibrium with partial delegation: since prices are strategic complements, a principal that expects its rival to be informed with a higher probability will be keener to delegate because it expects the rival's price to be more aligned to what upstream competition mandates, which in turn lowers the price charged by its own agent, mitigating the agency conflict. Therefore, since information sharing increases the probability that both principals are informed, the result follows immediately.

We can now state the main result of the article.

[16] That is, for $c \in (c_N, c_S]$ an equilibrium with partial delegation exists only with information sharing.

Theorem 1. *Suppose that $c \in (0, c_N]$ and $\alpha \in (0,1)$. There exist two thresholds, $c^\star > 0$ and $d^\star > 0$, such that principals gain from sharing information if and only if $d \leq d^\star$ and $c \in [c^\star, c^N] \neq \varnothing$. Sharing information has, instead, a neutral effect on principals' profits when they are fully informed ($\alpha = 1$), when they are uninformed ($\alpha = 0$) and when agents' preferences are fully aligned with (upstream) profit maximization ($c = 0$). The threshold $c^\star$ is decreasing in α.*

In addition, the expected market price is unambiguously lower with than without information sharing — i.e., the difference $p^N - p^S$ is positive, increasing in c and d, inverted-U shaped with respect to α, with a maximum at $\alpha = 0.5$ and it reaches zero at $\alpha \in \{0, 1\}$.

In words, when agents are granted partial discretion in both IS regimes, although communication reduces the expected market price and unambiguously increases consumer surplus, it benefits principals if products are sufficiently differentiated and the conflict of interest with agents is not negligible.

To gain intuition, it is helpful to discuss first why IS reduces expected prices and benefits consumers. When principals exchange information, it is enough that one of them is informed to have both informed. Hence, IS aligns vertical incentives by inducing expected prices to drop. The difference between the expected price in the two regimes is zero when principals are fully informed ($\alpha = 1$) and when they are uninformed ($\alpha = 0$). The non-monotone effect of α can be explained as follows. When α is high, the difference in prices is negligible because principals are fully informed. In this case, reducing α below 1 tends to increase the difference between expected prices because, by dealing with relatively less informed principals, agents will exploit strategic complementarity (via a higher pass-through rate) to increase their prices in the regime without IS.[17] When α is small, this difference is negligible too because principals are uninformed, and sharing information has little impact on prices. In this case, increasing principals' information accuracy spurs the difference between expected prices because prices fall in the event that principals are informed, and this effect more than compensates the indirect price reduction effect of α on $p^N(-\epsilon, s_i = \varnothing)$ described above. Finally, while the positive impact of c on the difference $p^N - p^S$ is evident because the conflict of interest between principals and agents is less intense with than without IS, the positive effect of d hinges on the fact that the pass-through rate is higher when products are closer substitutes, and IS reduces the probability that such a pass-through occurs in equilibrium.

Building on these insights, we can now explain the effect of IS on principals' expected profits. First, when principals share information, each learns the demand shock with greater probability than without information sharing. Other things being equal, this effect benefits principals because it mitigates the conflict of interest with their agents. Second, when principals share information, expected prices drop because agents are less likely to pass on their distribution costs to final consumers. The impact of this price-reducing effect on principals' expected profit is ambiguous. On the one hand, the fact that agents charge lower prices reduces profits because it intensifies competition. On the other hand, when distribution costs are relatively high, the pass-through is high, and IS mitigates the 'double marginalization' phenomenon, increasing sales and thus profits.

[17] Recall that $p^N(-\epsilon, s_i = \varnothing)$ is decreasing in α.

On the net, IS benefits principals if distribution costs are sufficiently high. In this case, its beneficial effect on firms' internal agency conflict and the positive effect on sales overwhelms the competition-strengthening effect associated with the drop of expected prices under IS. The reason why products need to be sufficiently differentiated (d low) for IS to be beneficial to principals is as follows. If products are relatively close substitutes (d high), softening competition is relatively more important than solving internal agency problems. Finally, notice that $c^{\star}$ is decreasing in α — i.e., information sharing is relatively more likely to benefit principals when their information accuracy rises — because a higher information accuracy increases rivalry by aligning prices with what upstream competition mandates.

4.4 List prices, 'price tailoring' and comparative statics

One interesting feature of the equilibria with partial delegation studied above is that the price cap can be interpreted as a list price. As a result, the difference between this cap and the price that agents charge in the low state of demand can be interpreted as the extent of 'price tailoring' that consumers enjoy when their willingness to pay turns out to be low. Notably, this suggests that the link between list and transaction prices is not deterministic, but it depends on the relative probability of low- and high-demand states. In what follows, we study the determinants of such price tailoring and how it depends on the IS regime.

When principals do not exchange information, the extent of price tailoring is

$$r^N \triangleq \overline{p}^N - p^N(-\epsilon, s_i = \varnothing) = \frac{2\epsilon}{2+d} - \frac{2+d(2-\alpha)}{2(2+d)}c,$$

which is decreasing in the agents' distribution cost c and the degree of product substitutability d, while it rises with the significance of demand uncertainty ϵ and the principals' information accuracy α.

With IS, instead, the extent of price tailoring is

$$r^S \triangleq \overline{p}^S - p^S(-\epsilon, \sigma = \varnothing) = \frac{2\epsilon}{2+d} - \frac{1+d}{2+d}c,$$

which is again increasing in ϵ and decreasing in c and d. However, in this case, the extent of price tailoring does not depend on the principals' information accuracy α.

Straightforward algebra implies the following result.

Corollary 1. *The extent of price tailoring is weaker with than without IS — i.e.,*

$$r^N - r^S = \frac{d\alpha}{2(2+d)} > 0.$$

Even though IS reduces expected prices, consumers obtain better deals when principals do not exchange information, provided they are uninformed. The reason is simple: without IS, each agent (say A_i) must take an expectation over the price charged by the rival when it is granted price authority. This expectation weights, with probability α, the event in which P_{-i} is informed and lowers its price compared to the case in which it is uninformed. With IS, instead, every agent knows whether the rival principal is informed or not. When both are uninformed, each agent knows that the rival agent will increase the price above the principals' ideal point in the low-demand state (simply because agents learn from the principals choices). Strategic complementarity then implies that, conditional on principals not being informed, agents charge a higher price in the low demand state with than without IS.

A novel implication of this result is that observing more extensive price tailoring in the no IS regime does not necessarily imply that consumers prefer this regime to one with IS from an ex-ante standpoint. Hence, even if ex post consumers enjoy more extensive price tailoring without IS, they prefer firms to exchange information ex ante. Interestingly, this may create a time inconsistency problem because consumers may favor firms' commitment to sharing information ex ante but complain and claim damages ex post.

5 Policy implications, robustness and related work

The analysis developed up to this point shows that when competing principals face a delegation dilemma, the emergence of information-sharing agreements, according to which principals share (directly or indirectly) demand information, benefits consumers if agents are (partially) delegated price authority before and after the introduction of the agreement (two conditions that are testable empirically with a simple before and after analysis). As a result, long-lasting information-sharing agreements may not necessarily have anticompetitive effects but express a genuine competitive conduct to reduce the conflict of interest between upstream and downstream divisions in vertical industries. A key question, however, concerns the robustness of these results. How robust is the insight that, when present together, price delegation and information sharing tend to generate pro-competitive effects? For completeness, in the following, we discuss the strengths and limitations of this conclusion.

Information structure. The result that the expected price is lower with than without information sharing does not rely on the hump-shaped pattern highlighted in Theorem 1. While the hump-shaped result follows from the fact that principals' signals are iid draws, the reason why information sharing reduces prices hinges on the fact that sharing information enables both principals to control their agents' behavior better. Irrespective of the information structure — i.e., whether principals' signals are iid or somewhat correlated — sharing demand information increases the probability that each is informed and can implement the complete information outcome. Of course, the fact that information sharing would still exert downward pressure on prices in these circumstances means that principals would still gain from sharing information when c is relatively large. A similar logic applies even when considering richer information structures — e.g., information structures in which the probability of learning depends on the demand state. When this is the case, principals update their posteriors when they do not learn demand (and potentially also about what the rival knows in case draws are correlated). Hence, although the possibility of learning makes information sharing relatively less appealing, it is still the case that sharing information allows principals to discipline their agents and align the conflict of interest with their agents when such learning is not perfect.

Acquiring or sharing information? A limitation of our analysis, worth exploring in future work, is that principals' learning technology is given — i.e., the probability α of discovering the state of demand is exogenous and cannot be influenced by firms' investment activities. Does the incentive to share information strengthen or weaken when firms can engage in costly information acquisition? Various effects are likely to be at play in this scenario. First, ceteris paribus, sharing information becomes less critical because principals can control the amount of learning via the information acquisition technology. Second, when they commit ex ante to sharing information, a free-riding problem is likely to emerge: each principal has an incentive to save on the cost of acquiring information, counting on the rival's investment. This free-riding problem also makes knowledge sharing less appealing. Yet, the bright side of information sharing remains. The point of sharing information is to eliminate uncertainty about the rival's strategy: when knowing the rival's information, principals can enforce the complete information benchmark, avoiding excessive pricing. Hence, when the endogenous probability of being informed is (strictly) lower than 1, sharing information still mitigates the price pressure induced by the agents' desire to pass on their cost to consumers.

Alternative demand specifications. The observation that agents are incentivized to pass on their distribution cost to consumers at the expense of principals' profit and consumer surplus is rather general and holds with a general demand structure. We have chosen a simple linear formulation to obtain closed-form solutions and relate the incentive to share information to market characteristics, such as demand uncertainty and product substitutability. Yet, one may wonder how robust our comparative statics results are. In the online Appendix, we argue that the qualitative conclusions of the analysis remain valid when considering alternative demand functions.

However, an interesting aspect worth studying more extensively in future work is the introduction of intra-brand competition. In the online Appendix, we show that while inter-brand competition exacerbates the conflict of interest between principals and agents, intra-brand competition tends to align upstream and downstream incentives. As a result, the equilibrium with partial delegation will be less likely to emerge in industries featuring stronger intra-brand competition for a given information-sharing regime. This implies that, while information sharing will still benefit consumers irrespective of the level of intra-brand competition, principals will be less likely to share information as intra-brand competition intensifies because the competition softening effect gains weight as agents within each distribution network compete more fiercely.

Pooling equilibria. So far, we have focused on equilibria featuring partial delegation. As explained above, the analysis of pooling equilibria — i.e., equilibria in which uninformed principals do not grant price discretion at all to their agents — is less interesting and somewhat trivial. The competitive effects of pooling equilibria have already been studied (indirectly) in the literature dealing with information exchanges in oligopoly. In our setting, uninformed principals de facto behave as integrated firms when they do not grant agents price discretion and tailor prices to demand only when they are informed. Hence, as argued by Khün and Vives (1995), while information sharing has a neutral impact on expected prices, it allows firms to gain flexibility and tailor prices to demand. Therefore, while being consumer welfare neutral, IS increases principals' expected profits. This is certainly true when $c > c_S$, where pooling equilibria prevail under both regimes. Yet, an interesting region of parameters is that in which $c \in (c_N, c_S]$. In this case, the no IS regime can only feature a pooling equilibrium while partial delegation prevails under IS. Hence, the comparison of principals' expected profits and prices is less obvious. In the online Appendix we show that, in this region of parameters, expected prices are higher with than without information sharing. Principals benefit from sharing information if d and/or ϵ are sufficiently large. Since IS leads to a change in the type of equilibrium, principals gain flexibility when moving from the no IS regime to that with IS. Yet, since agents have biased preferences, they will price too high in the low-demand state compared to what principals would like to do, making (ceteris paribus) information sharing less compelling for both principals and consumers. Finally, since agents raise prices above the competitive level, principals can soften competition when granting them price authority.

When products are sufficiently close substitutes — i.e., d relatively large — the competition-softening effect dominates the loss of control effect. In other words, softening competition is relatively more important than losing control of agents' choices. By contrast, when products are sufficiently differentiated — i.e., d is sufficiently low — the cost of giving up control is relatively stronger than the competition-softening effect. In this region of parameters, IS still enhances principals' expected profit if demand is sufficiently volatile, and the opposite holds otherwise. Hence, for intermediate values of the distribution cost, while information sharing agreements increase prices and reduce consumer surplus, principals have no incentive to enter into these agreements when products are sufficiently differentiated, and demand is not too uncertain. As hinted above, this result shows that the conclusions of the baseline model are restricted to instances where partial delegation prevails before and after the introduction of the information sharing agreement.

Implementation through disclosure of (list) price intentions. To be effective, information sharing agreements require some degree of coordination between their members, especially in oligopolistic markets (see, e.g., Ziv, 1993, who shows why these agreements fall apart in the absence of coordination). Most of the existing models (see, e.g., Raith, 1996) assume that firms can commit themselves either to reveal their private information to other firms or to keep it private before receiving any private information. The implicit hypothesis is that these agreements are organized by certification intermediaries — e.g., auditors, data analytic companies, marketing information services firms, trade associations, etc. — who own the technology to discover the private information of the participants to the agreement and can commit to disclosure rules that disseminate this knowledge among them (see, e.g., also Lizzeri, 1999). Following such a 'reduced-form' approach, thus far, we have assumed that principals can freely exchange their demand information without specifying the communication protocol or the 'language' through which this information is shared.

In reality, however, firms do not communicate through a vague 'word of mouth' process, but signal their private information to rivals via their market choices — i.e., prices, investment decisions, output, etc. When this is the case, these variables de facto form the language through which firms communicate. In the Appendix, we explain how an information-sharing agreement can be organized through the disclosure of price intentions in the context of our model. This may contribute to understanding how firms communicate in practice and shed some light on the simple logic of sharing information about (future) list prices. (see, e.g., Harrington and Ye, 2017, for a survey of recent cases where this practice is under investigation). More importantly, this observation may also help assess the competitive effects of recent exchanges of information on list prices that are under investigation in the US and EU — i.e., cases involving the exchange of information on prices that are different from those that customers eventually pay.[18] The relevance of examining this issue is also demonstrated by the ongoing EU trucks cartel case — the largest-ever EU cartel infringement by fine size that projected a wave of damage claims across the EU. In this case, the European Commission has concluded that executives of major truck manufacturers regularly met and exchanged gross list price information and that this conduct constitutes an infringement by object of Article 101 TFEU. Although derived in a stylized framework, our analysis shows that, from a static point of view, the welfare consequences of these exchanges of information are unclear and depend on the purpose of the exchange of information and its impact on the firms' organizational design.

Related literature. Our analysis borrows from and contributes to several strands of the IO literature dealing with competition in oligopolies, delegation, information sharing, and incentives within firms.

Several papers have investigated the strategic value of delegation in oligopoly games (see, e.g., Bonanno and Vickers, 1988, Fershtman and Judd, 1987, Sklivas, 1987, and more recently, Prat and Rustichini, 2003). In these models, delegating price and output decisions to agents can be a credible mechanism to soften competition. The crucial feature is the ability of principals to disclose contracts to rivals and therefore influence their conduct. The model developed in this paper differs in two fundamental ways from this literature. First, it considers a framework where monetary incentives cannot solve agency conflicts. Second, in our model, delegation is not explained by its strategic commitment value since principals' permission sets are unobservable as in the literature on secret contracting.

A few papers have extended the idea of strategic delegation to environments with secret contracts and different forms of information asymmetries. For example, Pagnozzi and Piccolo (2011) show that vertical delegation may occur at equilibrium even when contracts are secret, provided that agents do not hold passive beliefs off-equilibrium. Bhardwaj (2001), instead, shows that with competition in prices and effort, the strategic nature of delegation depends on the relative intensity of competition. In contrast to us, with unobservable contracts and risk-averse sales representatives, he finds that firms delegate the pricing decision when price competition is intense (see also Gal-Or, 1991, Blair and Lewis, 1994, Martimort and

[18] Harrington (2021) discusses some of these cases including high fructose corn syrup, urethane, cement, air freight, air passengers, and railroads.

Piccolo, 2010, for models with adverse selection, and Mishra and Prasad, 2005, for a moral hazard set-up).

In all these models, principals can align incentives (partially or in full) through monetary incentives. In contrast, we focus on cases where monetary incentives are not enforceable. In this sense, our model is closely related to and builds on the partial delegation literature initiated by Holmstrom (1977-1984). Following his seminal work, many scholars have investigated the determinants of delegation in the absence of monetary incentives and the conditions under which interval allocation is optimal (see, e.g., Amador and Bagwell, 2013, Alonso and Matouschek, 2008, Armstrong and Vickers, 2010, Dessein, 2002, Dessein and Santos, 2006, Frankel, 2014-2016, Martimort and Semenov, 2006, Melumad and Shibano, 1991, among many others). We contribute to this bulk of work by considering competing organizations and characterizing the equilibrium interval delegation with and without information sharing.

In this latter respect, our analysis has obvious connections with the traditional information-sharing literature in oligopoly. This literature shows that firms' incentives to share information about their common demand function (Novshek and Sonnenschein, 1982; Clarke, 1983; Vives, 1984; Gal-Or, 1985) or about their private costs of production (Fried, 1984; Gal-Or, 1986; Shapiro, 1986) depend on the nature of competition between them (Bertrand or Cournot). Raith (1996) rationalizes the results of this vast literature in a unified framework. Yet, this literature assumes that firms are profit-maximizing black boxes; therefore, it is silent on the interplay between information exchanges and agency conflicts within organizations. To the best of our knowledge, a few exceptions are Calzolari and Pavan (2006) and Maier and Ottaviani (2009), who consider common agency models in which two principals may share the information they obtain by contracting with a common agent, and Piccolo and Pagnozzi (2013) and Piccolo et al. (2015), who consider competing, vertical organizations where principals can share information about their exclusive agents. These models posit that monetary transfers between principals and agents are feasible. Hence, principals partly internalize agents' incentives to misreport their private information (adverse selection) and exert low effort (moral hazard) through fixed (lump sum) fees. In this paper, instead, we study information sharing in a framework where monetary transfers are unfeasible: principals cannot internalize agents' decisions, but they can only decide what agents are entitled to do by designing permission sets from which agents must select their choices.

For obvious reasons, our model relates to the ongoing literature on price caps and list prices. In a recent influential article, Rey and Tirole (2019) argue why authorities may want to consider allowing price-cap agreements when it is unclear whether products or services are substitutes or complements. The intuition for why price caps can be attractive is that they allow producers of complements to cooperate and solve Cournot's double-marginalization problem but do not allow competitors to collude and raise prices of substitutes. We find similar results in a different setting and, in addition, examine how the benefits of price caps depend on the accuracy of the information shared between principals. Harrington and Ye (2022) also develop a theory to explain the welfare effects of list price coordination on transaction prices. They assume a deterministic link between list and transaction prices. We explain why this link can be stochastic and responsive to the competitive environment, thereby complementing their work. Gill and Thanassoulis (2016) also consider upstream cooperation but, in contrast to Harrington and Ye (2022), assume that firms can coordinate on both list and transaction prices because both are verifiable in their framework (see also Raskovich, 2007, Lester et al., 2015, and Mallucci et al., 2019).

In an interesting paper, Harrington (2021) also shows that an information exchange of list prices can lead to higher retail prices. He considers two firms competing on price for customers. However, before the firms set their selling prices, they exchange information about the prices they want to set later for consumers. After this exchange, each firm can set a selling price different than the announced one — e.g., an unobservable discount. Yet, a deviation from the reported price leads to a cost for the deviating firm granting the discount. This cost is higher the more the actual selling price differs from the price communicated at the meeting. Harrington (2021) shows that such a two-step process, in which information exchange on prices takes place before the final sales price is set, can lead to a higher sales price if the exogenous deviation cost is high enough. Hence, in these circumstances, the sales prices that firms set are higher than the prices they would have charged in a situation without the possibility of information exchange. In his model, however, when an anticompetitive effect emerges, list prices are binding, as opposed to what happens in our model. In addition, while Harrington is interested in how the cost of 'reneging' on the original list price announcement impact final prices, our focus is on the endogenous emergence of such list prices.

Finally, Myatt and Ronayne (2019) consider a consumer search environment and study a two-stage game where firms set list prices in the first stage, and in the second stage, they may discount (but not raise) those prices. Their crucial hypothesis is that the announcement of list prices is public both to rivals and consumers, whose search strategy depends on the pattern of observed list prices. Under several specifications, they find a unique set of prices supported by a pure strategy equilibrium. Firms' prices differ (even for symmetric firms), and the opportunity to offer a second stage discount is not used (but the possibility determines equilibrium prices). When the asymmetry of firms is driven by the heterogeneity of costs so that the most aggressive firm has the lowest marginal cost, an equilibrium with on-path pure strategies in their two-stage model generates higher welfare and higher consumer surplus than in any equilibrium of a single-stage pricing game.

6 Concluding remarks

In this paper, we offered new insights into how competing principals delegate price authority to their privately informed agents and into how an information-sharing agreement between principals affects such decisions. Under the natural hypothesis that downstream agents are privately informed about demand conditions, but principals can learn this state of nature probabilistically, the equilibrium delegation form features binding list prices that prevent agents from passing on their distribution costs to consumers. When principals share their information about demand, agents are more likely to be granted price authority. By learning demand with greater probability, the agreement relaxes the trade-off between flexibility and loss of control, thereby making principals more willing to award agents with price authority. Hence, expected prices are lower with information sharing than without and profitable from the principals' point of view when distribution costs are neither too high nor too low and products are sufficiently differentiated. This suggests that the coexistence of information-sharing agreements, either about demand or price intentions, and list prices, intended to align upstream and downstream incentives, is not necessarily a symptom of consumer harm.

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7 Appendix

Proof of Lemma 1. Suppose that an equilibrium with full delegation exists. First, it is immediate that this cannot be true if a principal is informed. Therefore, if such an equilibrium exists, it must be such that agents are granted full price authority only when principals are uninformed — i.e., either $s_i = \varnothing$ or $\sigma = \varnothing$ depending on the IS regime.

Suppose that principals do not exchange information. Let $p^D(\theta, s_{-i})$ denote the equilibrium candidate. If an equilibrium with full delegation exists, then A_i solves the following maximization problem in every state θ:

$$\max_{p_i \geq 0} \mathbb{E}_{s_{-i}} \left[D\left(\theta, p_i, p^D(\theta, s_{-i})\right)(p_i - c)\right],$$

whose first-order condition yields

$$p_i(\cdot) = \frac{c}{2} + \frac{a + \theta + d\mathbb{E}_{s_{-i}}\left[p^D(\theta, s_{-i})\right]}{2(1+d)}, \quad \forall \theta \in \Theta,$$

with

$$\mathbb{E}_{s_{-i}}\left[p^D(\theta, s_{-i})\right] \triangleq \alpha p^D(\theta, s_{-i} = 1) + (1-\alpha) p^D(\theta, s_{-i} = \varnothing).$$

Hence, in a symmetric equilibrium of the pricing subgame, it must be

$$p^D(\theta, s_{-i} = \varnothing) = \frac{a + \theta + c(1+d) + d\alpha p^D(\theta, s_{-i} = 1)}{2 + d(1+\alpha)}, \quad \forall \theta \in \Theta.$$

An informed principal, instead, solves the following maximization problem

$$\max_{p_i \geq 0} \mathbb{E}_{s_{-i}} \left[D\left(\theta, p_i, p^D(\theta, s_{-i})\right) p_i\right],$$

whose first-order condition yields immediately

$$p^D(\theta, s_i = 1) = \frac{a + \theta}{2(1+d) - \alpha d} + \frac{1-\alpha}{2(1+d) - \alpha d} d p^D(\theta, s_{-i} = \varnothing), \quad \forall \theta \in \Theta.$$

In a symmetric equilibrium,

$$p^D(\theta, s_i = 1) = p^P(\theta) + \frac{d}{2(2+d)}(1-\alpha) c, \quad \forall \theta \in \Theta,$$

and

$$p^D(\theta, s_i = \varnothing) = p^P(\theta) + \frac{2(1+d) - d\alpha}{2(2+d)} c, \quad \forall \theta \in \Theta,$$

with $p^D(\theta, s_i = \varnothing) > p^D(\theta, s_i = 1)$.

Consider now a deviation from this candidate equilibrium by an uninformed principal (say P_i). Suppose, for example that it imposes a list price $\overline{p}_i$ binding in the high-demand state only. Then, this deviation is such that P_i solves

$$\max_{\overline{p}_i \geq 0} \mathbb{E}_{s_{-i}} \left[D\left(\epsilon, \overline{p}_i, p^D\left(\epsilon, s_{-i}\right)\right) \overline{p}_i \right],$$

whose first-order condition immediately yields the following deviation price

$$\overline{p}_i = p^P\left(\epsilon\right) + \frac{d}{2\left(2+d\right)}\left(1-\alpha\right)c < p^D\left(\epsilon, s_i = \varnothing\right),$$

implying that full delegation cannot be an equilibrium in the regime without IS.

Suppose now that principals share information. In this regime, it is straightforward to show that when principals are informed they set $p^P\left(\theta\right)$ for every $\theta \in \Theta$, while, in the equilibrium candidate with full delegation agents set

$$p^A\left(\theta\right) \triangleq p^P\left(\theta\right) + \frac{1+d}{2+d}c, \quad \forall \theta \in \Theta.$$

Consider, again, a deviation by an uninformed principal (say P_i again). The deviation price cap solves

$$\max_{\overline{p}_i \geq 0} D\left(\epsilon, \overline{p}_i, p^A\left(\epsilon\right)\right) \overline{p}_i,$$

yielding

$$\overline{p}_i = p^P\left(\epsilon\right) + \frac{d}{2\left(2+d\right)}c < p^A\left(\epsilon\right),$$

which shows that full delegation cannot be an equilibrium with IS either. ■

Partial-delegation equilibrium without IS. When P_i is informed, the first-order condition of its maximization problem yields the following best-reply function

$$p_i\left(\theta, s_i = 1\right) = \frac{a + \theta + d\mathbb{E}_{s-i}\left[p^N\left(\theta, s_{-i}\right)\right]}{2\left(1+d\right)}, \quad \forall \theta \in \Theta.$$

Under passive beliefs, the following expectation must hold true

$$\mathbb{E}_{s_{-i}}\left[p^N\left(\theta, s_{-i}\right)\right] \triangleq \alpha p^N\left(\theta, s_{-i} = 1\right) + \left(1-\alpha\right) p^N\left(\theta, s_{-i} = \varnothing\right).$$

Hence, for given $p^N\left(\theta, s_i = \varnothing\right)$, the equilibrium price $p^N\left(\theta, s_i = 1\right)$ is such that

$$p^N\left(\theta, s_i = 1\right) = \frac{a+\theta}{2\left(1+d\right) - \alpha d} + \frac{\left(1-\alpha\right)d}{2\left(1+d\right) - \alpha d} p^N\left(\theta, s_{-i} = \varnothing\right), \quad \forall \theta \in \Theta.$$

Suppose now that P_i is uninformed ($s_i = \varnothing$) and assume that it grants A_i price authority within the set $\mathcal{P}_i = [0, \overline{p}_i]$, with $\overline{p}_i$ being binding in state $\theta = \epsilon$ only. We will check ex post that these properties are indeed optimal from P_i's standpoint given the rival's equilibrium behavior. A_i solves the following maximization problem in state $\theta = -\epsilon$,

$$\max_{p_i \geq 0} \mathbb{E}_{s_{-i}} \left[D_i \left(-\epsilon, p_i, p^N \left(-\epsilon, s_{-i} \right) \right) \left(p_i - c \right) \right].$$

The first-order condition yields the best-reply function

$$p_i \left(-\epsilon, s_i = \varnothing \right) = \frac{c}{2} + \frac{a - \epsilon + d \mathbb{E}_{s_{-i}} \left[p^N \left(-\epsilon, s_{-i} \right) \right]}{2 \left(1 + d \right)},$$

which pins down A_i's optimal price when it expects the rival to face the equilibrium permission set $\mathcal{P}^N$ and to set the price $p^N \left(-\epsilon, s_{-i} \right) < \overline{p}^N$.

Again, under passive beliefs off the equilibrium path, the following holds

$$\mathbb{E}_{s_{-i}} \left[p^N \left(-\epsilon, s_{-i} \right) \right] \triangleq \alpha \underbrace{\left[\frac{a - \epsilon}{2 \left(1 + d \right) - \alpha d} + \frac{d \left(1 - \alpha \right)}{2 \left(1 + d \right) - \alpha d} p^N \left(-\epsilon, s_i = \varnothing \right) \right]}_{= p^N \left(-\epsilon, s_i = 1 \right)} + \left(1 - \alpha \right) p^N \left(-\epsilon, s_{-i} = \varnothing \right).$$

Substituting this expectation into $p_i \left(-\epsilon, s_i = \varnothing \right)$, the equilibrium price $p^N \left(-\epsilon, s_i = \varnothing \right)$ solves

$$\begin{aligned} p^N \left(-\epsilon, s_i = \varnothing \right) = & \frac{a - \epsilon + c \left(1 + d \right)}{2 \left(1 + d \right)} + \frac{\left(1 - \alpha \right) d}{2 \left(1 + d \right)} p^N \left(-\epsilon, s_i = \varnothing \right) \\ & + \frac{\alpha d}{2 \left(1 + d \right)} \left(\frac{a - \epsilon}{2 \left(1 + d \right) - \alpha d} + \frac{d \left(1 - \alpha \right)}{2 \left(1 + d \right) - \alpha d} p^N \left(-\epsilon, s_i = \varnothing \right) \right), \end{aligned}$$

which yields

$$p^N \left(-\epsilon, s_i = \varnothing \right) = p^P \left(-\epsilon \right) + \frac{2 + d \left(2 - \alpha \right)}{2 \left(2 + d \right)} c.$$

To complete the characterization we just need to find $\overline{p}^N$ and then check that it is higher than $p^N \left(-\epsilon, s_i = \varnothing \right)$ and lower than that A_i's ideal price in state $\theta = \epsilon$ (so that the price cap actually binds). Since P_i's profit in state $\theta = -\epsilon$ does not depend on the cap $\overline{p}_i$ imposed in the high-demand state, this value solves the following maximization problem

$$\max_{\overline{p}_i \geq 0} \mathbb{E}_{s_{-i}} \left[D_i \left(\epsilon, \overline{p}_i, p^N \left(\epsilon, s_{-i} \right) \right) \overline{p}_i \right],$$

whose solution yields the following best-reply function

$$\overline{p}_i \left(\epsilon, s_i = \varnothing \right) = \frac{a + \epsilon + d \mathbb{E}_{s_{-i}} \left[p^N \left(\epsilon, s_{-i} \right) \right]}{2 \left(1 + d \right)},$$

where, under passive beliefs off the equilibrium path, it must be

$$\mathbb{E}_{s_{-i}}\left[p^N(\epsilon, s_{-i})\right] \triangleq \alpha\underbrace{\left(\frac{a+\epsilon}{2(1+d)-\alpha d}+\frac{d(1-\alpha)}{2(1+d)-\alpha d}\overline{p}^N\right)}_{=p^N(\epsilon, s_i=1)}+(1-\alpha)\overline{p}^N.$$

Hence, in equilibrium, $\overline{p}^N$ is solution of

$$\overline{p}^N=\frac{a+\epsilon}{2(1+d)}+\frac{d}{2(1+d)}\left[\alpha\left(\frac{a+\epsilon}{2(1+d)-\alpha d}+\frac{d(1-\alpha)}{2(1+d)-\alpha d}\overline{p}^N\right)+(1-\alpha)\overline{p}^N\right],$$

yielding $\overline{p}^N=p^P(\epsilon)$ — i.e., under partial delegation principals force their ideal price in state $\theta=\epsilon$.

It is easy to show that A's unconstrained price choice in state $\theta=\epsilon$ would violate the cap $\overline{p}^N$. Indeed, given the equilibrium strategies, if A_i could set its price without restrictions, it would charge

$$p_i(\epsilon, s_i=\varnothing)=\frac{c}{2}+\frac{a+\epsilon+d\mathbb{E}_{s_{-i}}\left[p^N(\theta, s_{-i})\right]}{2(1+d)}=p^P(\epsilon)+\frac{c}{2}>p^P(\epsilon).$$

Moreover, simple algebra shows that

$$0\leq\overline{p}^M-p^N(-\epsilon, s_i=\varnothing)=\frac{2\epsilon}{2+d}-\frac{2+d(2-\alpha)}{2(2+d)}c \qquad \Leftrightarrow \qquad c\leq\overline{c}_N\triangleq\frac{4\epsilon}{2+d(2-\alpha)}.$$

In the proof of Proposition 1, we show that the condition $c\leq\overline{c}_N$ is always satisfied in the region of parameters where the equilibrium outcome characterized above is immune to deviations by the principals.

Finally, the expected equilibrium price with no IS is

$$\begin{aligned} p^N &\triangleq \frac{\alpha}{2}\left(p^P(\epsilon)+p^N(-\epsilon, s_i=1)\right)+\frac{1-\alpha}{2}\left(p^P(\epsilon)+p^N(-\epsilon, s_i=\varnothing)\right) \\ &= \mathbb{E}\left[p^P(\theta)\right]+\underbrace{\frac{2+d(2+\alpha)}{4(2+d)}(1-\alpha)c}_{\text{Price distortion without IS}}, \end{aligned}$$

which concludes the characterization.

Proof of Proposition 1. We now derive the conditions under which a deviation to full pooling is not profitable. When retaining full price control, the deviating principal (say P_i) solves the following maximization problem

$$\max_{p\geq 0}\mathbb{E}_{s_{-i}}\left[D_i\left(\epsilon, p, p^N(\epsilon, s_{-i})\right)+D_i\left(-\epsilon, p, p^N(-\epsilon, s_{-i})\right)\right]p.$$

The first-order condition yields the singleton (deviation) price

$$\hat{p}^N=\mathbb{E}\left[p^P(\theta)\right]+\frac{d}{2(2+d)}(1-\alpha)c.$$

The expected profit associated with such a deviation is therefore

$$\begin{aligned}\hat{\pi}^N &\triangleq \frac{1}{2}\mathbb{E}_{s_{-i}}\left[D_i\left(\epsilon, \hat{p}^N, p^N(\epsilon, s_{-i})\right) + D_i\left(-\epsilon, \hat{p}^N, p^N(-\epsilon, s_{-i})\right)\right]\hat{p}^N \\ &= \frac{(1+d)(2a + cd(1-\alpha))^2}{4(2+d)^2}.\end{aligned}$$

P_i's equilibrium profit is instead

$$\begin{aligned}\pi^N(s_i = \varnothing) &\triangleq \frac{1}{2}\mathbb{E}_{s_{-i}}\left[D_i\left(\epsilon, p^N(\epsilon, s_i = \varnothing), p^N(\epsilon, s_{-i})\right) p^N(\epsilon, s_i = \varnothing)\right] \\ &+ \frac{1}{2}\mathbb{E}_{s_{-i}}\left[D_i\left(-\epsilon, p^N(-\epsilon, s_i = \varnothing), p^N(-\epsilon, s_{-i})\right) p^N(-\epsilon, s_i = \varnothing)\right] \\ &= (1+d)\left(\frac{a^2+\epsilon^2}{(2+d)^2} + \frac{(1-\alpha)(a-\epsilon)d}{(2+d)^2}c - \frac{(2+d\alpha)(2(1+d)-d\alpha)}{2(2+d)^2}c^2\right).\end{aligned}$$

Taking the difference between these expressions, we have

$$\begin{aligned}\pi^S(s_i = \varnothing) - \hat{\pi}^N &= (1+d)\frac{4\epsilon^2 - 4cd\epsilon(1-\alpha) + \left(d^2\alpha^2 - 2d^2\alpha - \left(d^2 + 8(1+d)\right)\right)c^2}{4(2+d)^2} \geq 0 \\ &\Leftrightarrow \qquad c \leq c_N \triangleq \frac{2\epsilon}{d(1-\alpha) + (2+d)\sqrt{2}}.\end{aligned}$$

Notice that

$$\overline{c}_N - c_N = \frac{2\epsilon(2(2\sqrt{2}-1) + (2\sqrt{2}-\alpha)d)}{((1-\alpha)d + (2+d)\sqrt{2})(2 + d(2-\alpha))} > 0.$$

Hence, whenever the equilibrium characterized above exists, it also features partial delegation.

Finally, following the logic of the proof of Lemma 1, showing that a deviation to full delegation is not profitable is immediate because in the high-demand state P_i has always an incentive to cap A_i's choice and that this cap is binding. Uniqueness, instead, follows from linearity of the best-reply functions. The rest of the proof can be obtained with straightforward algebra. ■

Partial delegation equilibrium with IS. We have already characterized in the benchmark with full information the (Nash) equilibrium of the pricing subgame in which principals are informed. Therefore, let us assume $\sigma = \varnothing$. Suppose that P_i offers a permission set $\mathcal{P}_i = [0, \overline{p}_i]$, with $\overline{p}_i$ being binding in state $\theta = \epsilon$ only. Given property **S2,** A_i solves the following maximization problem in state $\theta = -\epsilon$

$$\max_{p_i \geq 0} D_i\left(-\epsilon, p_i, p^S(-\epsilon, \sigma = \varnothing)\right)(p_i - c).$$

The first-order condition yields the best-reply function

$$p_i(\cdot) = \frac{c}{2} + \frac{a - \epsilon + dp^S(-\epsilon, \sigma = \varnothing)}{2(1+d)}.$$

Hence, in a symmetric equilibrium, the following holds

$$p^S(-\epsilon, \sigma = \varnothing) = p^P(-\epsilon) + \frac{1+d}{2+d}c, \quad \forall i = 1, 2.$$

To complete the characterization we just need to find the price cap $\overline{p}^S$. Again, since P_i's profit in the low-demand state does not depend on the cap $\overline{p}_i$ imposed in the high-demand state, the maximization problem that determines this value is

$$\max_{\overline{p}_i \geq 0} D_i\left(\epsilon, \overline{p}_i, \overline{p}^S\right)\overline{p}_i,$$

whose first-order condition yields the best-reply function

$$p_i(\cdot) = \frac{a+\epsilon}{2(1+d)} + \frac{d}{2(1+d)}\overline{p}^S.$$

Thus, in a symmetric equilibrium, it holds that $\overline{p}^S = p^P(\epsilon)$. Once again, principals force their ideal price in the high demand state, while agents are free to increase prices above the competitive level in the low demand state. Finally, it is easy to show that A's unconstrained choice in state $\theta = \epsilon$ would violate the cap $\overline{p}^S$. Indeed, it follows immediately that

$$p_i(\cdot) = \frac{c}{2} + \frac{a+\epsilon}{2(1+d)} + \frac{d}{2(1+d)}p^S(\theta, \sigma = \varnothing) = p^P(\epsilon) + \frac{c}{2} > p^P(\epsilon).$$

Moreover,

$$0 \leq \overline{p}^M - p^S(-\epsilon, \sigma = \varnothing) = \frac{2\epsilon}{2+d} - \frac{1+d}{2+d}c \qquad \Leftrightarrow \qquad c \leq \overline{c}_S \triangleq \frac{2\epsilon}{1+d}.$$

In the proof of Proposition 2, we show that the condition $c \leq \overline{c}_S$ is always satisfied in the region of parameters where the equilibrium outcome characterized above is immune to deviations by the principals.

Finally, the expected market price that each principal sets in the information sharing regime is

$$\begin{aligned} p^S &\triangleq \frac{1-(1-\alpha)^2}{2}\left(p^P(\epsilon) + p^P(-\epsilon)\right) + \frac{(1-\alpha)^2}{2}\left(p^P(\epsilon) + p^S(-\epsilon, \sigma = \varnothing)\right) \\ &= \mathbb{E}\left[p^P(\theta)\right] + \underbrace{\frac{(1-\alpha)^2(1+d)}{2(2+d)}}_{\text{Price distortion with IS}} c, \end{aligned}$$

which concludes the characterization.

Proof of Proposition 2. Following the logic of the proof of Proposition 1, Suppose that a principal, say P_i, deviates to a singleton (pooling) price. Such a deviation solves

$$\max_{p \geq 0}\left[D_i\left(\epsilon, p_i, p^S(\epsilon, \sigma = 0)\right) + D_i\left(-\epsilon, p_i, p^S(-\epsilon, \sigma = 0)\right)\right]p_i.$$

The first-order condition yields a deviation price

$$\hat{p}^S = \mathbb{E}\left[p^P(\theta)\right] + \frac{d}{4(2+d)}c,$$

so that the expected profit from such a deviation is

$$\begin{aligned}\hat{\pi}^S &\triangleq \frac{1}{2}\left[D_i\left(\epsilon, \hat{p}^S, p^S(\epsilon, \sigma = 0)\right) + D_i\left(-\epsilon, \hat{p}^S, p^S(-\epsilon, \sigma = 0)\right)\right]\hat{p}^S \\ &= \frac{(1+d)(4a+cd)^2}{16(d+2)^2}.\end{aligned}$$

The equilibrium profit, instead, is

$$\begin{aligned}\pi^S(\sigma = \varnothing) &\triangleq \frac{1}{2}D_i\left(\epsilon, p^S(\epsilon, \sigma = \varnothing), p^S(\epsilon, \sigma = \varnothing)\right)p^S(\epsilon, \sigma = \varnothing) \\ &\quad + \frac{1}{2}D_i\left(-\epsilon, p^S(-\epsilon, \sigma = \varnothing), p^S(-\epsilon, \sigma = \varnothing)\right)p^S(-\epsilon, \sigma = \varnothing) \\ &= \frac{(1+d)\left(a^2+\epsilon^2\right)}{(2+d)^2} + \frac{d(a-\epsilon)(1+d)}{2(2+d)^2}c - \frac{(1+d)^2}{2(2+d)^2}c^2.\end{aligned}$$

Comparing these expressions, we have

$$\begin{aligned}\pi^S(\sigma = \varnothing) - \hat{\pi}^S &= (1+d)\frac{4\epsilon^2 - \left(2d(4+d\alpha) + 8d^2\left(1-\alpha^2\right)\right)c^2 - 4d\epsilon(1-\alpha)c}{4(2+d)^2} \geq 0 \\ \Leftrightarrow &\qquad c \leq c_S \triangleq \frac{4(\sqrt{2}-1)\epsilon}{4-2\sqrt{2}+d}.\end{aligned}$$

Notice that

$$\bar{c}_S - c_S = \frac{2(3-2\sqrt{2})(2+d)\epsilon}{(d+2(2-\sqrt{2}))(d+1)} > 0.$$

Hence, whenever the equilibrium characterized above exists, it also features partial delegation.

Finally, following the logic of the proof of Lemma 1, showing that a deviation to full delegation is not profitable is immediate because in the high-demand state P_i has always an incentive to cap A_i's choice and that this cap is binding. Uniqueness, instead, follows from linearity of the best-reply functions. The rest of the proof follows from straightforward comparative statics. ■

Proof of Lemma 2. Taking the difference between c_N and c_S we have

$$c_N - c_S = -2\epsilon\frac{4+d-2\sqrt{2}-2d\alpha(\sqrt{2}-1)}{(4+d-2\sqrt{2})(d(1-\alpha)+(2+d)\sqrt{2})},$$

which is negative since

$$4+d-2\sqrt{2}-2d\alpha(\sqrt{2}-1) \geq 4+d-2\sqrt{2}-2d(\sqrt{2}-1) > 0. \qquad ■$$

Proof of Theorem 1. Assume $c < c_N$ so that in both IS regimes there exists a unique symmetric equilibrium with partial delegation. To begin with, notice that the difference between expected prices is

$$p^N - p^S = \frac{\alpha (1-\alpha)(2+3d)}{4(2+d)} c,$$

which is positive and features the properties stated in the proposition.

We can now turn to evaluate whether principals prefer to share information or not. Let

$$\begin{aligned} \pi^N \triangleq & \ \alpha \mathbb{E}_\theta \mathbb{E}_{s_{-i}} \left[D_i \left(\theta, p^N (\theta, s_i = 1), p^N (\theta, s_{-i}) \right) p^N (\theta, s_i = 1) \right] \\ & + (1-\alpha) \mathbb{E}_\theta \mathbb{E}_{s_{-i}} \left[D_i \left(\theta, p^N (\theta, s_i = \varnothing), p^N (\theta, s_{-i}) \right) p^N (\theta, s_i = \varnothing) \right], \end{aligned}$$

be a principal's ex-ante expected profit without IS and, by the same token, let

$$\begin{aligned} \pi^S \triangleq & \ (1-(1-\alpha)^2) \mathbb{E} \left[D_i \left(\theta, p^P (\theta), p^P (\theta) \right) p^P (\theta) \right] \\ & + (1-\alpha)^2 \mathbb{E} \left[D_i \left(\theta, p^S (\theta, \sigma = \varnothing), p^S (\theta, \sigma = \varnothing) \right) p^S (\theta, \sigma = \varnothing) \right], \end{aligned}$$

be a principal's ex-ante expected profit with IS.

Substituting the equilibrium values into π^N and π^S and taking the difference, we obtain

$$\Delta \pi \triangleq \pi^S - \pi^N = \frac{c\alpha (1-\alpha) \left[(4(1+d)^2 + d^2 \alpha (1+d\alpha))c - 2d(3d+2)(a-\epsilon) \right]}{8(2+d)^2}.$$

This expression is zero when $c = 0$ and/or when $\alpha \in \{0, 1\}$. When none of these cases is verified, solving $\Delta\pi = 0$ with respect to c, we have

$$\Delta \pi \geq 0 \quad \Leftrightarrow \quad c \geq c^\star \triangleq \frac{2d(2+3d)(a-\epsilon)}{4(1+d)^2 + \alpha d^2 (1+\alpha d)},$$

with $c^\star$ being decreasing in α. Notice that $\lim_{d \to 0} c^\star = 0$ while

$$\lim_{d \to 0} c_N = \frac{\epsilon}{\sqrt{2}} > 0.$$

As a result, there exists a threshold $\overline{d} > 0$ such that $c^\star < c_N$ for all $d < \overline{d}$, and $\Delta\pi \geq 0$ if and only if $c \in [c^\star, \underline{c}_N)$, which concludes the proof. ■

Proof of Corollary 1. The proof follows immediately from taking the difference $r^N - r^S$. ■