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The Catalytic Effect of Blended Finance

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Abstract

Development finance institutions mobilize over $250 billion annually through blended finance operations, yet practitioners lack a unified framework to evaluate its catalytic effect and for choosing among instruments. I develop a model of investment multipliers under two canonical market failures—production externalities and credit market imperfections—and two instruments: subsidized loans and credit guarantees. Three results emerge. First, the catalytic multiplier is decreasing in the severity of the market failure, creating a fundamental tension: interventions targeting the largest distortions achieve the lowest leverage. Second, the relative efficiency of guarantees and subsidized loans depends on the accounting convention used to measure cost. The guarantee and subsidized loan yield equal multipliers for pure de-risking and for production externalities; the guarantee achieves a higher multiplier for financial frictions and, in most configurations, for credit rationing. Third, for subsidized loans, non-de-risking interventions always yield higher multipliers than interventions that fully eliminate default risk. For guarantees the ranking depends on the nature of the market failure. A practical rule of thumb emerges from the analysis: production externalities call for subsidized loans, while financial frictions are best addressed with guarantees. The paper shows that even though blended finance is not effective when default risk is high, full de-risking is rarely optimal.

Keywords: Blended Finance; Catalytic Effect; Subsidized Loans; Credit Guarantees; Market Failures; Development Finance

JEL Codes: D62; G18; G32; H23; H41; Q01

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1 Introduction

The recognition that the Sustainable Development Goals cannot be financed through official development assistance alone led to the "Billions to Trillions" agenda: concessional public capital must function as a catalyst for private investment. Blended finance, the deliberate use of concessional funds to crowd in commercial capital for projects that markets would otherwise underprovide, is the operational expression of that agenda. Reflecting this shift, blended finance operations in developing and emerging economies have grown from roughly $60 billion to over $250 billion annually over the past decade.[1]

This expansion in blended finance operations has outpaced our analytical understanding. We know that blended finance has the potential to mobilize private capital; we know far less about when it outperforms a direct public subsidy, and less still about which instrument delivers the greatest leverage per dollar of fiscal cost. To address these questions, I take as given that a market failure justifies some form of public support and develop a simple model to evaluate the catalytic effect of different instruments. The evaluation criterion is deliberately narrow: the catalytic multiplier, defined as the increase in total project size per dollar of expected fiscal cost (throughout the paper I use the terms "multiplier" and "catalytic effect" interchangeably). This is not a welfare measure. A low multiplier does not imply that a project should be rejected: the social returns to closing even a small investment gap may be large (see Pegon, 2023, for a framework that centres on social returns rather than leverage ratios). The multiplier is, however, the metric that institutions use in practice to rank interventions, justify budgetary allocations, and report to donors.[2] This is why understanding its determinants has direct operational value.

A multiplier greater than one means that each dollar of public expenditure catalyzes more than one dollar of increase in total project size. A multiplier equal to one indicates that public funds do not mobilize additional private financing: the entire increase in project size is accounted for by the public contribution itself. A multiplier below one is worse still: the private investment mobilized by the intervention is lower than the fiscal cost of the intervention. To be clear, addressing market failures when the multiplier is at or below one may still be justified on welfare grounds, but such operations do not catalyze additional private finance.

I compare two canonical instruments: subsidized loans and credit guarantees, each bench-

[1] Data are from Convergence https://www.convergence.finance/blended-finance.

[2] According to the OECD, blended finance is evaluated principally by its ability to "mobilize additional commercial capital towards sustainable development" (OECD, 2025), and leverage ratios dominate the operational reporting of development finance institutions.

marked against a plain-vanilla direct subsidy of equal expected fiscal cost (this is the denominator of the multiplier). The direct subsidy serves as the natural baseline because it requires no financial intermediary, involves no delegation to banks, and is more transparent to monitor. Martin, Mayordomo, and Vanasco (2025) show that delegating guarantee allocation to banks allows them to extract rents and tilt allocations toward less productive, highly indebted firms, reinforcing the case for treating the direct subsidy as the benchmark.

For simplicity, I assume that interventions fully close the investment gap in targeted projects, bringing production to the social optimum. This assumption keeps the analysis tractable, but raises the question of whether results carry over to a portfolio setting in which an agency allocates a fixed budget across multiple projects. I show that they do: when the agency equalizes marginal multipliers across projects, the optimal allocation implies partial gap-closing rather than full financing of fewer projects, but the comparative statics on how multipliers vary with the severity of the market failure are unchanged. I also abstract from the fiscal cost of raising the funds used for subsidies or guarantees. This is not an innocuous assumption: even a multiplier greater than one does not guarantee that an intervention is welfare-improving if the required public expenditure is raised through distortionary taxation. However, the presence of distortionary taxes does not affect the analysis conditional on the intervention being worthwhile. If the agency has already determined that the social benefits outweigh the full costs, including the deadweight loss from taxation, then the comparison between a plain-vanilla subsidy, a subsidized loan, and a credit guarantee is unaffected by how the funds are raised.

The paper yields three results. The first concerns how the multiplier varies with the size of the market failure. I show that the catalytic multiplier is decreasing in the severity of the distortion. This finding creates a fundamental tension at the heart of blended finance: the interventions that are most needed—those targeting large externalities or severe financial frictions—are precisely those that achieve the lowest private capital leverage. The mechanism is straightforward: correcting a large distortion requires a proportionally large subsidy relative to the induced investment, compressing the ratio of additional private capital to public expenditure. An important corollary is that large reported multipliers are not, by themselves, evidence that blended finance is addressing serious market failures; they may instead reflect interventions in markets that are nearly efficient. The paper also shows that blended finance interventions are rarely effective (as they often yield multipliers below one) when default risk is high.

The second result concerns instrument choice. Under the accounting convention adopted in this paper, guarantees and subsidized loans carry equal expected fiscal costs for pure de-risking and for production externalities, so instrument choice in those cases rests on operational and institutional considerations. For financial frictions that lead to high interest rates, guarantees are cheaper because the friction has a component that persists in both default and no-default states. For credit rationing, guarantees are cheaper in most configurations.

The third result concerns the choice between addressing only the market failure and also eliminating default risk. I compare two intervention types: Type I, which targets only the distortions while leaving default risk intact, and Type II, which additionally absorbs default risk and brings production to the risk-free social optimum. For subsidized loans, Type I achieves a greater multiplier than Type II for all positive default probabilities: the higher cost of absorbing default risk always outweighs the larger investment response from eliminating default risk. For guarantees, the ranking depends on the nature of the market failure. When the distortion takes the form of a production externality or of a financial friction that leads to higher interest rates, a guarantee without full de-risking (Type I) achieves a greater multiplier for all practically relevant default probabilities. When the distortion is credit rationing, the guarantee with full de-risking (Type II) can have a greater multiplier when the probability of default is sufficiently high.

The three results together yield a practical rule of thumb for instrument choice. Production externalities are best addressed through subsidized loans. Financial frictions are instead often best addressed with guarantees. Full de-risking is rarely optimal as it is warranted only when credit rationing is the dominant friction and default risk is high. Taken together, these results suggest a simple decision hierarchy: diagnose the nature of the market failure first, match the instrument to the friction, and use the non-de-risking variant unless there is a specific case for absorbing default risk.

Two caveats are in order. The first concerns the type of market failure studied. I focus on production externalities and credit market imperfections because these are the most commonly cited rationales for blended finance. However, blended finance operations may also be justified on other grounds, including reducing agency problems in public investment, introducing private-sector discipline to limit corruption, or exploiting comparative advantages in project management. Each of these rationales requires a distinct analytical framework. The second caveat concerns the instrument set. I focus on subsidized loans and credit guarantees, but in practice blended finance also employs currency hedges, political risk insurance, technical assistance facilities, and performance-based incentives to address off-take risk, regulatory risk, and other non-credit distortions. The model's conclusions should be read with these limitations in mind.

Literature. This paper connects three strands of literature. The first is the theoretical literature on credit market intervention. The classic results of Stiglitz and Weiss (1981) and Gale (1990) establish that asymmetric information justifies credit market intervention but provide limited guidance on instrument design. Kim and Park (2020) show that guarantees are most effective when targeted toward high-risk but socially valuable borrowers; I complement their analysis by deriving closed-form multiplier expressions that characterize the efficiency ranking across instruments for any given combination of market failures. This paper is closely related to Flammer, Giroux, and Heal (2025, 2024), who provide the most systematic empirical analysis of blended finance operations to date. Their work documents the prevalence and variety of blending instruments; I provide a theoretical framework that can be helpful in interpreting their findings and ranking instruments.

The second strand is the empirical literature on mobilization and leverage. A growing body of work documents wide heterogeneity in mobilization effects across programs and institutional settings (Bachas and Yannelis, 2021; Akcigit, Ünal Seven, İbrahim Yarba, and Yılmaz, 2024; Broccolini, Lotti, Maffioli, Presbitero, and Stucchi, 2020). In this paper, I provide a theoretical benchmark for interpreting this heterogeneity: the model predicts that programs addressing smaller market failures should generate larger multipliers, and that guarantees should dominate subsidized loans in most configurations, the main exception being when the market failure takes the form of a production externality in a low-risk environment. The model also clarifies why reported leverage ratios can be misleading when computed at face value, as emphasized by cautionary assessments of mobilization measurement (World Bank Independent Evaluation Group, 2019; Kenny and Yang, 2020).

The third strand relates to the literature on instrument design in development finance. Fernández-Arias and Xu (2020) discuss how national development finance institutions should decide between loans and guarantees. The OECD (OECD, 2021, 2025) and IFC (IFC, 2021) have developed operational principles emphasizing minimum concessionality and financial additionality. This paper formalizes those principles: the requirement of minimum concessionality corresponds to finding the smallest subsidy consistent with the social optimum, and the additionality condition maps to whether the multiplier exceeds one.

Layout. The paper proceeds as follows. Section 2 analyzes the benchmark case of risk-free projects, establishing the key multiplier formulas for subsidized loans and guarantees under production externalities and financial frictions. Section 3 introduces default risk, characterizes Type I (without full de-risking) and Type II (with full de-risking) interventions, and shows how accounting conventions can affect the multiplier. Section 4 characterizes full de-risking interventions in an environment without market failures. Section 5 studies different types of interventions in settings characterized by default risk and market failures. Section 6 concludes.

2 Baseline with Risk-free Projects

Consider a project that has a social return which is higher than its private return. Specifically, a project of size K has a social return of λK^{α} and the social planner maximizes:

$$\lambda K^{\alpha} - RK \tag{1}$$

In Equation (1), $\lambda \geq 1$ is a production externality that creates a wedge between social and private returns (I do not consider $\lambda < 1$, which would indicate a negative externality), $0 < \alpha < 1$ is a productivity parameter that captures the presence of decreasing returns to scale (this assumption guarantees that the optimal project has finite size), and R is the gross risk-free interest rate ($R \equiv 1 + r$, where r is the risk-free rate).

The socially optimal size of the project matches the marginal social return to the cost of capital: $\alpha\lambda K^{\alpha-1} = R$. Solving for K gives the socially optimal project size:

$$K^* = \left(\frac{\lambda\alpha}{R}\right)^{\frac{1}{1-\alpha}} \tag{2}$$

The project is assumed to be risk-free throughout this section. In a perfect capital market, the entrepreneur should therefore be able to borrow at the risk-free interest rate r (default risk is introduced in Section 3).

I assume that there are two differences between private and social returns: (i) entrepreneurs do not internalize the production externality λ and (ii) because of financial frictions, even in the absence of default risk, the interest rate faced by entrepreneurs exceeds the risk-free rate (more on this in Section 5.2).

Financial frictions stem from various causes, the most common are inadequate contract enforcement, limited ability to post collateral, asymmetric information, lenders' excessive risk aversion, and insufficient competition in the banking system. Because of these financial frictions, I assume that the market gross interest rate is

$$\Psi = 1 + \delta + r \geq R \tag{3}$$

(for all practical purposes, this is equivalent to assuming a private sector hurdle rate greater than R). Private returns of an investment of size K are K^α and profits are given by:

$$K^\alpha - \Psi K$$

The entrepreneur maximizes profits by equalizing marginal private returns to the cost of capital: $\alpha K^{\alpha-1} = \Psi$. Solving for K gives the entrepreneur's desired production level:

$$K' = \left(\frac{\alpha}{\Psi}\right)^{\frac{1}{1-\alpha}}$$

To capture a second financial imperfection, I also assume that entrepreneurs have no internal funds and must borrow the full project cost K.[3] Entrepreneurs face a borrowing limit of $\theta K'$, where K' is their unconstrained credit demand and $0 \leq \theta \leq 1$.

While high borrowing costs and credit constraints are both manifestations of financial market imperfections, they operate through different channels. When $\delta > 0$, entrepreneurs face elevated interest rates but can borrow any desired amount at the market rate. Under credit rationing, the interest rate equals Ψ up to the limit $\theta K'$, beyond which additional credit is unavailable at any price: the effective marginal cost of borrowing becomes unbounded. Real-world market distortions often combine both features and, over some range, generate interest rates that increase with the loan-to-value ratio. The two frictions are kept separate here for analytical tractability.

In the presence of credit rationing, entrepreneurs would like to borrow $K' = \left(\frac{\alpha}{\Psi}\right)^{\frac{1}{1-\alpha}}$ (this is credit demand for a given interest rate), but banks will only lend $\theta K'$. The largest possible project is:

$$K^m = \theta \left(\frac{\alpha}{\Psi}\right)^{\frac{1}{1-\alpha}} \tag{4}$$

[3] This assumption is made for tractability; introducing entrepreneur equity would add a loan-to-value dimension to the credit constraint without changing the qualitative results.

A comparison of Equation (2) and Equation (4) clarifies that production externalities are an issue in the real economy and not a deficiency of the financial system. Even a fully efficient financial system ($\theta = 1$, and $\delta = 0$) would not lead to K^* when $\lambda > 1$.

In summary, there are three reasons why the market outcome delivers suboptimally low investment: (i) production externalities ($\lambda > 1$); (ii) high interest rates ($\delta > 0$); and (iii) credit constraints ($\theta < 1$). The first market failure operates through credit demand and the other two through credit supply. This is the minimal set of assumptions needed to study the two principal market failures that blended finance is designed to address.

2.1 The Catalytic Effect of Different Instruments

I now evaluate two possible blended finance interventions aimed at addressing the market failures described above: a subsidized loan and a credit guarantee.

For each market failure, I derive the perfectly targeted intervention (the intervention that brings production to the social optimum) under the assumption of no default risk and then measure its catalytic effect. The analysis assumes that the agency always fully closes the investment gap. Appendix A.1 shows that the key comparative statics carry over to a portfolio setting in which a fixed budget is allocated across multiple projects without fully closing any individual gap.

2.2 Subsidized Loans

I begin by deriving the interest rate subsidy that would move the market equilibrium to the socially optimal project size K^*.[4] With an interest rate subsidy of size s, private returns become $K^\alpha - \Psi K + sK$ and, if there are no credit constraints, project size is:

$$K^s = \left(\frac{\alpha}{\Psi - s} \right)^{\frac{1}{1-\alpha}}$$

[4]For tractability, this paper parameterizes all forms of loan concessionality as an equivalent interest rate reduction. In practice, concessional loans may embed subsidies through extended grace periods, reduced collateral requirements, longer maturities, or non-monetary technical assistance. Under standard discounting assumptions, each of these can be expressed as a reduction in the effective cost of capital, so the interest rate subsidy s captures the economic substance while abstracting from institutional variety.

In the presence of credit constraints, the entrepreneur can only borrow a fraction θ of the subsidized market outcome and project size is:

$$K^{ms} = \theta \left(\frac{\alpha}{\Psi - s} \right)^{\frac{1}{1-\alpha}} \tag{5}$$

Proposition 1 (Perfectly targeted subsidy)**.** *To derive the interest rate subsidy which leads to the socially optimal project size, I set $K^{ms} = K^*$ and solve for s. The perfectly targeted subsidy is:*

$$s^* = \Psi - \frac{R\theta^{1-\alpha}}{\lambda} \tag{6}$$

Moreover, s^* is increasing in λ and δ and decreasing in θ.

Proof. See Appendix A.2 □

Equation (6) verifies that the market equilibrium is equal to the social optimum (and $s = 0$) when there are no market failures (i.e., when $\theta = \lambda = 1$ and $\delta = 0$). Equation (6) also shows that if $\theta = 0$ the subsidy needs to be equal to the full gross interest rate Ψ. The same applies when λ is large (when λ goes to infinity s converges to Ψ). This is the standard result that there is no private provision of pure public goods (in this model, a pure public good corresponds to $\lambda = \infty$).[5]

The subsidy s is expressed as a share of total investment, so its monetary value is:

$$C^s = s^* K^* = \left(\Psi - \frac{R\theta^{1-\alpha}}{\lambda} \right) \left(\frac{\lambda\alpha}{R} \right)^{\frac{1}{1-\alpha}} \tag{7}$$

If the interest rate subsidy is chosen optimally, the multiplier of the subsidized loan is given by the ratio between the increase in project size linked to the subsidy (i.e., the level of additionality brought about by the subsidized loan) and the size of the subsidy:

$$M^S = \frac{K^* - K^m}{s^* K^*} = \frac{1 - \frac{K^m}{K^*}}{s^*} \tag{8}$$

The multiplier is not defined in the absence of market imperfections because without market

[5] The model approach taken here sets s^* to bring production to the social optimum K^*. Note that this is not the same as maximizing the catalytic multiplier: because the investment response is concave in fiscal cost, the average multiplier is highest at the smallest intervention and falls as the subsidy is increased toward s^*. The optimal subsidy thus maximizes investment while accepting a lower multiplier than would be achieved by a smaller, partial intervention.

failures, the optimal subsidy is zero. Using Equations (2), (4), and (6), we obtain:

$$M^S = \frac{1 - \theta \left(\frac{R}{\lambda \Psi}\right)^{\frac{1}{1-\alpha}}}{\Psi - \frac{R\theta^{1-\alpha}}{\lambda}} \tag{9}$$

A blended finance intervention has a catalytic effect if and only if this multiplier is greater than one.[6] Interventions with a multiplier equal to one are not effective in mobilizing private financial resources and interventions with a multiplier which is smaller than one are a waste of public or philanthropic resources because it would have been possible to obtain a larger effect with a grant with the same fiscal cost (alternatively, the same effect with a smaller grant).

I now consider one market failure at a time.

Production externalities. First, I focus on the case in which the only market failure is the production externality (i.e., $\lambda > 1$, $\delta = 0$, and $\theta = 1$). In this case, the catalytic effect is:

$$M^{S1} = \frac{1 - \left(\frac{1}{\lambda}\right)^{\frac{1}{1-\alpha}}}{R\left(1 - \frac{1}{\lambda}\right)} \tag{10}$$

Result 1 (Benchmark cases and monotonicity)**.** *The catalytic effect is decreasing in λ and converges to $\frac{1}{R}$ as λ goes to infinity. Hence, the catalytic effect reaches a minimum (and is smaller than one if $r > 0$) when production externalities are large. The catalytic effect is instead large when production externalities are positive but small. Formal proofs of monotonicity and the limiting behavior are provided in Appendix A.3.*

The top left panel of Figure 1 (panel (a)) illustrates the behavior of Equation (10) for three values of α and when $r = 5\%$. In all cases, the multiplier converges to $\frac{1}{R}$ as the externality λ increases. The figure also shows that, other things equal, higher values of α are associated with larger multipliers.

Credit rationing. Consider next the case in which the only market failure is credit rationing

[6] It is worth being precise about terminology. In the model, the multiplier measures the total increase in project size (ΔK) per dollar of fiscal cost (C). Because the project is privately owned, ΔK is sometimes referred to as "additional private investment." In the development finance literature, however, "private financing mobilized" often refers to the net commercial capital contributed ($\Delta K - C$). Under the model's definition, a multiplier greater than one implies $\Delta K > C$ but not necessarily $\Delta K - C > C$; the latter would require a multiplier greater than two. The subsequent analysis uses the $\Delta K/C$ definition throughout, consistent with standard mobilization reporting practice.

(i.e., $\lambda = 1$, $\delta = 0$, and $\theta < 1$). The catalytic effect is:

$$M^{S2} = \frac{1-\theta}{R(1-\theta^{1-\alpha})} \tag{11}$$

The credit-rationing multiplier likewise converges to $\frac{1}{R}$ when market failures are at their maximum ($\theta = 0$) and is larger when market failures are smaller (see panel (b) of Figure 1; note that the market failure is $1-\theta$, so the multiplier increases moving left on the x-axis). As in the production externality case, the multiplier is increasing in α.

Financial frictions. Finally, consider the case in which financial imperfections lead to high interest rates but there are no production externalities or credit rationing (i.e., $\delta > 0$ and $\lambda = \theta = 1$):

$$M^{S3} = \frac{1-\left(\frac{R}{\Psi}\right)^{\frac{1}{1-\alpha}}}{\Psi - R} = \frac{1-\left(\frac{1+r}{1+r+\delta}\right)^{\frac{1}{1-\alpha}}}{\delta} \tag{12}$$

As in the previous two cases, the catalytic effect is decreasing in the market imperfection. In this case, however, the multiplier goes to zero when δ goes to infinity. The top right panel of Figure 1 (panel (c)) shows that the multiplier becomes smaller than one when $\delta > 0.45$ and $\alpha = 0.4$ or when $\delta > 0.75$ and $\alpha = 0.6$. When financial frictions are pervasive, a subsidized loan has no catalytic effect and can lead to wasted resources.

Panel (d) of Figure 1 jointly considers a production externality and credit rationing. In the limit as θ approaches zero (extreme credit rationing), the multiplier converges to $1/R < 1$ for all values of $\lambda \geq 1$. For small positive values of θ, large production externalities (λ far from one) also push the multiplier below one.

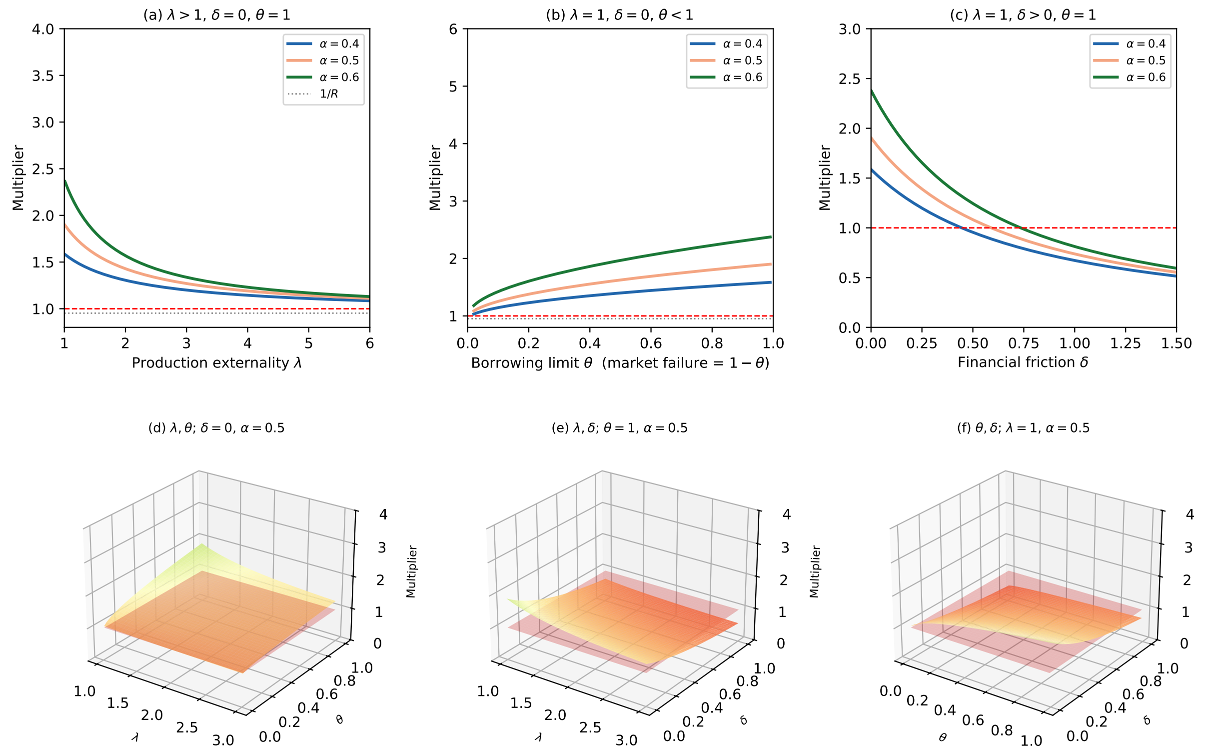
Panel (e) of Figure 1 considers a production externality when $\delta > 0$. Here too the catalytic effect reaches a minimum when both market failures are large. For the parameters illustrated in the figure, variations in credit market imperfections exert greater influence than changes in production externalities in determining the magnitude of the multiplier and there is a large range of parameters for which the multiplier is smaller than one.

Panel (f) of Figure 1 combines the two financial market failures and corroborates the previous findings that the multiplier reaches a minimum when credit market imperfections are large.

In sum, while subsidized loans are unnecessary in the absence of market failures, their effectiveness in catalyzing private capital decreases with the severity of the distortion. When financial frictions are large the multiplier can fall below one, indicating that a direct grant would

Figure 1: **Catalytic effect of a subsidized loan with risk-free projects**

Panels (a)–(c): multiplier as a function of the market failure parameter for $\alpha \in \{0.4, 0.5, 0.6\}$ and $R = 1.05$. Panel (a): production externality only ($\lambda > 1$, $\delta = 0$, $\theta = 1$); panel (b): credit rationing only ($\lambda = 1$, $\delta = 0$, $\theta < 1$; note that the market failure is $1 - \theta$, so the multiplier increases moving left on the x-axis); panel (c): financial friction only ($\delta > 0$, $\lambda = \theta = 1$). Panels (d)–(f): 3D surfaces with $\alpha = 0.5$; the red horizontal plane marks the multiplier $= 1$ threshold. Panel (d): joint variation in λ and θ (note that for any fixed $\theta > 0$, sufficiently large λ will drive the multiplier below one outside the range depicted here); panel (e): joint variation in λ and δ; panel (f): joint variation in θ and δ.



achieve the same investment outcome at lower fiscal cost.

2.3 Credit Guarantees

The analysis above showed that a subsidy is effective in dealing with production externalities but may yield small multipliers—even below one—in the presence of certain types of credit market failures. I now show that the picture is different for credit guarantees.

With a credit guarantee that fully eliminates both the borrowing spread (bringing Ψ down to R) and the credit constraint, the maximum project size is:

$$K' = \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}}$$

If the only market failure is the production externality, the interest rate is already R and the *credit guarantee is useless in addressing the production externality.*

If the market failure is instead only driven by credit market imperfections (i.e., $\lambda = 1$) and the guarantee $G = \gamma K$ (with $0 \leq \gamma \leq 1$) is set at a level that fully compensates for credit rationing (i.e., $\gamma = 1 - \theta$) and induces risk-averse financiers to lend at the risk-free interest rate, the guarantee allows reaching the socially optimal level of production.[7] Moreover, under the assumption that the project is indeed risk-free, the guarantee will never be drawn. Hence, the ex-post (and ex-ante, if we are certain that the probability of default is zero) cost of the guarantee is zero and the catalytic effect is unboundedly large: the guarantee mobilizes positive private investment at zero expected public cost.

The conclusion that one should always issue a guarantee when a project carries no true default risk but financiers perceive it as risky may appear trivial. However, it has meaningful practical implications. Arrow and Lind (1970) provides the classical argument for the higher risk-bearing capacity of the public sector, and empirical evidence supports the view that private lenders systematically overprice emerging-market risk relative to realized outcomes. The Global Emerging Markets Risk Database (GEMs Consortium, 2025)—drawing on over three decades of lending data from 29 multilateral development banks and development finance institutions—documents an average default rate of 3.54 percent for loans to private entities in emerging markets, comparable to non-investment-grade firms in advanced economies, alongside average

[7] Note that the partial guarantee $\gamma = 1 - \theta$ removes the quantity constraint associated with credit rationing, but does not eliminate the interest rate spread on the remaining unguaranteed fraction. If $\delta > 0$ full elimination of the spread requires $\gamma = 1$.

recovery rates of 72.9 percent that surpass global benchmarks including Moody's Global Loans (70 percent) and emerging market bonds (38 percent). Chari, Henry, Lee, and Mauro (2025) report analogous outperformance for infrastructure equity investments specifically. These findings are consistent with commercial lenders applying excessive risk premia to emerging market borrowers—precisely the friction under which guarantees generate the largest fiscal efficiency advantage over subsidies. Given the pervasiveness of this tendency to overestimate risk, the domain of parameter configurations in which guarantees generate high multipliers is considerably larger than one might initially assume. Of course default risk is never zero. Here I set it to zero to isolate the role of market failures; Section 5 shows that the main results carry over once default risk and its interaction with financial frictions are introduced.

Note, however, that if the wedge between the risk-free rate and the rate charged by banks ($\delta > 0$) is driven by lack of competition in the banking system, the guarantee could be ineffective (but will also have no cost if the project is truly free of default risk).

Unlike production externalities, credit market imperfections are best addressed with a credit guarantee rather than a subsidized loan, provided the project is risk-free. When both a production externality and a financial friction co-exist, the efficient strategy is to deploy a guarantee to reduce the interest rate to R and then apply a subsidy only to compensate for the remaining externality. Using a subsidy to address the financial friction directly is wasteful by comparison. Flammer, Giroux, and Heal (2024) document that the International Finance Corporation often combines different types of blending instruments in practice.

3 Projects with Default Risk

I now introduce default risk. A positive probability of default ($0 < \pi < 1$) leads to an interest rate $\rho > r$. This is not a market failure; it is simply the reflection that default risk is nonzero.

In the next two sections, I consider two types of interventions: Type I interventions address market failures without eliminating default risk, and Type II (or *de-risking*) interventions additionally absorb default risk. Consequently, I use two notations for the social optimum. The *risk-free social optimum*, $K_R^* \equiv (\lambda\alpha/R)^{1/(1-\alpha)}$, is identical to K^* defined in Section 2 and corresponds to the production target when default risk is either zero or fully absorbed by a de-risking intervention.[8] The *risky social optimum*, $K_H^* \equiv (\lambda\alpha/H)^{1/(1-\alpha)}$, is the socially optimal level of

[8] K_R^* is what a fully insured entrepreneur would choose and the institutional target of Type II blended finance operations. It is not a welfare benchmark: if default reflects genuine output loss, the expected social return

production when default risk is positive and left uncompensated. Type I interventions target K_H^*; Type II interventions target K_R^*. Since $H = R/(1-\pi) > R$, it follows that $K_H^* < K_R^*$.

Before deriving multipliers it is necessary to specify how the fiscal cost of each instrument is measured. The ranking between subsidies and guarantees depends on this choice. Two conventions are often in use: the *face-value convention* more prevalent in development finance institutions and the *probability-weighted expected-cost convention* which is standard in the economics and finance literature. Under the former, the cost of the subsidy is its full contracted obligation, provisioned regardless of whether default occurs. Under the latter, the subsidy's cost is probability-weighted—recognising that a rate-buydown subsidy flows through the borrower's income statement and disappears in default, so its true economic cost is discounted by the probability that it is actually paid.

To fix ideas, consider a project of size $K = 1$ with private returns equal to one when successful, a risk-free interest rate of zero, and a default probability of 50%. Under these assumptions, risk-neutral lenders will demand a 100% interest rate, and the project will not be realized. An agency wishing to see this project implemented through a blended finance operation has two options. First, it can provide a conditional interest rate subsidy—one that is paid only in the no-default state—sufficient to induce lending at the risk-free rate. The required subsidy rate is $s = \pi R/(1-\pi) = 1$. Second, it can provide a full guarantee, rendering the project risk-free from the lender's perspective; the expected cost of the guarantee is $\pi RK = 0.5$. The two instruments achieve the same investment outcome, so their relative efficiency depends entirely on how one measures the subsidy's cost. Under the expected-cost convention, the subsidy's cost is $(1-\pi)\,sK = 0.5$, equal to the guarantee's cost; both instruments yield a multiplier of 2. Under the face-value convention, the subsidy is provisioned at its full contracted obligation $sK = 1$; the subsidy multiplier is then 1 while the guarantee multiplier remains 2, so the guarantee dominates.

Throughout the paper I adopt the expected-cost convention, which is standard in the economics and finance literature (results under the face-value convention are derived in the Ap-

remains $(1-\pi)\lambda K^\alpha$, the welfare-maximizing optimum is K_H^*, and pushing production to K_R^* merely transfers physical risk to the public sector. I use K_R^* descriptively, as the target an agency pursues when it aims to eliminate default risk, without endorsing that objective on welfare grounds. Throughout the paper I follow the standard Arrow-Lind assumption that the public sector can bear project risk at zero cost, so the expected fiscal cost of the guarantee equals its expected payout with no additional risk premium. Under this assumption, the welfare cost of transferring default risk to the public sector is zero and the comparison between Type I and Type II instruments rests entirely on the multiplier. If instead the public sector faces a non-trivial cost of risk-bearing, Type II instruments would be relatively less attractive than the analysis suggests.

pendix). A further advantage of this convention is that it does not stack the cards against subsidized loans: the face-value convention disadvantages subsidies by provisioning them at their full contracted obligation while pricing guarantees at expected loss, whereas the expected-cost convention applies probability-weighting symmetrically to both instruments. Under this convention, the guarantee and the subsidized loan carry equal expected costs for pure de-risking (Section 4), so instrument choice in that case rests on operational considerations rather than on differences in catalytic efficiency. The equality persists when I introduce production externalities in Section 5. For financial frictions, however, the guarantee yields higher multipliers under both accounting conventions, because the friction has a component that raises the subsidy's expected cost above the guarantee's even after probability-weighting.[9]

4 No Market Failures

To clarify the relationship between default risk and the two instruments, I begin with the case in which there is default risk but no market failures. In this setting, the quantity $K^0 = (\alpha/R)^{1/(1-\alpha)}$ equals K_R^* evaluated at $\lambda = 1$.

With perfect capital markets and risk-neutral lenders, the risky interest rate ρ satisfies $\pi(1+\rho)m + (1-\pi)(1+\rho) = 1+r$, where $0 \leq m \leq 1$ is the recovery rate in default and $\pi(1-m)$ is the expected loss. For simplicity, I set the recovery rate to zero, so $(1-\pi)(1+\rho) = 1+r$.[10] Thus:

$$\rho = \frac{r+\pi}{1-\pi} \tag{13}$$

The corresponding gross interest rate is:

$$H = 1+\rho = \frac{1+r}{1-\pi} = \frac{R}{1-\pi} \tag{14}$$

and production is given by:

$$K' = \left(\frac{\alpha}{H}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha(1-\pi)}{R}\right)^{\frac{1}{1-\alpha}} \tag{15}$$

Suppose that providers of blended finance wish to fully de-risk the project and bring default risk to zero. This can be done with an interest rate subsidy or with a guarantee.

[9] This extends the result of Section 2 to a situation with default risk.

[10] The results are qualitatively similar for $m > 0$; the appropriate modification is to replace π with the expected loss rate $\Pi = \pi(1-m)$ throughout.

De-risking subsidy. An interest rate subsidy that eliminates default risk sets $1+\rho-s=1+r$, which yields $s=\pi R/(1-\pi)$ (see Appendix A.5). The cost of the subsidy is:

$$C^s = \frac{\pi R}{1-\pi}\left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}} \tag{16}$$

where $K^0 = \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}}$ is production without default risk. Under the expected-cost convention the multiplier is:

$$M^S = \frac{1-(1-\pi)^{\frac{1}{1-\alpha}}}{\pi R}$$

De-risking guarantee. The only guarantee that fully eliminates default risk is a full guarantee covering 100% of the loan value ($\gamma=1$; the derivation is in Appendix A.5). The cost of the guarantee is:

$$C^g = \pi R\left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}} \tag{17}$$

and the multiplier is:

$$M^G = \frac{1-(1-\pi)^{\frac{1}{1-\alpha}}}{\pi R}$$

Under probability-weighted expected-cost accounting, the expected cost of the subsidy is $(1-\pi)\,sK^0 = \pi R K^0 = C^g$. Since both instruments achieve the same increase in production and carry equal expected costs, they yield identical multipliers for pure de-risking. They are increasing in α and converge to $1/R$ as $\pi \to 1$. Under the face-value convention, instead, the guarantee is strictly cheaper and yields a higher multiplier for all $0<\pi<1$.[11]

5 Market Failures with Default Risk

I now reintroduce the three market failures discussed in Section 2: one affecting credit demand and two affecting credit supply. The demand-side market failure is the same production externality (λ) as before. The two credit supply failures are instead different because they interact financial frictions with default risk, though they reduce to those of Section 2 when $\pi=0$.

The first financial market imperfection ($\phi>0$) amplifies default risk through a direct effect on the interest rate. It has two components: a spread ϕ that the lender charges regardless of the default risk, and a multiplicative amplification of the default risk. Specifically, I set the

[11] All face-value convention results are derived in Appendix A.4. Figure A1 in the Appendix illustrates the multipliers under the face-value convention, under which the guarantee dominates throughout.

gross interest rate faced by the entrepreneur to:

$$\Phi = 1 + r + \phi + (1 + \phi)(\rho - r) = H(1 + \phi\pi) + \phi, \qquad \phi \geq 0 \tag{18}$$

where $H(1+\phi\pi)$ is the amplified risky rate and ϕ is the spread that persists even when default risk is zero (this is the same as in Section 2). Note that, at the cost of carrying one more parameter, I could have written the gross interest rate as $\Phi = 1 + r + \phi_1 + (1 + \phi_2)(\rho - r)$ with $\phi_1 \neq \phi_2$. The dominance of the guarantee over the subsidy derived in this section requires $\phi_1 > 0$ (when $\phi_1 = 0$, the friction is purely multiplicative and under expected-cost accounting the guarantee and subsidy carry equal expected costs). Other qualitative results are similarly robust as long as $\phi_1 > 0$ and $\phi_2 > 0$. For convenience, I define $\tilde{\Phi} \equiv R + \phi(1 + \pi r)$; one can verify that $\Phi = \tilde{\Phi}/(1 - \pi)$.[12]

The second market failure allows default risk to amplify credit rationing. Specifically, I assume that firms can only borrow a fraction $1 - (1 + \pi)\vartheta$ of the desired project size, where $\vartheta \geq 0$ and $\vartheta < 1/(1+\pi)$ ensures the constraint is non-trivial. As $\vartheta \to 1/(1+\pi)$, the borrowable fraction tends to zero and $K^m \to 0$: the credit constraint becomes so severe that no investment takes place, and the multiplier approaches zero, consistent with the general monotonicity result. This formulation combines a level-shift component ϑ, which (as in Section 2) binds even when $\pi = 0$, and a risk-amplifying component $\pi\vartheta$, which tightens as default risk rises.

The parameterization differs from Section 2 on conceptual grounds. There, financial imperfections were modeled as level-shift frictions because the project was risk-free: a constant spread δ raised the borrowing cost unconditionally, and a fixed parameter $\theta < 1$ capped credit independently of any default probability that did not exist. In a setting where default risk is the central object, it is natural to model financial imperfections as forces that interact with and amplify that risk.

Note that setting $\phi = \vartheta = 0$ recovers perfect credit markets with default risk, that is, the model of Section 3. Setting $\pi = 0$ instead recovers Section 2, isolating the role of financial frictions in a risk-free setting.[13]

[12] $\tilde{\Phi} = R+\phi(1+\pi r) = (1+r)+\phi(1+\pi r)$. Then $\Phi = 1+r+\phi+(1+\phi)(\rho-r) = (1+\phi)H-\phi r = (1+\phi)R/(1-\pi)-\phi r$. Multiplying out: $(1 + \phi)R - \phi r(1 - \pi) = R + \phi R - r\phi(1 - \pi) = R + \phi(R - r(1 - \pi)) = R + \phi(1 + r - r + r\pi) = R + \phi(1 + r\pi) = \tilde{\Phi}$. Hence $\Phi = \tilde{\Phi}/(1 - \pi)$.

[13] With $\pi = 0$ (consequently $\rho = r$), the two credit market imperfections become $\Phi = 1 + r + \phi$ and $1 - \vartheta$, which, setting $\phi = \delta$ and $\vartheta = 1 - \theta$, are identical to those of Section 2.

Given these assumptions, the risky social optimum is:

$$K_H^* = \left(\frac{\lambda\alpha}{H}\right)^{\frac{1}{1-\alpha}} \tag{19}$$

and the market outcome is:[14]

$$K^m = (1-(1+\pi)\vartheta)\left(\frac{\alpha(1-\pi)}{\tilde{\Phi}}\right)^{\frac{1}{1-\alpha}} \tag{20}$$

I now evaluate two types of subsidies and guarantees. Type I interventions only compensate for the market failures λ, ϕ, and ϑ, leaving default risk unaddressed; their target is the risky social optimum K_H^*. Type II interventions also absorb the default risk of the project, bringing production to the risk-free social optimum K_R^* (i.e., the same outcome as when firms borrow at rate R).

5.1 Subsidized Loans

I start with subsidies that only address market failures (Type I subsidies). Then, I study subsidies that also compensate for default risk (Type II subsidies).

Type I Subsidies. The perfectly targeted subsidy that addresses market failures without eliminating default risk (see Appendix A.6 for the derivation) is:

$$s_I = \frac{1}{1-\pi}\left(\tilde{\Phi} - \frac{R\,(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda}\right) \tag{21}$$

The subsidy is increasing in default risk (π) and in the market imperfections λ, ϕ, and ϑ. Using the definition of the catalytic effect (Equation (8)), we obtain:

$$M_I^S = \frac{(1-\pi)\left[1-(1-(1+\pi)\vartheta)\left(\frac{R}{\lambda\tilde{\Phi}}\right)^{\frac{1}{1-\alpha}}\right]}{\tilde{\Phi} - \frac{R\,(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda}} \tag{22}$$

Figure 2 shows the multiplier for different combinations of the parameters in Equation (22) when $R = 1.05$. When the probability of default is high (50% or higher) and the productivity parameter α is not very large (0.5 or lower), the multiplier is always smaller than one, indepen-

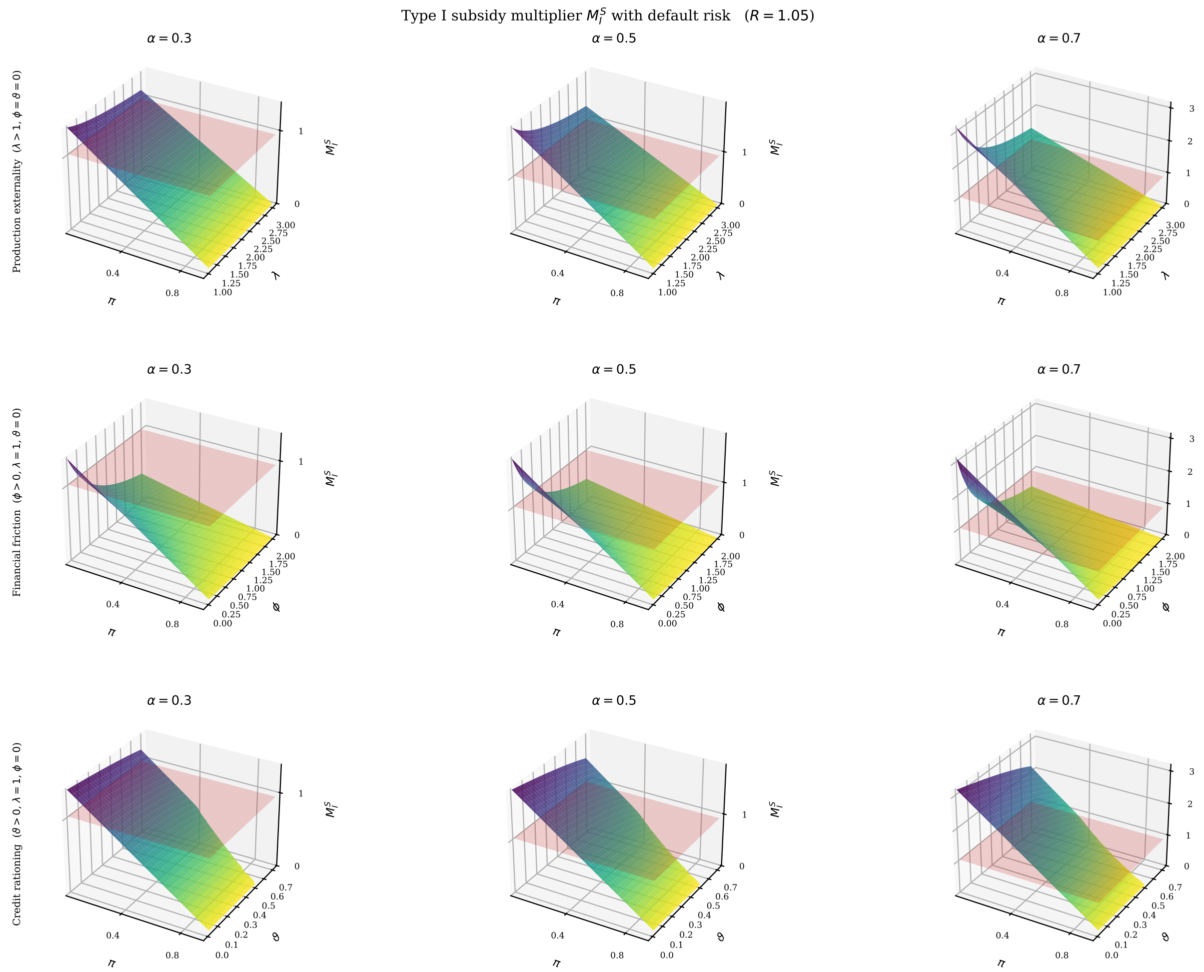
[14] To derive Equation (20), note that the entrepreneur facing interest rate Φ maximises $K^\alpha - \Phi K$, yielding unconstrained credit demand $\hat{K} = (\alpha/\Phi)^{1/(1-\alpha)}$. Using $\Phi = \tilde{\Phi}/(1-\pi)$ gives $\hat{K} = (\alpha(1-\pi)/\tilde{\Phi})^{1/(1-\alpha)}$. The borrowing constraint then caps the loan at $(1-(1+\pi)\vartheta)\hat{K}$, so $K^m = (1-(1+\pi)\vartheta)(\alpha(1-\pi)/\tilde{\Phi})^{1/(1-\alpha)}$.

dently of the levels of λ, ϕ, and ϑ. Subsidized loans are unlikely to have a catalytic effect in the presence of high default risk. When the probability of default is low ($\pi \leq 0.3$) and market imperfections are not very large (λ is close to one and ϕ and ϑ are close to zero), the multiplier becomes larger than one.

When default risk is high (above 70%), multipliers fall below one even for large values of the productivity parameter α, suggesting that subsidized loans are unlikely to have a catalytic effect in high-risk environments regardless of project productivity.

Figure 2: **Type I subsidy multiplier with default risk**

This figure plots the Type I subsidy multiplier with default risk (M_I^S, Equation (22)). Rows correspond to the three market failure types: row 1, production externality only ($\phi = \vartheta = 0$, $\lambda > 1$ on the y-axis); row 2, financial friction only ($\lambda = 1$, $\vartheta = 0$, $\phi > 0$ on the y-axis); row 3, credit rationing only ($\lambda = 1$, $\phi = 0$, $\vartheta > 0$ on the y-axis). Columns correspond to $\alpha \in \{0.3, 0.5, 0.7\}$. The front axis is the default probability $\pi \in [0.01, 0.95]$. The red horizontal plane marks the multiplier $= 1$ threshold. $R = 1.05$.



Closed-form expressions for the multiplier with one market failure at a time—$M_{I,\lambda}^S$, $M_{I,\phi}^S$, and $M_{I,\vartheta}^S$—are derived in Appendix A.7. In all cases the probability of default and the productivity parameter α are the key drivers. Figure A2 in the Appendix illustrates each case.

Type II Subsidies. The previous section showed that blended finance multipliers are small when default risk is high. One might therefore ask whether a subsidized loan that eliminates default risk could improve matters. The answer is no. The perfectly targeted subsidy that addresses market failures and fully compensates for default risk (derivation in Appendix A.6) is:

$$s_{II} = \frac{1}{1-\pi}\left(\tilde{\Phi} - \frac{R(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda}\right) \tag{23}$$

Since this subsidy compensates for both market failures and default risk, it exceeds s_I.[15] The multiplier is:

$$M_{II}^{S} = \frac{(1-\pi)\left[1-(1-(1+\pi)\vartheta)\left(\frac{R(1-\pi)}{\lambda\tilde{\Phi}}\right)^{\frac{1}{1-\alpha}}\right]}{\tilde{\Phi} - \frac{R(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda}} \tag{24}$$

We can now show that, as long as $\pi > 0$, the subsidy multiplier without full de-risking is always larger than the subsidy multiplier with full de-risking.

Proposition 2. *$M_I^S \geq M_{II}^S$ for all admissible parameter values such that $s_I > 0$, with equality if and only if $\pi = 0$.*[16]

Proof. See Appendix A.8. □

There are two competing forces: a Type II subsidy induces a larger increase in production (which raises the multiplier) but also carries a higher cost (which lowers it). Proposition 2 establishes that the cost effect always dominates. This result is driven by the concavity of the production function: because returns to reducing financing costs are diminishing, the no-de-risking subsidy, which operates over a shorter distance toward the optimum, is always more efficient per dollar spent. The Type II subsidy extends the intervention into regions of lower marginal product, pulling down the average multiplier over the entire interval from K^m to K_R^*. Appendix Figure A3 illustrates the ratio $\frac{M_I^S}{M_{II}^S}$ across a wide range of parameter values and confirms that the ratio is always above one.

5.2 Credit Guarantees

As in the case of subsidized loans, I now consider two types of credit guarantees: guarantees that only address market failures and guarantees that also compensate for default risk.

[15] The gap is $s_{II} - s_I = \frac{R(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda} \cdot \frac{\pi}{1-\pi} > 0$.

[16] The condition $s_I > 0$ is equivalent to $K^m < K_H^*$, i.e. the market outcome falling short of the risky social optimum. This holds whenever at least one market failure is present and $\pi < 1$, which covers all economically relevant cases considered in this paper.

Type I Credit Guarantees. To examine the perfectly targeted guarantee, I start with production externalities and then move to credit market failures.

Production externalities. As established in Section 2, a guarantee cannot address production externalities when there is no default risk. Does the presence of default risk change this? The answer is yes. To see why, recall that in the presence of default risk but perfect capital markets, entrepreneurs will set production at:

$$K' = \left(\frac{\alpha(1-\pi)}{R}\right)^{\frac{1}{1-\alpha}}$$

(see Equation (15)), while the risky social optimum with production externalities is:

$$K_H^* = \left(\frac{\lambda\alpha(1-\pi)}{R}\right)^{\frac{1}{1-\alpha}}$$

A guarantee can compensate for default risk and push production up to $K'' = \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}}$. However, note that:

$$\frac{K_H^*}{K''} = (\lambda(1-\pi))^{\frac{1}{1-\alpha}}$$

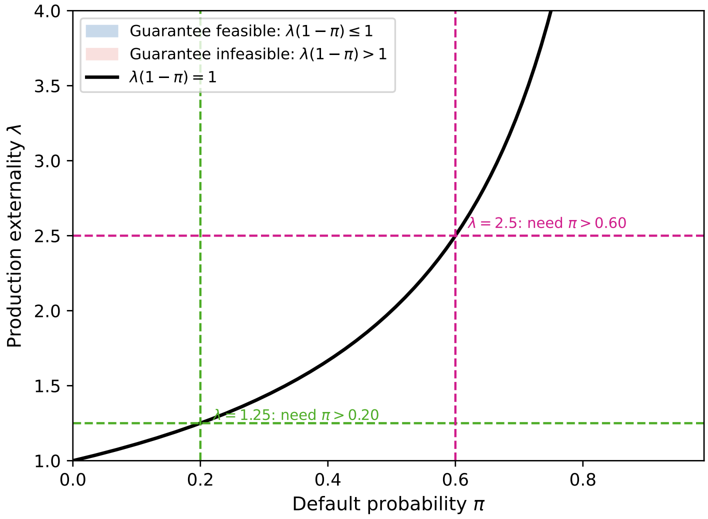
Therefore, if $\lambda(1-\pi) > 1$, the guarantee cannot fully compensate for production externalities even when it is set at its maximum level ($\gamma = 1$). Instead, if $\lambda(1-\pi) \leq 1$, a guarantee can move production to the risky social optimum K_H^*. Intuitively, this condition requires that the reduction in the interest rate achievable through a full guarantee ($\gamma = 1$) is large enough to compensate for the externality wedge—which in turn requires default risk to be sufficiently high relative to the externality. For instance, if $\lambda = 1.25$ a credit guarantee can lead to the social optimum as long as default risk is more than 20%. However, if $\lambda = 2.5$ a guarantee will be effective only if default risk is above 60% (see Figure 3).

If the condition $\lambda(1-\pi) \leq 1$ is satisfied, a perfectly targeted guarantee leads to a gross interest rate H' which sets:

$$K_H^* = \left(\frac{\lambda\alpha}{H}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha}{H'}\right)^{\frac{1}{1-\alpha}}$$

Figure 3: **Feasibility region for the Type I guarantee targeting a production externality**

In the blue shaded region ($\lambda(1-\pi) \leq 1$) a guarantee can fully close the gap to the risky social optimum K_H^*; in the red shaded region ($\lambda(1-\pi) > 1$) even a full guarantee ($\gamma = 1$) is insufficient. Dashed lines mark the specific examples discussed in the text: $\lambda = 1.25$ (requires $\pi > 0.20$) and $\lambda = 2.5$ (requires $\pi > 0.60$).



Then $H' = \frac{H}{\lambda}$ and the perfectly targeted guarantee is:[17]

$$\gamma = \frac{(1-\pi)(\lambda-1)}{\pi}$$

Note that we are working under the assumption that $\lambda(1-\pi) \leq 1$. Thus, $\gamma \leq 1$. The expected cost of this guarantee is computed as $\pi\gamma H' K_H^*$, where $H' = H/\lambda = R/(\lambda(1-\pi))$ is the risky interest rate consistent with production at K_H^*:[18]

$$C^g = \frac{R(\lambda-1)}{\lambda}\left(\frac{\lambda\alpha(1-\pi)}{R}\right)^{\frac{1}{1-\alpha}}$$

Under the assumption that $\lambda(1-\pi) \leq 1$, the multiplier is:

$$M_{I,\lambda}^G = \frac{K_H^* - K'}{C^g} = \frac{\lambda\left(1-\lambda^{-\frac{1}{1-\alpha}}\right)}{R(\lambda-1)}$$

As usual, the multiplier reaches a maximum when market failures are small (λ close to one). Notably, π cancels in the ratio: the guarantee multiplier for a production externality equals

[17] To see this, set $1+\rho' = \frac{1+\rho}{\lambda}$ (this is $H' = \frac{H}{\lambda}$) and use the interest parity condition: $\pi(1+\rho')\gamma + (1-\pi)(1+\rho') = (1-\pi)(1+\rho)$. Substitute the first equation into the second: $\pi(\frac{1+\rho}{\lambda})\gamma + (1-\pi)(\frac{1+\rho}{\lambda}) = (1-\pi)(1+\rho)$ and solve for γ.

[18] To see that $H' = R/(\lambda(1-\pi))$, note that $K_H^* = (\lambda\alpha/H)^{1/(1-\alpha)} = (\alpha/H')^{1/(1-\alpha)}$ requires $H' = H/\lambda = R/(\lambda(1-\pi))$.

exactly the risk-free subsidy benchmark M^{S1} from Equation (10).

As we are focusing on production externalities, we can compare the fiscal cost of the guarantee with that of the subsidy (both instruments bring production to K_H^*, so the cost comparison fully determines the multiplier ranking) by setting $\phi = \vartheta = 0$ in Equation (21). Under the expected-cost convention, both instruments carry equal expected costs and therefore yield equal multipliers.[19] The cost is the same, but subsidies are effective even when default risk is too low for a guarantee to be feasible. Thus, under the expected-cost convention, subsidies are no less efficient than guarantees for production externalities and are the more versatile instrument in this setting.

Financial frictions. I now turn to credit market imperfections, starting with $\phi > 0$. With the parameterisation $\Phi = H(1 + \phi\pi) + \phi$, the Type I guarantee must bring the borrower's effective rate down from Φ to H (the actuarially fair risky rate), which requires the lender's charged rate to fall by a precisely calibrated amount. The perfectly targeted guarantee is (see Appendix A.9):

$$\gamma_{I,\phi} = \frac{(1-\pi)\phi(1+\pi r)}{\pi\big(R + r(1-\pi)\phi\big)} \tag{25}$$

This guarantee is feasible (i.e. $\gamma \leq 1$) when $\phi \leq \pi R/(1-\pi)$, the same condition that ensures the friction ϕ does not exceed the default premium.[20] If this condition holds, the cost of the guarantee is:

$$C^g = \frac{\phi(1+\pi r)}{1+\phi} \left(\frac{(1-\pi)\alpha}{R} \right)^{\frac{1}{1-\alpha}} \tag{26}$$

The multiplier is:

$$M_{I,\phi}^G = \frac{(1+\phi)\left[1 - \left(\frac{R}{\tilde{\Phi}}\right)^{\frac{1}{1-\alpha}}\right]}{\phi(1+\pi r)} \tag{27}$$

The multiplier is decreasing in ϕ and in π. As shown in Appendix A.9, the guarantee is cheaper than the equivalent subsidy under both accounting conventions. Under the expected-cost convention, $C^g/C^s_{EL} = 1/(1+\phi) < 1$; under the face-value convention, $C^g/C^s = (1-\pi)/(1+\phi) < 1$. This convention-independent dominance reflects the nature of ϕ: the subsidy must compensate for the spread in every no-default period, whereas the guarantee is triggered

[19]Under the face-value convention, the guarantee dominates: $C^g/C^s = 1 - \pi < 1$ (see Appendix A.4).

[20]When $\phi > \pi R/(1-\pi)$, the friction exceeds the entire default premium and a Type I guarantee alone cannot restore the actuarially fair rate. Therefore, a Type II (de-risking) instrument or a hybrid guarantee-subsidy combination is required.

only in default.[21]

The guarantee's cost advantage under financial frictions maps conceptually to real-world first-loss mechanisms. A first-loss guarantee creates a discrete improvement in the risk-return profile for senior lenders, unlocking access to a much larger pool of capital at lower cost than an equivalent interest-rate subsidy. The guarantee directly removes downside risk from senior investors' perspective, creating a two-tier capital structure whose structural subordination—not just income support—is what enables it to mobilise private capital more efficiently. While these institutional features are not explicitly modelled here, the cost-advantage result provides a stylized theoretical foundation for this well-documented mechanism. Note that the standard Modigliani and Miller (1958) irrelevance result does not apply here because financial frictions drive a wedge between the costs of different financing instruments even when they achieve the same investment outcome.

Credit rationing. In the presence of credit rationing ($\vartheta > 0$), the maximum borrowable fraction is $1 - (1+\pi)\vartheta$. A perfectly targeted guarantee covers the constrained portion: $\gamma = (1+\pi)\vartheta$ (derivation in Appendix A.9). Its cost is:

$$C^g = \frac{\pi(1+\pi)\vartheta R}{1-\pi} \left(\frac{(1-\pi)\alpha}{R} \right)^{\frac{1}{1-\alpha}} \tag{28}$$

The multiplier is:

$$M^G_{I,\vartheta} = \frac{K^*_H - K^m}{C^g} = \frac{1-\pi}{\pi R} \tag{29}$$

The multiplier exceeds one if and only if $(1-\pi) > \pi R$, i.e. $\pi < 1/(1+R) = 1/(2+r)$: a guarantee loses its catalytic effect when the probability of default exceeds approximately 50%. Notably, the size of credit rationing ϑ is irrelevant for the multiplier, since both additionality ($K^*_H - K^m = (1+\pi)\vartheta K^*_H$) and cost ($C^g \propto \pi(1+\pi)\vartheta$) are proportional to $(1+\pi)\vartheta$, which cancels in the ratio.

This independence from ϑ is an exception to Result 1: for the Type I guarantee under credit rationing, the catalytic multiplier does not decline as the market failure becomes more severe. When the guarantee is perfectly targeted to relax the borrowing constraint, a larger rationing parameter ϑ simultaneously widens the investment gap (increasing additionality) and requires

[21] Under expected cost convention the guarantee and the subsidy would have had the same cost (and multiplier) if the financial friction had been modeled as $\Phi = 1 + r + (1+\phi)(\rho - r)$ instead of $\Phi = 1 + r + \phi + (1+\phi)(\rho - r)$. Under the face value convention the guarantee would have been cheaper under both modeling strategies.

a proportionally larger guarantee (increasing cost), so the two effects exactly offset. As a result, the multiplier $(1-\pi)/(\pi R)$ is governed entirely by default risk and carries no information about how binding the credit constraint is. This also means that, unlike other instruments, the Type I guarantee for credit rationing offers no additional leverage for addressing more severe rationing: the fiscal efficiency per dollar of investment gap is constant regardless of constraint severity. This independence result is specific to credit rationing as the sole market failure: when multiple failures co-exist (e.g., $\lambda > 1$ and $\vartheta > 0$), the guarantee must simultaneously address both distortions and the clean cancellation no longer obtains.

The cost comparison between the guarantee and the equivalent subsidy is ambiguous under the expected-cost convention. The ratio is (see Appendix A.10):

$$\frac{C^g}{C^s_{EL}} = \frac{\pi(1+\pi)\vartheta}{(1-\pi)\left(1-(1-(1+\pi)\vartheta)^{1-\alpha}\right)} \tag{30}$$

Two limiting cases clarify the conditions under which each instrument dominates. When rationing is mild ($\vartheta \to 0$), a first-order expansion gives $C^g/C^s_{EL} \approx \pi/[(1-\pi)(1-\alpha)]$, so the guarantee dominates if and only if $\pi < (1-\alpha)/(2-\alpha)$. This threshold is decreasing in α: it ranges from approximately 0.44 when $\alpha = 0.2$ to 0.17 when $\alpha = 0.8$, implying that with highly productive capital a subsidized loan is relatively efficient and the guarantee only dominates at low default probabilities. When rationing is severe ($\vartheta \to 1/(1+\pi)$), the ratio converges to $\pi/(1-\pi)$ and α drops out entirely: the guarantee dominates whenever $\pi < 0.5$, regardless of the productivity parameter. In general, one can show that C^g/C^s_{EL} is strictly decreasing in ϑ, so the guarantee becomes relatively cheaper as rationing becomes more binding. The subsidized loan is most likely to dominate when default risk is high, productivity is high, and credit rationing is mild. Figure 4 illustrates the dominance regions across the full parameter space.[22]

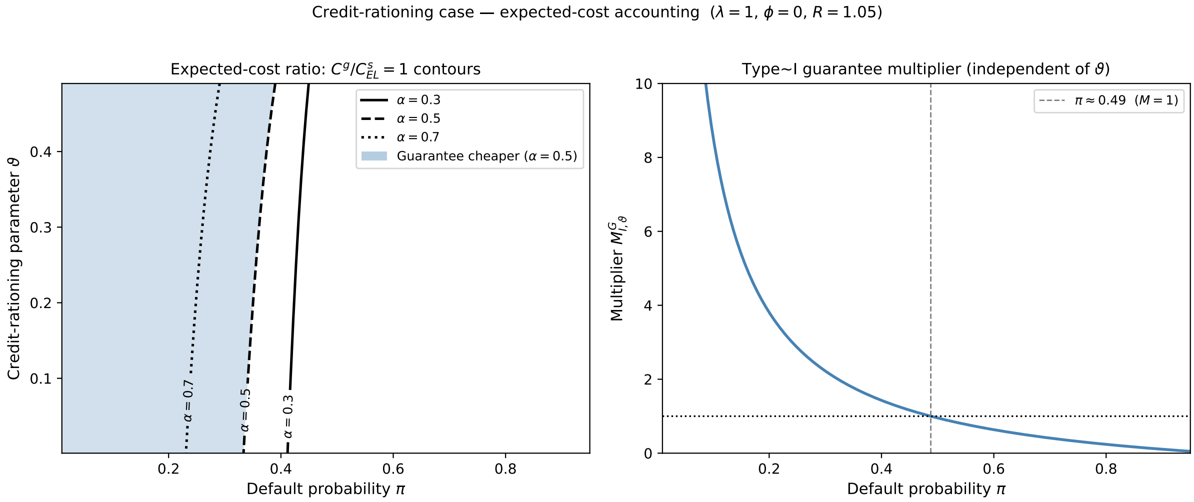
Type II Credit Guarantees. I now discuss credit guarantees that address market failures and eliminate (or reduce) default risk.

Production externalities. As shown for the Type I guarantee, a guarantee can address a production externality as long as default risk is large enough relative to the externality ($\lambda(1-\pi) \leq 1$). When this condition holds, the guarantee reduces borrowing costs sufficiently to compensate

[22] The face-value convention ratio, $C^g/C^s = \pi(1+\pi)\vartheta/(1-(1-(1+\pi)\vartheta)^{1-\alpha})$, equals the expected-cost ratio multiplied by $(1-\pi)$ and yields a qualitatively similar but less restrictive condition for guarantee dominance (see Appendix Figure A5).

Figure 4: **Credit-rationing case ($\lambda = 1$, $\phi = 0$, $R = 1.05$).**

Left panel: contours where the expected-cost ratio $C^g/C^s_{EL} = 1$ in (π, ϑ) space for $\alpha \in \{0.3, 0.5, 0.7\}$ (line styles), where $C^g/C^s_{EL} = \pi(1+\pi)\vartheta/[(1-\pi)(1-(1-(1+\pi)\vartheta)^{1-\alpha})]$; the blue shaded region shows where the guarantee is cheaper ($C^g < C^s_{EL}$) for $\alpha = 0.5$. Right panel: Type I guarantee multiplier $M^G_{I,\vartheta} = (1-\pi)/(\pi R)$, which does not depend on ϑ; the horizontal dotted line marks $M = 1$, crossed at $\pi \approx 0.49$. The corresponding face-value cost ratio is shown in Appendix Figure A5.



for the wedge between private and social returns, and it is cheaper than the equivalent subsidy because it is only triggered in default. This mechanism is no longer at work if the intervention also aims at eliminating default risk. Once default risk is brought to zero, the guarantee can no longer reduce interest rates and thus increase the demand for funds. We are back to the case without default risk where a guarantee is ineffective in addressing the production externality.

The formal argument is given in Appendix A.11. When both $\lambda > 1$ and $\pi > 0$, the required combination is: (i) a full guarantee ($\gamma = 1$) covering the entire loan K^*_R, which eliminates default risk and allows the borrower to face the risk-free rate R; and (ii) a subsidy $s = R(\lambda - 1)/\lambda$ that reduces the effective borrowing cost further and closes the gap to K^*_R. Writing $K^*_R = \lambda^{1/(1-\alpha)} K''$, the total expected fiscal cost of this combination is:

$$C^{\text{hybrid}} = \pi R \lambda^{\frac{1}{1-\alpha}} K'' + (1-\pi) \frac{R(\lambda - 1)}{\lambda} \lambda^{\frac{1}{1-\alpha}} K'' = R(\lambda - 1 + \pi) \lambda^{\frac{\alpha}{1-\alpha}} K''$$

The additionality is $K^*_R - K' = \left(\lambda^{1/(1-\alpha)} - (1-\pi)^{1/(1-\alpha)}\right) K''$. The multiplier is therefore:

$$M^{G+S}_{II,\lambda} = \frac{\lambda^{\frac{1}{1-\alpha}} - (1-\pi)^{\frac{1}{1-\alpha}}}{R(\lambda - 1 + \pi)\lambda^{\frac{\alpha}{1-\alpha}}}$$

This multiplier is decreasing in both π and λ, consistent with the general monotonicity result.

Financial frictions. For financial frictions ($\lambda = 1$, $\vartheta = 0$, $\phi > 0$), the Type II objective requires the guarantee to eliminate not only default risk but also the excess borrowing spread created by ϕ. Because ϕ has a component that persists even at zero default risk, eliminating it requires coverage exceeding 100% of the loan face value. The required coverage ratio is (see Appendix A.11):

$$\omega = 1 + \frac{\phi(1 + \pi r)}{\pi R}$$

Since $\phi > 0$, this always exceeds 1, indicating that a full guarantee is not enough. Reaching K_R^* in practice therefore requires pairing a full guarantee with a complementary interest-rate subsidy. The multiplier expressions should be read as characterising a theoretical composite instrument. Its multiplier is:

$$M_{II,\phi}^G = \frac{1 - \left(\frac{(1-\pi)R}{\tilde{\Phi}}\right)^{\frac{1}{1-\alpha}}}{\tilde{\Phi} - (1-\pi)R}$$

where $\tilde{\Phi} - (1 - \pi)R = \pi R + \phi(1 + \pi r)$. Under the expected-cost convention, the de-risking guarantee and the de-risking subsidized loan carry equal costs ($C^g/C^s_{EL} = 1$) and therefore yield identical multipliers (under the face-value convention, the guarantee dominates, see Appendix A.4).

For practically all default probabilities ($\pi \gtrsim 0.02$), the simpler non-de-risking Type I guarantee $M^G_{I,\phi}$ has a greater multiplier than the Type II de-risking guarantee: the Type II instrument bears a much higher cost because it must compensate for both default risk and the full friction ϕ, and the additional production gain from pushing all the way to the risk-free optimum K_R^* does not justify that extra cost. Appendix Figure A4 illustrates this comparison across a range of parameter values.

The requirement $\omega > 1$ may appear paradoxical: why must an agency subsidise a project even after fully eliminating its default risk? The answer connects directly to the message of Section 2. Financial market frictions are not simply a restatement of default risk, and they do not disappear once default risk is removed. In practice, private investors routinely demand returns above the risk-free rate for emerging-market projects even when those projects carry negligible default risk. Several distinct frictions sustain this premium. Illiquidity commands a separate price: emerging-market assets cannot be unwound quickly without significant cost, so investors require compensation for holding an asset whose value may be difficult to realise at short notice. Search and due-diligence costs are non-trivial for cross-border transactions, where identifying counterparties, verifying project fundamentals, and structuring agreements absorbs time and resources that must be recovered in the yield. Once committed, investors face ongoing monitoring and effort costs that are not eliminated by a guarantee on the underlying loan. Regulatory capital requirements assign positive risk weights to emerging-market exposures irrespective of their guaranteed status, raising the effective funding cost for institutional lenders independently of actual credit quality. Residual country risk—covering legal uncertainty, the enforceability of contracts, and the possibility of regulatory or political intervention—is not extinguished by a credit guarantee and constitutes a permanent wedge between the risk-free rate and the rate at which private capital will flow. Finally, institutional constraints such as investment-mandate restrictions, home bias, and limited partner agreements can exclude entire asset classes from institutional portfolios regardless of risk-adjusted returns. Taken together, these frictions imply that $\phi > 0$ is a realistic feature of emerging-market finance: full de-risking is not a sufficient condition for attracting private capital at the risk-free rate.

Credit rationing. In the presence of credit rationing ($\lambda = 1$, $\phi = 0$, $\vartheta > 0$) the Type II guarantee faces an additional complication. With the parameterisation $1 - (1+\pi)\vartheta$, a full de-risking guarantee ($\omega = 1$) eliminates the default-risk component of the constraint but leaves a residual rationing level of ϑ that persists even at zero default risk. Specifically, a full guarantee achieves production $(1-\vartheta)K_R^* < K_R^*$; reaching K_R^* requires an additional subsidy to relax the residual constraint. Since the guarantee covers the actual loan $(1-\vartheta)K_R^*$, its expected cost is $\pi R(1-\vartheta)K_R^*$, and the multiplier is:

$$M^G_{II,\vartheta} = \frac{(1-\vartheta) - (1-(1+\pi)\vartheta)(1-\pi)^{\frac{1}{1-\alpha}}}{\pi R(1-\vartheta)} \tag{31}$$

The multiplier is increasing in α and ϑ and decreasing in π. Under the expected-cost convention, the Type II guarantee always dominates the equivalent Type II subsidy.[23]

The comparison between the Type I and Type II guarantees for credit rationing is, however, not monotone and contrasts sharply with the financial friction case. Using Equations (29) and (31), the ratio of multipliers is (see Appendix A.11):

$$\frac{M^G_{II,\vartheta}}{M^G_{I,\vartheta}} = \frac{(1-\vartheta) - (1-(1+\pi)\vartheta)(1-\pi)^{\frac{1}{1-\alpha}}}{(1-\vartheta)(1-\pi)}$$

[23] $\frac{C^g}{C^s_{II,EL}} = \frac{\pi(1-\vartheta)}{1-(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha}} < 1$ because $1-(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha} > 1-(1-\pi) = \pi > \pi(1-\vartheta)$.

This ratio is increasing in π: the Type I guarantee has a higher multiplier when default risk is low, while the Type II de-risking guarantee has a higher multiplier when default risk is high. The intuition is straightforward. When π is low, credit rationing is the binding distortion and default risk plays a minor role. The Type I guarantee precisely targets the rationing constraint at low cost, and the extra expenditure required to also eliminate default risk does not yield a commensurate gain in investment. When π is high, default risk severely depresses investment and eliminating it unlocks a large additional production gain, making the de-risking instrument worthwhile despite its higher cost. Figure 5 illustrates this mechanism. The left panel shows both multipliers as functions of π for $\alpha = 0.5$ and three values of ϑ: $M^G_{I,\vartheta}$ is a single downward-sloping line (independent of ϑ), while $M^G_{II,\vartheta}$ shifts upward as ϑ rises. The two multipliers always cross, with the crossover moving to lower π as credit rationing becomes more severe. The right panel maps the crossover locus in (π, ϑ) space for different values of α: to the left of the locus the Type I guarantee has a higher multiplier; to the right the Type II guarantee has a higher multiplier.

This result stands in contrast to the financial friction case, where the Type I guarantee has a higher multiplier for practically all values of π. The difference in ranking arises from a structural asymmetry: for financial frictions, the Type II instrument requires coverage exceeding the loan face value because the friction ϕ has a permanent component that persists even at zero default risk, making the de-risking guarantee not feasible. For credit rationing, the Type II instrument is a standard full guarantee, so its cost is not structurally inflated in the same way, and the comparison is governed by the relative size of default risk and the rationing distortion.

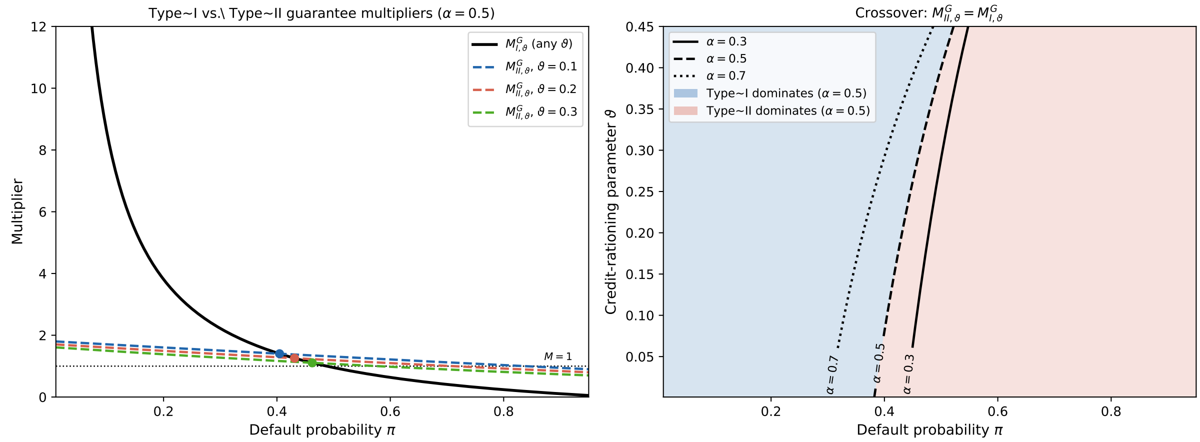
6 Conclusions

This paper develops a tractable framework for evaluating the catalytic effect of blended finance instruments. Taking as given that a market failure justifies public support, it asks how the multiplier—the increase in total project size per dollar of expected fiscal cost—varies with the nature and severity of the distortion, and which instrument delivers the greatest leverage. The framework considers two canonical instruments, subsidized loans and credit guarantees, across three market failures (production externalities, financial frictions, and credit rationing) and two intervention types (addressing only market failures, or also absorbing default risk).

The model identifies three sources of underinvestment: production externalities, which cause

Figure 5: **Type I vs. Type II guarantee multipliers under credit rationing ($\lambda = 1$, $\phi = 0$, $R = 1.05$).**

Left panel: $M^G_{I,\vartheta} = (1-\pi)/(\pi R)$ (solid black line, independent of ϑ) and $M^G_{II,\vartheta}$ (dashed coloured lines) as functions of π for $\alpha = 0.5$ and $\vartheta \in \{0.10, 0.20, 0.30\}$. Dots mark the crossover point where $M^G_{II,\vartheta} = M^G_{I,\vartheta}$. The dotted horizontal line marks the multiplier = 1 threshold. $M^G_{II,\vartheta}$ shifts upward as ϑ rises, reflecting the fact that higher credit rationing improves the additionality-to-cost ratio of the de-risking guarantee. Right panel: locus of crossover points ($M^G_{II,\vartheta} = M^G_{I,\vartheta}$) in (π, ϑ) space for $\alpha \in \{0.3, 0.5, 0.7\}$ (line styles). Blue shading: region where the Type I guarantee has a higher multiplier ($\alpha = 0.5$); red shading: region where the Type II guarantee has a higher multiplier ($\alpha = 0.5$). The locus shifts to the left as ϑ increases: higher credit rationing makes the de-risking guarantee relatively more attractive, reducing the default-probability threshold at which Type II begins to have a higher multiplier.



entrepreneurs to undervalue the social return of their projects; high interest rates stemming from financial frictions, which raise the cost of capital above the risk-adjusted rate; and credit rationing, which prevents entrepreneurs from borrowing the full amount needed to reach the social optimum. For each combination of market failures and default risk, the paper derives the perfectly targeted instrument and its catalytic multiplier.

The paper has three main results.

The multiplier is decreasing in the severity of the market failure. For all instruments except one, the catalytic effect declines as market failures become more severe. The exception is the Type I guarantee under credit rationing, whose multiplier $(1-\pi)/(\pi R)$ is independent of the severity of credit rationing ϑ (though it remains decreasing in default risk π). For all other instruments and market failures, the monotonicity result holds regardless of whether default risk is present or absent. This result, which survives the extension to a portfolio of projects, has a direct operational implication: large reported multipliers are more likely to reflect modest market failures than exceptional instrument design. Conversely, agencies operating in highly distorted environments should not expect large multipliers, and should not be penalized for reporting modest ones.

Guarantees and subsidized loans under two accounting conventions. The relative fiscal efficiency of guarantees and subsidized loans depends on the accounting convention used to measure instrument cost. Under probability-weighted expected-cost accounting (the convention adopted throughout this paper), a guarantee and a subsidized loan carry equal expected costs—and hence equal multipliers—for pure de-risking and for production externalities, because in both cases the subsidy flows through the borrower's income statement only in the no-default state and its expected cost equals the guarantee's. Under this convention, instrument choice for pure de-risking and production externalities should rest on operational and institutional considerations. For financial frictions, however, the permanent spread ϕ inflates the subsidy's expected cost above the guarantee's even after probability-weighting: the guarantee dominates under both conventions. For credit rationing, the guarantee is cheaper in most configurations under either convention.

Non-de-risking interventions often have higher multiplier than full de-risking ones. For subsidized loans, absorbing default risk makes the subsidy more expensive without a sufficient increase in production to compensate, because diminishing returns to reducing financing costs mean that the extra output from reaching the risk-free optimum is always worth less than the extra cost incurred. For guarantees, the ranking between instruments with and without de-risking is more nuanced and depends on the nature of the market failure. Under financial frictions that lead to high interest rates, the non-de-risking guarantee has a higher multiplier for all practically relevant default probabilities. Under credit rationing, the full de-risking guarantee can have a higher multiplier when default risk is high, because eliminating default risk then unlocks a sufficiently large increase in production.

The results of the paper can be distilled into a practical rule of thumb for development finance practitioners. The appropriate instrument depends on the nature of the market failure rather than on the size of the project or the level of default risk alone. When the primary distortion is a production externality, the appropriate instrument is a subsidized loan. The reason is not that subsidies are cheaper than guarantees in this setting; under the expected-cost convention the two instruments carry equal expected costs and yield equal multipliers when both are feasible. Rather, a guarantee is simply ineffective when the project has low risk because there is no interest rate premium to absorb. When default risk is present and large enough relative to the externality, a guarantee becomes feasible and at least as efficient, but the subsidy remains reliable across all risk environments.

When the distortion instead originates in the credit market—whether through a borrowing spread that inflates the cost of capital or through credit rationing that caps the scale of investment—a guarantee is the preferred instrument. For financial frictions with a permanent component, the guarantee dominates the subsidized loan under both accounting conventions, because the subsidy must compensate for the spread in every no-default period whereas the guarantee is triggered only in default. For credit rationing, the guarantee dominates in most configurations. However, full de-risking is rarely optimal. It is warranted only when credit rationing is the dominant friction and default risk is sufficiently high that eliminating it unlocks a large increase in production. The practical implication is straightforward: match the instrument to the market failure and use full de-risking for situations where credit rationing and high default risk coincide.

Several important caveats accompany these results. The multiplier is a narrow metric of fiscal efficiency and is not a welfare measure: an intervention with a multiplier below one may still be justified if the social return to closing the investment gap is large. At the same time, the model does provide clear guidance for cases where multipliers fall below one. When a blended finance instrument yields a multiplier below one, a direct grant of equal expected cost would achieve a larger increase in investment. In those configurations—which the model shows arise under high default risk, severe financial frictions, or aggressive de-risking through subsidized loans—the appropriate response is not to abandon intervention, but to reconsider whether blended finance is the optimal instrument.

Also note that the paper's qualitative results decompose into two groups. The first group is robust across functional forms: the negative relationship between the multiplier and the severity of the market failure; the strict dominance of the Type I subsidy over the Type II subsidy (Proposition 2); and the general multiplier–market-failure monotonicity all hold for any production function exhibiting diminishing returns. The second group depends on the specific parameterisation adopted here. The convention-independent dominance of the guarantee over the subsidy under financial frictions relies on the additive structure of $\tilde{\Phi}$, specifically the spread ϕ that survives even when default risk is zero; a purely multiplicative friction would restore equality under expected-cost accounting. The ϑ-independence of the Type I guarantee multiplier under credit rationing (equation (29)) depends on the linearity of the credit constraint, which makes additionality and cost proportional to the same factor $(1 + \pi)\vartheta$; a non-linear constraint would break this cancellation.

Finally, the analysis also abstracts from adverse selection and moral hazard in guarantee design, and from the administrative and institutional costs that differ across instruments. These considerations matter in practice and may shift the optimal instrument choice in ways the model does not capture.

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A Appendix

A.1 Portfolio of Projects and Robustness of the Monotonicity Result

Assume an agency with budget B that must allocate resources across a portfolio of N projects. For each project i, the market outcome satisfies $K_i^m < K_i^*$, where K_i^m denotes the market level of investment and K_i^* the socially optimal level. A fiscal intervention of size s_i raises investment to $\bar{K}_i(s_i)$, with

$$K_i^m \le \bar{K}_i(s_i) \le K_i^*,$$

with at least one inequality strict. Each project is also characterized by a market failure of size x_i (where x_i could be the externality λ_i, the financial friction ϕ_i, or the credit-rationing parameter ϑ_i).

The utility derived from allocating funds to project i is $U_i\big(\bar{K}_i(s_i) - K_i^m\big)$, where $U_i'(x) > 0$ and $U_i(0) = 0$. Utility functions may differ across projects. The agency's problem is:

$$\max_{\{s_i\}_{i=1}^N} \quad \sum_{i=1}^N U_i\big(\bar{K}_i(s_i) - K_i^m\big) \tag{32}$$

$$\text{s.t.} \quad \sum_{i=1}^N s_i \le B. \tag{33}$$

For any interior allocation $s_i > 0$, the first-order condition equates the marginal return across projects:

$$U_i'\big(\bar{K}_i(s_i) - K_i^m\big)\ \bar{K}_i'(s_i) = \mu \quad \forall\, i, \tag{34}$$

where μ is the shadow cost of the budget constraint. Condition (34) implies *partial gap-closing* in general: the agency allocates more resources to projects where the marginal investment response is highest, and stops short of fully closing any individual gap.

Define the *project-level multiplier* as

$$M_i = \frac{\bar{K}_i(s_i) - K_i^m}{s_i}, \tag{35}$$

which measures the investment mobilized per unit of fiscal cost in project i. The aggregate multiplier is the budget-share-weighted average:

$$M = \sum_{i=1}^N \frac{s_i M_i}{B} = \sum_{i=1}^N \frac{\bar{K}_i(s_i) - K_i^m}{B}. \tag{36}$$

Introducing a binding budget constraint therefore does not alter the definition of the multiplier; it only requires replacing the first-best target K_i^* with the intervention-induced level $\bar{K}_i(s_i)$.

Robustness of the monotonicity result. The main text derives multipliers under the assumption that the agency always fully closes the investment gap. Here I explore whether this assumption drives the result that the catalytic effect decreases in the size of the market failure (Result 1) and I show that it does not.[24]

In the model, $\bar{K}_i(s)$ is concave in s for all i, which follows from the decreasing-returns production function. The marginal multiplier $\bar{K}_i'(s_i)$ is therefore decreasing in s_i. The mechanism through which a larger market failure lowers the multiplier differs by friction type. For financial frictions and credit rationing, a larger imperfection (higher ϕ_i or ϑ_i) requires a larger subsidy to achieve any given production level, which is equivalent to shifting the $\bar{K}_i(s)$ schedule downward: the marginal dollar of fiscal cost induces a smaller increase in investment. Formally, $\partial^2 \bar{K}_i / \partial s_i \, \partial x_i < 0$ for these frictions. For a production externality, λ_i does not alter the private return or the subsidy required to reach any given production level; instead, a higher λ_i raises the socially optimal target K_i^*, requiring the agency to move further along the same concave $\bar{K}_i(s)$ curve. In both cases the result is the same: a larger distortion is associated with a lower average multiplier.

This cross-partial is precisely the source of Result 1: for a given fiscal expenditure, the investment response is smaller when the market failure is larger. Under the optimal portfolio allocation, the agency endogenously assigns a smaller budget share to low social return projects (since their marginal multiplier is lower) and a larger share to high social return projects. The ranking of marginal multipliers by market failure size therefore coincides with the ranking of average multipliers in the full-gap-closing case. Hence the qualitative conclusion survives the portfolio extension: large market failures are associated with low multipliers. The absolute magnitudes differ because partial gap-closing implies $\bar{K}_i(s_i) < K_i^*$, but the monotonicity result is unchanged.

[24] I would like to thank Eduardo Fernández-Arias for raising this point.

A.2 Proof of Proposition 1

Set $K^{ms} = K^*$ using (2)–(5):

$$\theta \left(\frac{\alpha}{\Psi - s} \right)^{\frac{1}{1-\alpha}} = \left(\frac{\lambda\alpha}{R} \right)^{\frac{1}{1-\alpha}}.$$

Raising both sides to $(1-\alpha)$ yields $\frac{\theta^{1-\alpha}\alpha}{\Psi - s} = \frac{\lambda\alpha}{R}$, so $\Psi - s = \frac{R\theta^{1-\alpha}}{\lambda}$ and (6) follows.

Comparative statics: $\partial s^*/\partial\lambda = R\theta^{1-\alpha}/\lambda^2 > 0$, $\partial s^*/\partial\delta = 1$, and $\partial s^*/\partial\theta = -(R/\lambda)(1-\alpha)\theta^{-\alpha} < 0$.[25]

A.3 Monotonicity proofs

A.3.1 Monotonicity of M^{S1} in λ

Recall

$$M^{S1}(\lambda) = \frac{1 - \lambda^{-p}}{R(1 - \lambda^{-1})}, \qquad p \equiv \frac{1}{1-\alpha} > 1.$$

Let $f(\lambda) = \frac{1-\lambda^{-p}}{1-\lambda^{-1}}$. Then

$$f'(\lambda) = \frac{p\lambda^{-p-1}(1 - \lambda^{-1}) - (1 - \lambda^{-p})\lambda^{-2}}{(1 - \lambda^{-1})^2}.$$

Multiply numerator by $\lambda^{p+2} > 0$:

$$\lambda^{p+2} \cdot \text{num} = p\lambda(1 - \lambda^{-1}) - (\lambda^{p+2} - \lambda^2)\lambda^{-2} = p(\lambda - 1) - (\lambda^p - 1).$$

Since $p > 1$ and $\lambda > 1$, convexity of λ^p implies $\lambda^p - 1 > p(\lambda - 1)$, hence the numerator is negative and $f'(\lambda) < 0$. Therefore M^{S1} is decreasing in λ.

A.3.2 Monotonicity of M^{S2} in the credit-rationing wedge

With $\lambda = 1$ and $\phi = 0$, the risk-free multiplier M^{S2} from Section 2 uses the borrowing fraction $\theta = 1 - \vartheta$. Substituting into $M^{S2} = \frac{1-\theta}{R(1-\theta^{1-\alpha})}$ gives $1 - \theta = \vartheta$ and $\theta^{1-\alpha} = (1-\vartheta)^{1-\alpha}$, so:

$$M^{S2}(\vartheta) = \frac{\vartheta}{R(1 - (1 - \vartheta)^{1-\alpha})}.$$

[25] The relationship between s and λ is concave ($\partial^2 s/\partial\lambda^2 < 0$), between s and θ is convex ($\partial^2 s/\partial\theta^2 > 0$), and between s and δ is linear.

Let $q \equiv 1 - \alpha \in (0,1)$ and $\theta = 1 - \vartheta$. Define $g(\theta) = \frac{1-\theta}{1-\theta^q}$. Then

$$g'(\theta) = \frac{-(1-\theta^q) - (1-\theta)(-q\theta^{q-1})}{(1-\theta^q)^2}.$$

The numerator equals $-1 + \theta^q + q\theta^{q-1}(1-\theta) = -1 + q\theta^{q-1} + (1-q)\theta^q$. As $\theta \to 0^+$, the term $q\theta^{q-1} \to +\infty$ (since $q - 1 < 0$), so the numerator diverges to $+\infty$. At $\theta = 1$, the numerator equals $-1 + q + (1-q) = 0$. Since the numerator is strictly decreasing on $(0,1)$ (its derivative is $q(q-1)\theta^{q-2}(1-\theta) < 0$) and equals zero at $\theta = 1$, it is strictly positive for all $\theta \in (0,1)$. Hence $g'(\theta) > 0$: M^{S2} decreases as θ falls, i.e. as ϑ rises (imperfection increases), consistent with Result 1.

A.3.3 Limit of M^{S1} as $\lambda \to \infty$

As $\lambda \to \infty$, $\lambda^{-p} \to 0$ and $\lambda^{-1} \to 0$, so $M^{S1} \to 1/R$.

A.4 Results under the Face-Value Convention

This appendix restates the main instrument-comparison results of Sections 4 and 5 under the *face-value convention*, in which the subsidy is provisioned at its full contracted obligation $C^s = sK^*$ rather than at expected loss $C^s_{EL} = (1-\pi)sK^*$.[26] Under this convention the guarantee's expected cost $C^g = \pi\gamma H'K^*$ is a fraction $(1-\pi)$ of the subsidy cost whenever both instruments achieve the same investment outcome. The face-value multiplier for the subsidy is therefore $(1-\pi)$ times the expected-cost multiplier; the guarantee multiplier is unchanged.

Pure de-risking (no market failures). The subsidy costs $s = \pi R/(1-\pi)$ per unit of capital, giving $C^s = \pi R/(1-\pi) \cdot K^0$. The guarantee costs $C^g = \pi R K^0$. Hence:

$$\frac{C^g}{C^s} = 1 - \pi < 1, \qquad \frac{M^G}{M^S} = \frac{1}{1-\pi} > 1.$$

Under the face-value convention, the guarantee strictly dominates for all $\pi \in (0,1)$.

Production externality ($\phi = \vartheta = 0$, $\lambda > 1$, $\lambda(1-\pi) \leq 1$). Setting $\phi = \vartheta = 0$ in s_I gives $C^s = R(\lambda-1)/(\lambda(1-\pi)) \cdot K^*_H$. The guarantee cost is $C^g = R(\lambda-1)/\lambda \cdot K^*_H$. Hence:

$$\frac{C^g}{C^s} = 1 - \pi < 1, \qquad M^G_{I,\lambda} > M^S_{I,\lambda}.$$

[26] The guarantee is always provisioned at expected loss.

The guarantee dominates the subsidized loan under the face-value convention. Under the expected-cost convention the two costs are equal and the instruments yield the same multiplier (as noted in the main text).

Financial friction ($\lambda = 1$, $\vartheta = 0$, $\phi > 0$). Under face-value: $C^g/C^s = (1-\pi)/(1+\phi) < 1$. Under expected-cost: $C^g/C^s_{EL} = 1/(1+\phi) < 1$. The guarantee dominates under both conventions.

Type I multiplier ratio:

$$\left.\frac{M^G_{I,\phi}}{M^S_{I,\phi}}\right|_{\text{FV}} = \frac{1+\phi}{1-\pi}.$$

Credit rationing ($\lambda = 1$, $\phi = 0$, $\vartheta > 0$). Under face-value: $C^g/C^s = \pi(1+\pi)\vartheta/(1-(1-(1+\pi)\vartheta)^{1-\alpha})$. Under expected-cost: $C^g/C^s_{EL} = \pi(1+\pi)\vartheta/[(1-\pi)(1-(1-(1+\pi)\vartheta)^{1-\alpha})]$, which equals the face-value ratio divided by $(1-\pi)$ and is therefore always larger. The condition for guarantee dominance is more restrictive under the expected-cost convention: the blue region of Figure 4 (main text) is smaller than that of Figure A5 (this appendix).

Figure A1 illustrates the multipliers under the face-value convention, under which the guarantee dominates throughout (under the expected-cost convention—the paper's primary convention—both curves coincide). When π is high the multiplier of either instrument is low, converging to $1/R$ as $\pi \to 1$. Under face-value accounting the subsidy multiplier falls below one when the probability of default is between approximately 40% and 65%, depending on α.

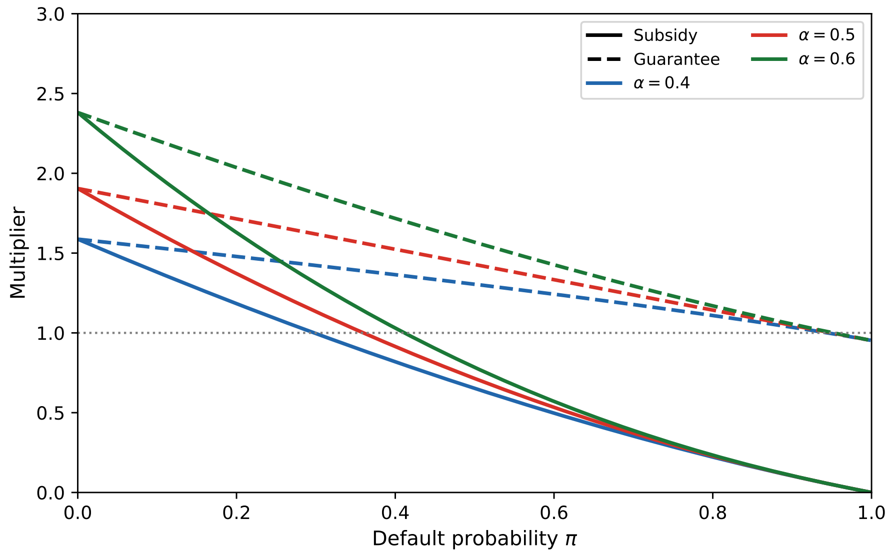
Summary table: cost ratios C^g/C^s under the two conventions.

Case	**Expected-cost** C^g/C^s_{EL}	**Face-value** C^g/C^s
Pure de-risking	1	$1-\pi$
Production externality	1	$1-\pi$
Financial friction	$1/(1+\phi) < 1$	$(1-\pi)/(1+\phi) < 1$
Credit rationing	$\pi(1+\pi)\vartheta/[(1-\pi)(1-(1-(1+\pi)\vartheta)^{1-\alpha})]$	$\pi(1+\pi)\vartheta/[1-(1-(1+\pi)\vartheta)^{1-\alpha}]$

The key takeaway is threefold. First, for pure de-risking and production externalities the convention drives the result: the guarantee dominates only under face-value accounting. Second, for financial frictions the guarantee dominates under both conventions, with the cost ratio C^g/C^s under face-value accounting being $(1-\pi)$ times as large as under expected-cost accounting, making the guarantee relatively cheaper by a factor of $1/(1-\pi)$. Third, for credit rationing the two cost ratios differ by a factor of $(1-\pi)$: the expected-cost ratio is larger (less favourable for

Figure A1: **De-risking multipliers for guarantee and subsidized loan**

The figure plots de-risking multipliers for guarantee (dashed lines) and subsidized loan (solid lines), as a function of the default probability π, for $\alpha \in \{0.4, 0.5, 0.6\}$. No market failures: $\lambda = 1$, $\phi = \vartheta = 0$, $R = 1.05$. Under expected-cost accounting (the main text convention) both multipliers are equal and converge to $1/R$ as $\pi \to 1$. Under face-value accounting the guarantee multiplier is above the subsidy multiplier; as $\pi \to 1$ the guarantee multiplier converges to $1/R$ while the subsidy multiplier converges to 0. The figure illustrates the face-value case.



the guarantee) because the expected-cost convention reduces the subsidy benchmark by $(1 - \pi)$ while leaving the guarantee cost unchanged.

A.5 Derivations for Section 4

De-risking subsidy. A subsidy s that induces risk-neutral behaviour sets $1 + \rho - s = 1 + r$, so $s = \rho - r$. Using Equation (13), we obtain:

$$s = \frac{\pi R}{1 - \pi}.$$

De-risking guarantee. A guarantee γ (the share of the loan recovered in default) that equalises risky and risk-free production satisfies:

$$\pi(1 + r)\gamma + (1 - \pi)R = (1 - \pi)(1 + \rho).$$

Solving with (13) gives $\gamma = 1$: full elimination of default risk requires a 100% guarantee.

A.6 Derivation of Type I and Type II Subsidy Formulas

Type I subsidy. Setting subsidized market production equal to the risky social optimum K_H^*:

$$(1-(1+\pi)\vartheta)\left(\frac{\alpha}{\Phi - s}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\lambda\alpha}{H}\right)^{\frac{1}{1-\alpha}}$$

where $\Phi = \tilde{\Phi}/(1-\pi)$. Raising both sides to the power $1-\alpha$:

$$(1-(1+\pi)\vartheta)^{1-\alpha} \cdot \frac{\alpha}{\Phi - s} = \frac{\lambda\alpha}{H}$$

Solving: $\Phi - s = \frac{H(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda}$, so:

$$s_I = \Phi - \frac{H(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda} = \frac{\tilde{\Phi}}{1-\pi} - \frac{R(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda(1-\pi)} = \frac{1}{1-\pi}\left(\tilde{\Phi} - \frac{R(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda}\right)$$

which yields Equation (21).

Type II subsidy. Setting subsidized market production equal to the risk-free social optimum $K_R^* = (\lambda\alpha/R)^{1/(1-\alpha)}$, and noting that the subsidy is set to achieve production at K_R^* rather than to equalise the borrower's effective financing cost to R (the actual effective cost after the subsidy is $\Phi - s = R(1-(1+\pi)\vartheta)^{1-\alpha}/\lambda < R$, as the equation that follows makes clear), the condition is:

$$(1-(1+\pi)\vartheta)\left(\frac{\alpha}{\Phi - s}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\lambda\alpha}{R}\right)^{\frac{1}{1-\alpha}}$$

Raising both sides to $1-\alpha$ and solving for s: $\Phi - s = R(1-(1+\pi)\vartheta)^{1-\alpha}/\lambda$, so:

$$s_{II} = \frac{\tilde{\Phi}}{1-\pi} - \frac{R(1-(1+\pi)\vartheta)^{1-\alpha}}{\lambda}$$

which is Equation (23). The gap $s_{II} - s_I = \pi R(1-(1+\pi)\vartheta)^{1-\alpha}/(\lambda(1-\pi)) > 0$ confirms that the Type II subsidy exceeds the Type I subsidy.

A.7 Type I Subsidy Multiplier: One Failure at a Time

Production externality only ($\vartheta = \phi = 0$). Setting $\vartheta = \phi = 0$ gives $\tilde{\Phi} = R$ and $(1-(1+\pi)\cdot 0) = 1$. Substituting into Equation (22):

$$M_{I,\lambda}^S = (1-\pi)\frac{\lambda\left(1-\lambda^{-\frac{1}{1-\alpha}}\right)}{R(\lambda-1)} = (1-\pi)M^{S1}$$

Default risk reduces the multiplier by the factor $(1-\pi)$. The multiplier goes to zero as $\pi \to 1$.

Financial friction only ($\lambda = 1$, $\vartheta = 0$, $\phi > 0$). With $\lambda = 1$, $\vartheta = 0$: $\tilde{\Phi} = R + \phi(1+\pi r)$. Substituting into Equation (22):

$$M^S_{I,\phi} = \frac{(1-\pi)\left[1-\left(\frac{R}{\tilde{\Phi}}\right)^{\frac{1}{1-\alpha}}\right]}{\tilde{\Phi}-R} = \frac{(1-\pi)\left[1-\left(\frac{R}{R+\phi(1+\pi r)}\right)^{\frac{1}{1-\alpha}}\right]}{\phi(1+\pi r)}$$

The multiplier goes to zero as $\pi \to 1$, and as $\phi \to 0$, $M^S_{I,\phi} \to (1-\pi)/((1-\alpha)R)$. The latter follows from a first-order Taylor expansion: for small $\varepsilon = \phi(1+\pi r)$, $(R/(R+\varepsilon))^{1/(1-\alpha)} \approx 1 - \varepsilon/((1-\alpha)R)$, so the numerator behaves as $(1-\pi)\varepsilon/((1-\alpha)R)$ and the limit follows on dividing by ε.

Credit rationing only ($\lambda = 1$, $\phi = 0$, $\vartheta > 0$). With $\lambda = 1$, $\phi = 0$: $\tilde{\Phi} = R$, $(1-(1+\pi)\vartheta)^{1-\alpha}/\lambda = (1-(1+\pi)\vartheta)^{1-\alpha}$. Substituting:

$$M^S_{I,\vartheta} = \frac{(1-\pi)(1+\pi)\vartheta}{R\,(1-(1-(1+\pi)\vartheta)^{1-\alpha})}$$

Again the multiplier goes to zero as $\pi \to 1$. The level-shift structure means that even a small $\vartheta > 0$ binds at $\pi = 0$.

A.8 Simplification of M^S_I/M^S_{II} and Proof of Proposition 2

Define $p = 1/(1-\alpha) > 1$, $q = 1-\pi$, $B = \tilde{\Phi}/R$, and $A = (1-(1+\pi)\vartheta)^{1-\alpha}/\lambda$. (Note: since $1-\alpha = 1/p$, we have $A = (1-(1+\pi)\vartheta)^{1/p}/\lambda$.) Setting $x \equiv A/B \in (0,1)$ (which follows from $s_I > 0$) and using $(1-(1+\pi)\vartheta) = (\lambda A)^p$, the two multiplier expressions reduce to:

$$M^S_I = \frac{q(1-x^p)}{R\cdot B(1-x)}, \qquad M^S_{II} = \frac{q(1-(xq)^p)}{R\cdot B(1-xq)},$$

so

$$\frac{M^S_I}{M^S_{II}} = \frac{(1-x^p)(1-xq)}{(1-x)(1-(xq)^p)} = \frac{g(x)}{g(xq)},$$

where $g(x) \equiv (1-x^p)/(1-x)$. Although the definitions of A and B differ from those in the risk-free version of the paper—there $B = 1+\delta/R$ and $A = \theta^{1/p}/\lambda$—the reduced form is identical because $\tilde{\Phi}/R$ and the credit-rationing term play exactly the same structural roles. The proof therefore carries through unchanged.

Proof of Proposition 2. It suffices to show that g is strictly increasing on $(0,1)$ for $p > 1$. Differentiating gives $g'(x) = h(x)/(1-x)^2$, where $h(x) \equiv 1 - px^{p-1} + (p-1)x^p$. We have $h(1) = 0$ and $h'(x) = p(p-1)x^{p-2}(x-1) < 0$ for $x \in (0,1)$, so h is strictly decreasing with $h(1) = 0$, implying $h(x) > 0$ throughout $(0,1)$. Hence $g'(x) > 0$. Since $q < 1$ implies $xq < x$, it follows that $g(x) > g(xq)$ and $M_I^S/M_{II}^S > 1$. The ratio equals one only when $\pi = 0$. □

A.9 Credit Guarantee Derivations

Type I guarantee for financial friction ($\phi > 0$). The guarantee γ must bring the borrower's effective rate from $\Phi = H(1+\phi\pi) + \phi$ down to H (the actuarially fair risky rate with no friction). With guarantee coverage γ, the lender breaks even at rate $1+\rho'$ satisfying $(1-\pi+\pi\gamma)(1+\rho') = R$. Setting the borrower's effective rate to H:

$$1 + r + \phi + (1+\phi)(\rho' - r) = H$$

gives $\rho' - r = [(\rho - r) - \phi]/(1+\phi)$, hence $1+\rho' = R + [(\rho-r)-\phi]/(1+\phi)$. Feasibility requires $\rho - r > \phi$, i.e. $\pi R/(1-\pi) > \phi$. Substituting $1+\rho'$ into the break-even condition and solving:

$$\gamma_{I,\phi} = \frac{(1-\pi)\phi(1+\pi r)}{\pi\big(R + r(1-\pi)\phi\big)} \tag{37}$$

The expected cost is $C^g = \pi\gamma_{I,\phi}(1+\rho')K_H^*$. After substitution and simplification:

$$C^g = \frac{\phi(1+\pi r)}{1+\phi}\,K_H^*$$

Under the face-value convention, the cost ratio with the Type I subsidy ($s_I = \phi(1+\pi r)/(1-\pi)$ at $\lambda = 1$, $\vartheta = 0$) is:

$$\frac{C^g}{C^s} = \frac{\phi(1+\pi r)/(1+\phi)}{\phi(1+\pi r)/(1-\pi)} = \frac{1-\pi}{1+\phi} < 1.$$

Under the expected-cost convention, $C_{EL}^s = (1-\pi)s_I K_H^* = \phi(1+\pi r)K_H^*$, so:

$$\frac{C^g}{C_{EL}^s} = \frac{1}{1+\phi} < 1.$$

The guarantee is always cheaper under *both* conventions for any $\phi > 0$.

Type I guarantee for credit rationing ($\vartheta > 0$). The maximum borrowable fraction is $1-(1+\pi)\vartheta$. The guarantee $\gamma = (1+\pi)\vartheta$ covers the constrained portion, restoring full access to the optimal loan size. With $\lambda = 1$, $\phi = 0$, the Type I subsidy is $s_I = R(1-(1-(1+\pi)\vartheta)^{1-\alpha})/(1-\pi)$, so the face-value cost ratio is:

$$\frac{C^g}{C^s} = \frac{\pi(1+\pi)\vartheta R/(1-\pi)}{R(1-(1-(1+\pi)\vartheta)^{1-\alpha})/(1-\pi)} = \frac{\pi(1+\pi)\vartheta}{1-(1-(1+\pi)\vartheta)^{1-\alpha}}.$$

Under the expected-cost convention, $C^s_{EL} = (1-\pi)s_I K^*_H = R(1-(1-(1+\pi)\vartheta)^{1-\alpha})K^*_H$, so:

$$\frac{C^g}{C^s_{EL}} = \frac{\pi(1+\pi)\vartheta/(1-\pi)}{1-(1-(1+\pi)\vartheta)^{1-\alpha}}.$$

The expected-cost ratio equals the face-value ratio divided by $(1-\pi)$ and is therefore always larger. Both ratios can be greater or less than one depending on ϑ, π, and α, as illustrated in Figures 4 and A5.

Type I guarantee for production externality ($\lambda > 1$). When $\lambda(1-\pi) \leq 1$, the perfectly targeted guarantee satisfies:

$$\gamma = \frac{(1-\pi)(\lambda-1)}{\pi}.$$

The expected cost, using $\pi\gamma H' K^*_H$ with $H' = R/(\lambda(1-\pi))$, is:

$$C^g = \pi \cdot \frac{(1-\pi)(\lambda-1)}{\pi} \cdot \frac{R}{\lambda(1-\pi)} \cdot K^*_H = \frac{R(\lambda-1)}{\lambda} K^*_H.$$

Under face-value: $C^g/C^s = 1-\pi < 1$. Under expected-cost: $C^g/C^s_{EL} = 1$ (both instruments carry equal expected costs for production externalities).

A.10 Cost Comparison: Type I Guarantee vs. Subsidized Loan under Credit Rationing

This appendix derives the cost ratio in Equation (30), establishes that it is strictly decreasing in the rationing parameter ϑ, and characterizes the two limiting cases discussed in the main text.

Setup. Under credit rationing only ($\lambda = 1$, $\phi = 0$, $\vartheta > 0$), both the Type I guarantee and the Type I subsidy bring production to K^*_H, so their multiplier ranking is determined entirely by their costs. Define the effective rationing parameter $u \equiv (1+\pi)\vartheta \in (0,1)$. The guarantee cost is (Equation (28)):

$$C^g = \frac{\pi u\, R}{1-\pi}\left(\frac{(1-\pi)\alpha}{R}\right)^{\frac{1}{1-\alpha}}$$

The expected cost of the subsidy is $C^s_{EL} = (1-\pi)s_I K^*_H$ with s_I evaluated at $\lambda = 1$, $\phi = 0$:

$$C^s_{EL} = R\left(1-(1-u)^{1-\alpha}\right)\left(\frac{(1-\pi)\alpha}{R}\right)^{\frac{1}{1-\alpha}}$$

Both C^g and C^s_{EL} share the factor $\left((1-\pi)\alpha/R\right)^{1/(1-\alpha)}$, which cancels entirely from the ratio, leaving Equation (30) unchanged. Dividing gives Equation (30):

$$\frac{C^g}{C^s_{EL}} = \frac{\pi u}{(1-\pi)(1-(1-u)^{1-\alpha})} \equiv \frac{\pi}{1-\pi}\,h(u)$$

where $h(u) \equiv u/\left(1-(1-u)^{1-\alpha}\right)$.

Monotonicity of h in u.

Lemma 1. *$h(u)$ is strictly decreasing on $(0,1)$ for any $\alpha \in (0,1)$.*

Proof. Differentiating, $h'(u) \propto g(u) \equiv \left(1-(1-u)^{1-\alpha}\right) - (1-\alpha)u(1-u)^{-\alpha}$. Substituting $v = 1-u$:

$$g = 1 - v^{1-\alpha} - (1-\alpha)(1-v)v^{-\alpha} = 1 - \alpha v^{1-\alpha} - (1-\alpha)v^{-\alpha}.$$

By the weighted AM–GM inequality with weights α and $1-\alpha$:

$$\alpha v^{1-\alpha} + (1-\alpha)v^{-\alpha} \;\geq\; v^{\alpha(1-\alpha)-(1-\alpha)\alpha} = v^0 = 1,$$

with equality only at $v = 1$ (i.e. $u = 0$). Hence $g \leq 0$ on $(0,1)$, with strict inequality for $u \in (0,1)$, so $h'(u) < 0$. □

Since h is strictly decreasing, C^g/C^s_{EL} is strictly decreasing in ϑ: the guarantee becomes relatively cheaper as rationing becomes more severe. Consequently, if the guarantee is cheaper than the subsidy at some ϑ_0, it is also cheaper for all $\vartheta > \vartheta_0$.

Limiting case 1: mild rationing ($u \to 0$). Applying a first-order Taylor expansion, $(1 - u)^{1-\alpha} \approx 1 - (1-\alpha)u$, so $1 - (1-u)^{1-\alpha} \approx (1-\alpha)u$ and:

$$h(u) \approx \frac{1}{1-\alpha}.$$

The guarantee dominates if and only if $\pi/(1-\pi) < 1 - \alpha$, i.e.:

$$\pi < \frac{1-\alpha}{2-\alpha}. \tag{38}$$

This threshold is strictly decreasing in α, ranging from $4/9 \approx 0.44$ at $\alpha = 0.2$ to $1/6 \approx 0.17$ at $\alpha = 0.8$. Higher productivity makes the subsidy relatively more efficient by generating a larger investment response, narrowing the range of default probabilities over which the guarantee dominates.

Limiting case 2: severe rationing ($u \to 1$). As $u \to 1$, $(1-u)^{1-\alpha} \to 0$, so $h(u) \to 1$ and:

$$\frac{C^g}{C^s_{EL}} \to \frac{\pi}{1-\pi}.$$

The guarantee dominates if and only if $\pi < 1/2$, independently of α. When the borrowing constraint is nearly binding, the productivity parameter ceases to affect the relative cost of the two instruments.

Summary. Combining the two limits and the monotonicity result: (i) for any α, the guarantee dominates at all levels of rationing whenever $\pi < (1-\alpha)/(2-\alpha)$ (the stricter mild-rationing threshold); (ii) for $\pi \in ((1-\alpha)/(2-\alpha), 1/2)$, the guarantee dominates for sufficiently severe rationing but not for mild rationing—there exists a unique crossing point $\vartheta^*(\pi, \alpha)$ below which the subsidy dominates; (iii) for $\pi \geq 1/2$, the subsidy dominates for all levels of rationing. The subsidized loan is therefore most likely to be the preferred instrument when default risk is high (π close to or above 1/2), productivity is high (large α), and rationing is mild.

A.11 Type II Guarantee Derivations

Production externality: formal proof that a de-risking guarantee cannot address $\lambda > 1$. When a de-risking guarantee eliminates default risk, the interest rate falls to R and production becomes $K'' = (\alpha/R)^{1/(1-\alpha)}$. The risk-free social optimum is $K^*_R = (\lambda\alpha/R)^{1/(1-\alpha)}$.

For the guarantee alone to reach K_R^*, we would need $K'' = K_R^*$, which requires $\lambda = 1$. Hence a de-risking guarantee cannot address a production externality. When both $\lambda > 1$ and $\pi > 0$, the required combination is a full guarantee ($\gamma = 1$) plus a subsidy $s = R(\lambda - 1)/\lambda$, which is exactly s^* from Equation (6) at $\delta = 0$ and $\theta = 1$.

Financial friction ($\phi > 0$): derivation of ω. The de-risking guarantee ω must bring the borrower's effective financing cost to R, which requires the lender to charge exactly R in the no-default state. With $\Phi = H(1 + \phi\pi) + \phi$, the lender's break-even at rate R gives:

$$\pi R\omega + (1 - \pi)R = (1 - \pi)\Phi$$

Solving: $\pi R\omega = (1 - \pi)(\Phi - R) = (1 - \pi)\big(\tilde{\Phi}/(1 - \pi) - R\big) = \tilde{\Phi} - R(1 - \pi)$, so:

$$\omega = 1 + \frac{\phi(1 + \pi r)}{\pi R}$$

Since $\phi > 0$, we have $\omega > 1$ for all π, $r > 0$.

The cost ratio with the Type II subsidy (Equation (23) with $\lambda = 1$, $\vartheta = 0$, giving $s_{II} = [\tilde{\Phi}/(1 - \pi)] - R = (\pi R + \phi(1 + \pi r))/(1 - \pi)$) under the face-value convention is:

$$\frac{C^g}{C^s} = \frac{\pi R\omega K_R^*}{s_{II} K_R^*} = \frac{\pi R\omega}{s_{II}} = \frac{\pi R + \phi(1 + \pi r)}{(\pi R + \phi(1 + \pi r))/(1 - \pi)} = 1 - \pi < 1,$$

so the de-risking guarantee dominates the de-risking subsidy under the face-value convention. Under the expected-cost convention, $C^s_{II,EL} = (1 - \pi)s_{II}K_R^* = (\pi R + \phi(1 + \pi r))K_R^*$, and since $\pi R\omega = \pi R + \phi(1 + \pi r)$ (from the derivation of ω above), we obtain:

$$\frac{C^g}{C^s_{II,EL}} = \frac{\pi R\omega}{\pi R + \phi(1 + \pi r)} = 1.$$

Under the expected-cost convention the two instruments are therefore equally expensive and yield identical multipliers.

Comparing the de-risking Type II guarantee with the non-de-risking Type I guarantee:

$$\frac{M^G_{II,\phi}}{M^G_{I,\phi}} = \frac{1 - \left(\frac{(1-\pi)R}{\tilde{\Phi}}\right)^{\frac{1}{1-\alpha}}}{[\tilde{\Phi} - (1 - \pi)R] \cdot \frac{(1+\phi)[1-(R/\tilde{\Phi})^{1/(1-\alpha)}]}{\phi(1+\pi r)}}.$$

Note that $\tilde{\Phi} - (1-\pi)R = \pi R + \phi(1+\pi r)$ and $\phi(1+\pi r) = \tilde{\Phi} - R$. For $\pi \gtrsim 0.02$, this ratio is strictly less than one, so $M^G_{I,\phi} > M^G_{II,\phi}$.

Credit rationing ($\vartheta > 0$): instrument comparisons. With the parameterisation $1-(1+\pi)\vartheta$, a full de-risking guarantee ($\omega = 1$) eliminates the default-risk component of the constraint but leaves a residual rationing ϑ active (the constraint at $\pi_{eff} = 0$ becomes $1-\vartheta$). Consequently, $\omega = 1$ achieves production $(1-\vartheta)K^*_R$, not K^*_R. The guarantee covers the actual loan $(1-\vartheta)K^*_R$, so its expected cost is $C^g = \pi R(1-\vartheta)K^*_R$. The multiplier is therefore:

$$M^G_{II,\vartheta} = \frac{(1-\vartheta)K^*_R - K^m}{\pi R(1-\vartheta)K^*_R} = \frac{(1-\vartheta) - (1-(1+\pi)\vartheta)(1-\pi)^{\frac{1}{1-\alpha}}}{\pi R(1-\vartheta)}$$

Comparing with the non-de-risking Type I guarantee (Equation (29)):

$$\frac{M^G_{II,\vartheta}}{M^G_{I,\vartheta}} = \frac{(1-\vartheta) - (1-(1+\pi)\vartheta)(1-\pi)^{\frac{1}{1-\alpha}}}{(1-\vartheta)(1-\pi)}.$$

This ratio is increasing in π: the de-risking guarantee has a higher multiplier when default risk is high; the Type I guarantee has a higher multiplier when default risk is low.

Comparing the Type II guarantee with the Type II subsidy, the Type II subsidy cost is $s_{II}K^*_R$ with $s_{II} = R(1-(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha})/(1-\pi)$ (Equation (23) with $\lambda = 1$, $\phi = 0$). Under the face-value convention:

$$\frac{C^g}{C^s_{II}} = \frac{\pi R(1-\vartheta)K^*_R}{s_{II}K^*_R} = \frac{\pi(1-\vartheta)(1-\pi)}{1-(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha}},$$

which can be greater or less than one. Under the expected-cost convention, $C^s_{II,EL} = (1-\pi)s_{II}K^*_R = R(1-(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha})K^*_R$, giving:

$$\frac{C^g}{C^s_{II,EL}} = \frac{\pi(1-\vartheta)}{1-(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha}} < 1,$$

since $(1-(1+\pi)\vartheta)^{1-\alpha} < 1$ implies $1-(1-\pi)(1-(1+\pi)\vartheta)^{1-\alpha} > 1-(1-\pi) = \pi > \pi(1-\vartheta)$. The Type II guarantee therefore always dominates the Type II subsidy under the expected-cost convention, while the comparison is ambiguous under face-value accounting.

Figure A2: **Type I subsidy multiplier with default risk, one imperfection at a time**

Rows: production externality ($\lambda > 1$, $\phi = \vartheta = 0$); financial friction ($\phi > 0$, $\lambda = 1$, $\vartheta = 0$); credit rationing ($\vartheta > 0$, $\lambda = 1$, $\phi = 0$). Columns: $\alpha \in \{0.2, 0.5, 0.8\}$. The front axis is the default probability π; the side axis is the market failure parameter. The red horizontal plane marks the multiplier $= 1$ threshold. $R = 1.05$.

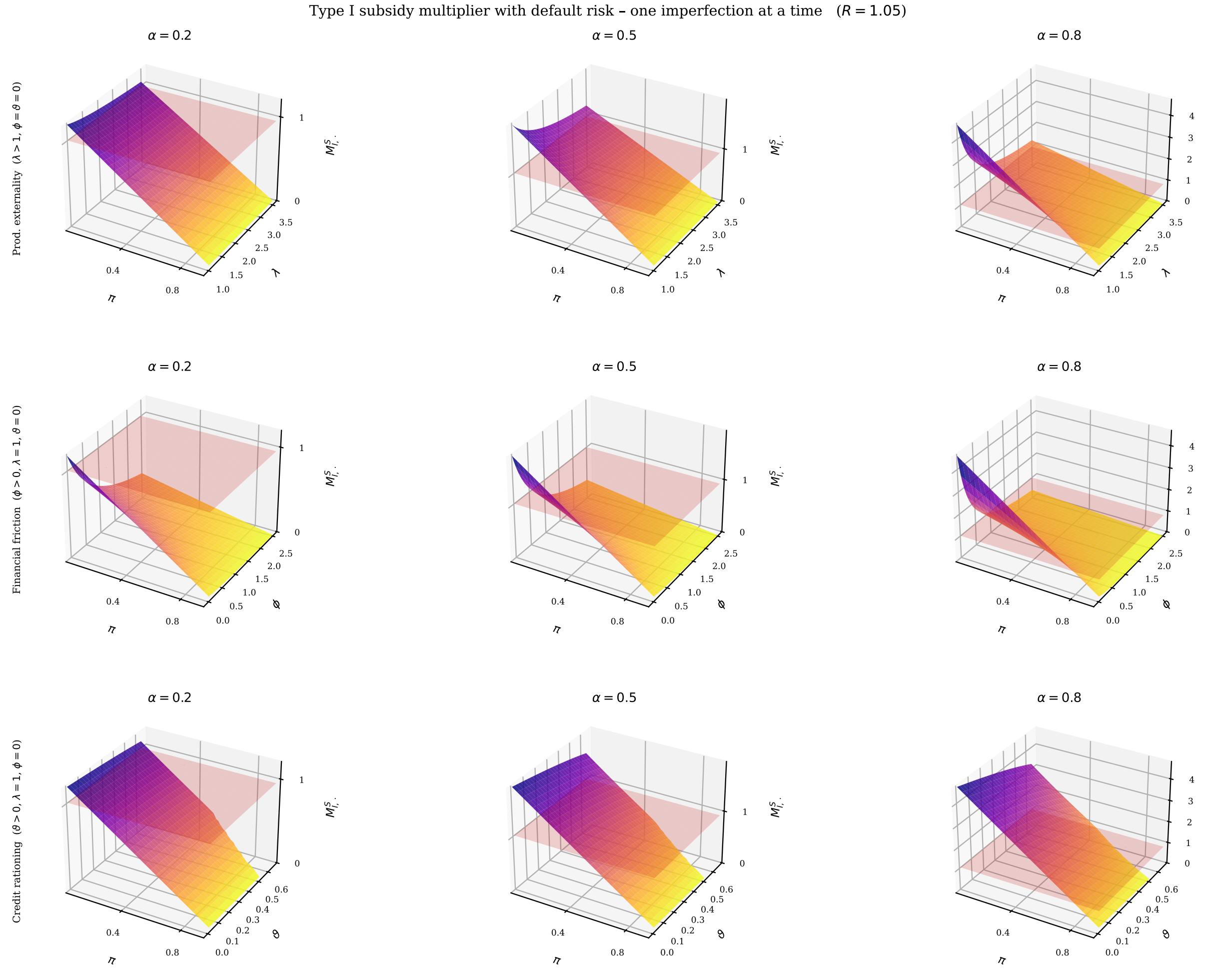


Figure A3: **Ratio of subsidy multipliers without and with de-risking**

This figure plots M_I^S/M_{II}^S. By Proposition 2, the ratio is always strictly greater than 1, confirming that the Type I (no de-risking) subsidy always dominates the Type II (de-risking) subsidy. Rows vary $\alpha \in \{0.3, 0.5, 0.7\}$; columns vary the financial friction ϕ and credit rationing ϑ. The λ and π parameters vary along the two horizontal axes. The black horizontal planes mark the ratio $= 0$ and ratio $= 1$ thresholds. $R = 1.05$.

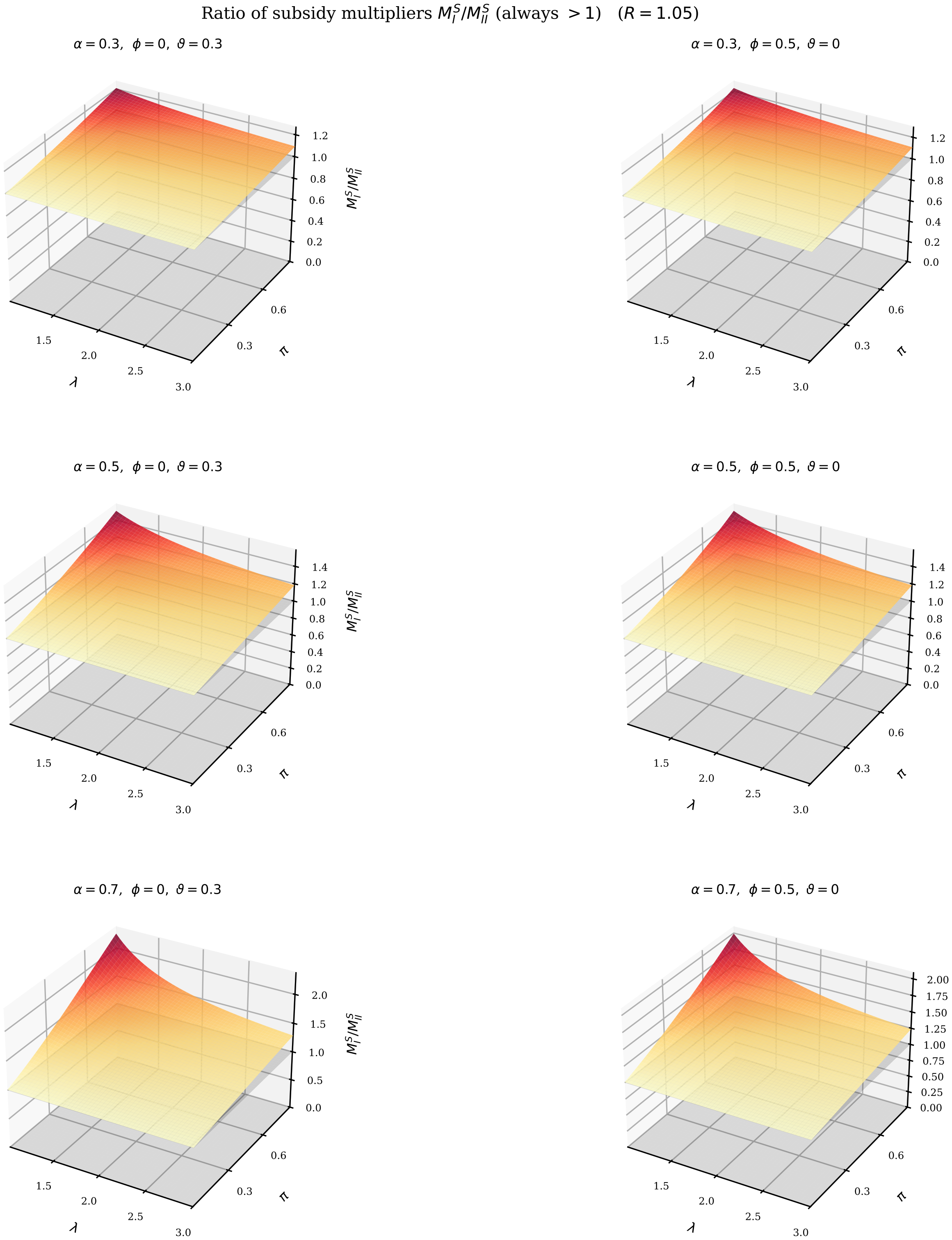


Figure A4: **Type I and Type II guarantee multipliers**

$M^G_{I,\phi}$ (solid lines, $\gamma = \gamma_{I,\phi}$ as in Equation (25)) and Type II de-risking guarantee multiplier $M^G_{II,\phi}$ (dashed lines), for $\phi \in \{0.5, 1, 2\}$ and $\alpha \in \{0.2, 0.4, 0.6, 0.8\}$. Each panel fixes α; π varies along the x-axis. $\lambda = 1$, $\vartheta = 0$, $R = 1.05$. The horizontal dotted line marks the multiplier = 1 threshold. The non-de-risking Type I guarantee dominates for all practically relevant default probabilities.

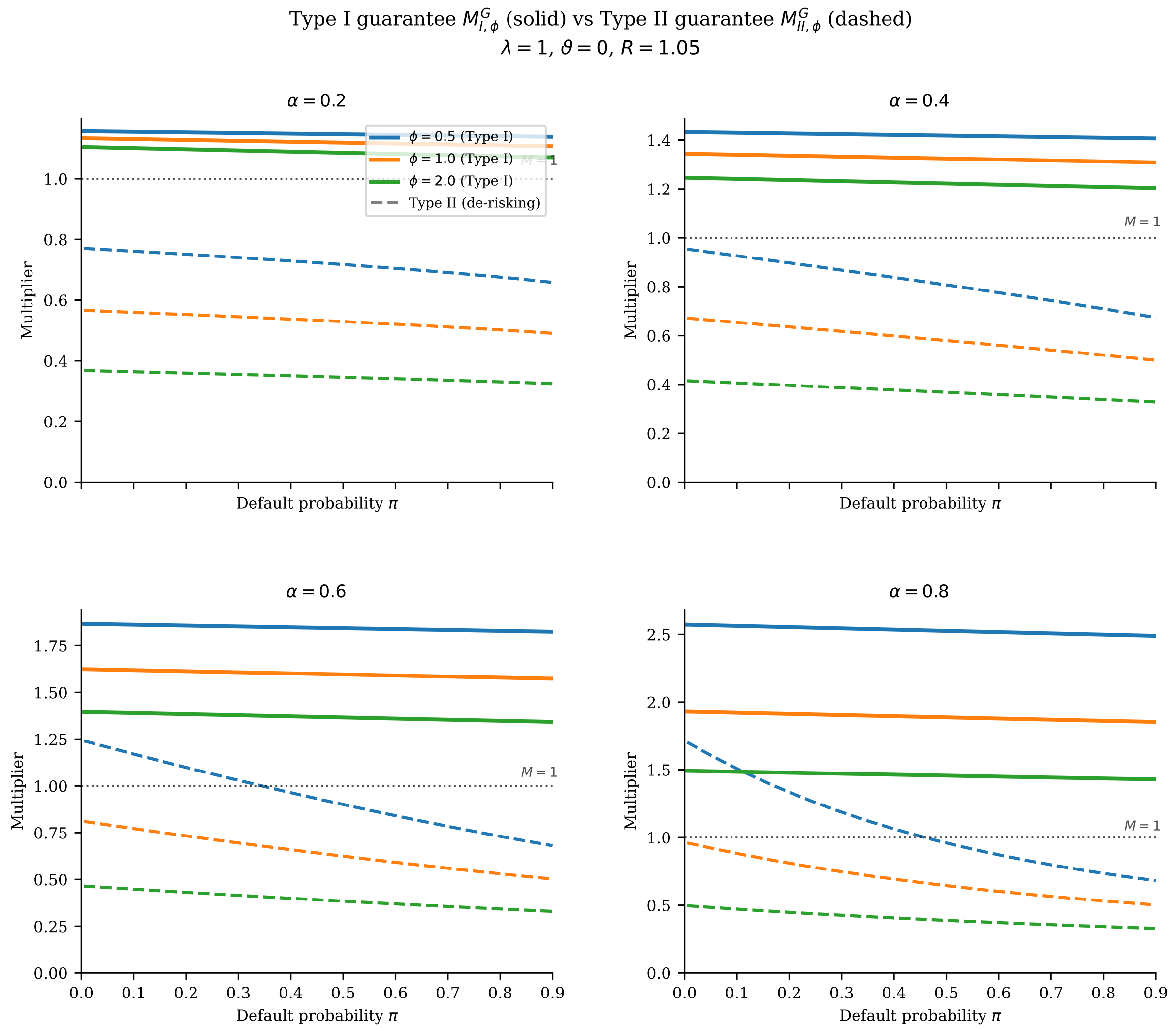
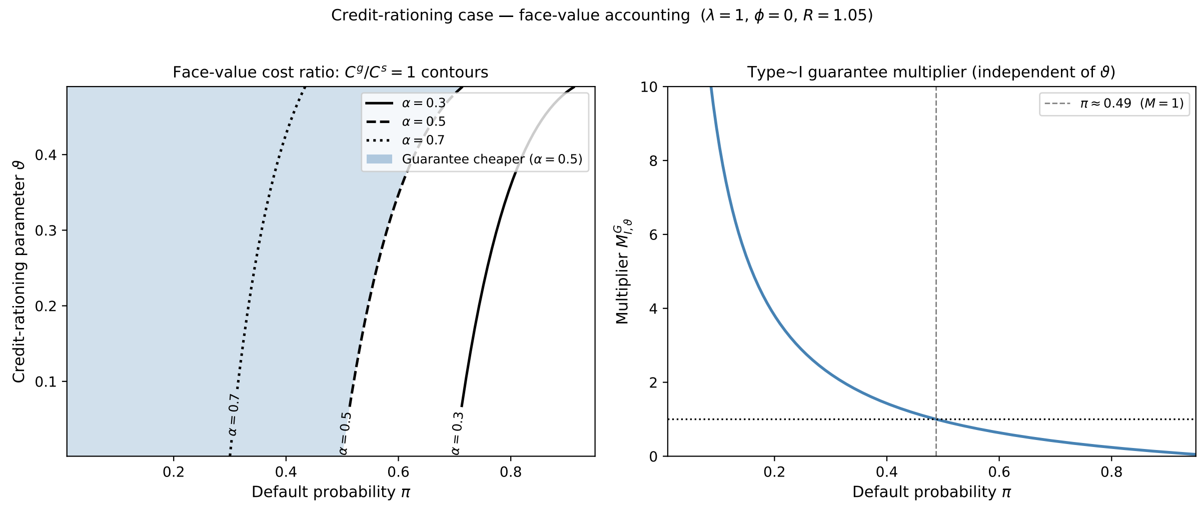


Figure A5: **Credit-rationing case: face-value cost ratio ($\lambda = 1$, $\phi = 0$, $R = 1.05$).**

Left panel: contours where the face-value cost ratio $C^g/C^s = 1$ in (π, ϑ) space for $\alpha \in \{0.3, 0.5, 0.7\}$ (line styles), where $C^g/C^s = \pi(1+\pi)\vartheta/(1-(1-(1+\pi)\vartheta)^{1-\alpha})$; the blue shaded region shows where the guarantee is cheaper ($C^g < C^s$) for $\alpha = 0.5$. Right panel: Type I guarantee multiplier $M^G_{I,\vartheta} = (1-\pi)/(\pi R)$, which does not depend on ϑ; the horizontal dotted line marks $M = 1$, crossed at $\pi \approx 0.49$. The expected-cost ratio equals this face-value ratio divided by $(1-\pi)$, so the blue region of guarantee dominance is smaller in the expected-cost version (Figure 4 in the main text).



A.12 Derivations

A.12.1 Closed form for the subsidy multiplier under default risk (Type I)

Using (19) and (20), and defining $C^s_I = s_I K^*_H$ with (21), we obtain the expression in Equation (22), which is used in the main text for calibration and figures.

A.12.2 Closed form for the de-risking subsidy multiplier (Type II)

Define $C^s_{II} = s_{II} K^*_R$ with (23) and $K^*_R = (\lambda\alpha/R)^{1/(1-\alpha)}$. Then the expression in Equation (24) follows. The ratio M^S_I/M^S_{II} is derived in Appendix A.8.

A.13 Additional clarifications

(i) Near $\lambda \downarrow 1$, M^{S1} is a 0/0 expression; applying L'Hôpital gives $\lim_{\lambda\downarrow 1} M^{S1} = \frac{p}{R} = \frac{1}{(1-\alpha)R}$.

(ii) In (29), ϑ cancels mechanically: additionality $K^*_H - K^m = (1+\pi)\vartheta K^*_H$ and cost $C^g \propto \pi(1+\pi)\vartheta$ are both proportional to $(1+\pi)\vartheta$, which cancels in the ratio.