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GLOBAL POPULATION GROWTH, TECHNOLOGY, AND MALTHUSIAN CONSTRAINTS: A QUANTITATIVE GROWTH THEORETIC PERSPECTIVE

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Global population growth, technology and Malthusian constraints: A quantitative growth theoretic perspective*

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Abstract

We structurally estimate a two-sector Schumpeterian growth model with endogenous population and finite land reserves to study the long-run evolution of global population, technological progress and the demand for food. The estimated model closely replicates trajectories for world population, GDP, sectoral productivity growth and crop land area from 1960 to 2010. Projections from 2010 onwards show a slowdown of technological progress, and, because it is a key determinant of fertility costs, significant population growth. By 2100 global population reaches 12.4 billion and agricultural production doubles, but the land constraint does not bind because of capital investment and technological progress.

Keywords: Global population; Technological progress; Economic growth; Agriculture; Malthusian constraints; Land conversion; Structural estimation
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1 Introduction

World population has doubled over the last fifty years and quadrupled over the past century (United Nations, 1999). During this period and in most parts of the world, productivity gains in agriculture have confounded Malthusian predictions that population growth would outstrip food supply. Population and income have determined the demand for food and thus agricultural production, rather than food availability determining population. However, recent evidence suggests a widespread slowdown of growth in agricultural output per unit of land area (i.e. agricultural yields, see Alston et al., 2009), and the amount of land that can be brought into the agricultural system is physically finite. For reasons such as these, several prominent contributions from the natural sciences have recently raised the concern that a much larger world population cannot be fed (e.g. Godfray et al., 2010; Tilman et al., 2011). Our aim in this paper is to study how population and the demand for land interacted with technological progress over the past fifty years, and derive some quantitative implications for the years to come.

Despite the importance of these issues, few economists have contributed to the debate about the role of Malthusian constraints in *future* population growth. This is especially surprising given the success of economic theories in explaining the (past) demographic transition in developed countries in the context of their wider development paths (e.g. Galor and Weil, 2000; Jones, 2001; Bar and Leukhina, 2010; Jones and Schoonbroodt, 2010, and other contributions reviewed below). Empirical evidence emphasises the role of technology, education and per-capita income in long-run fertility development (e.g. Rosenzweig, 1990; Herzer et al., 2012), and it documents a complementarity between technological progress and the demand for human capital (Goldin and Katz, 1998). Furthermore, per-capita income is an important determinant of the demand for food (e.g. Subramanian and Deaton, 1996; Thomas and Strauss, 1997), just as technological progress is of food production and associated demand for land (Alston and Pardey, 2014).

The role of economic incentives and technology in the long-run evolution of population and per capita income, and the associated demand for food and land, is, however, absent from leading international assessments of population growth and agricultural production. In addition, while the evolution of population and agriculture are inherently interconnected, they are considered separately. On the one hand, the *de facto* standard source of demographic projections is the United Nations' series of *World Population Prospects*. The UN works from the basic demographic identity that the change in population, at the global level, is equal to the number of births less the number of deaths, with exogenous trajectories assumed for fertility and mortality. Implications for food demand and supply are not explicitly considered, although it is implicitly assumed that the projected population can be supported by agricultural production. On the other hand, agricultural projections by the Food and Agriculture Organisation of the UN (FAO) use exogenous trajectories for population, per-capita income and agricultural yields (see Alexandratos and Bruinsma, 2012). Clearly, considering outcomes separately makes the assessment of potential Malthusian constraints difficult.

In this paper we propose to use an integrated, quantitative approach to study the interactions between global population, technological progress, per-capita income, demand for food and agricultural land expansion. More specifically, we formulate a model of endogenous growth with an explicit behavioural representation linking child-rearing decisions to technology, per-capita income and availability of food. In the tradition of Barro and Becker (1989), households in the model have preferences over own consumption, the number of children they have and the utility of their children. Child-rearing is time-intensive, and fertility competes with other labour-market activities. As in Galor and Weil (2000), technological advances are associated with a higher demand for human capital, capturing the aforementioned complementarity between human capital and the level of technology, so that the cost of educating children increases with technological progress. It follows that, over time, technological process gradually increases the cost of population increments (or additions to the stock of effective labour units) both directly (as human capital requirements and education costs increase) and indirectly (as wages and the opportunity cost of time increases), which induces a gradual transition to low-fertility regime.

Besides the cost of rearing and educating children, the other key driver of population growth in our model is food requirements. As in Strulik and Weisdorf (2008), Vollrath (2011) and Sharp et al. (2012), we make agricultural output a necessary condition to sustain the contemporaneous level of population. In addition, the demand for food is increasing in per-capita income (albeit at a declining rate, see Subramanian and Deaton, 1996), reflecting empirical evidence on how diet changes as affluence rises. An agricultural sector, which meets the demand for food, requires land as an input, and agricultural land has to be converted from a stock of natural land. Therefore, as population and income grow, the demand for food increases, raising the demand for agricultural land. In the model land is treated as a scarce form of capital, which has to be converted from a finite resource stock of natural land. The cost of land conversion and the fact that it is physically finite generate a potential Malthusian constraint to long-run economic development.

In our model technology plays a central role in both fertility and land conversion decisions. On the one hand, technological progress raises the opportunity and human-capital cost of children. On the other hand, whether land conversion acts as a constraint on population growth mainly depends on technological progress. We model the process of knowledge accumulation in the Schumpeterian framework of Aghion and Howitt (1992), where the growth rate of total factor productivity (TFP) increases with labour hired for R&D activities. A well-known drawback of such a representation of technological progress is the population scale effect (see Jones, 1995a).[1] This is important in a setting with endogenous population, as it would imply that accumulating population would increase long-run technology and income growth. By contrast, our representation of technological progress falls in the class of Schumpeterian growth models that dispose of the scale effect by considering that innovation applies to a growing number of differentiated products ('product lines', see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998), so that long-run growth is not proportional to the level or growth rate of population.[2]

To fix ideas, we start with a simple theoretical illustration of the mechanism underlying fertility and land conversion decisions in our model. However, the main contribution of our work is to structurally estimate the model and use it to study the quantitative behaviour of the system. More specifically, most of the parameters of the model are either imposed or calibrated from external sources, but those determining the marginal cost of population, labour productivity in sectoral R&D and labour productivity in agricultural land conversion are structurally estimated with simulation methods. We use 1960-2010 data on world population, GDP, sectoral TFP growth and crop land area to define a minimum distance estimator, which compares observed trajectories with those simulated from the model. We show that trajectories simulated with the estimated vector of parameters closely replicate observed data for 1960 to 2010, and that the estimated model also provides a good account of non-targeted moments over the estimation period, notably

[1] The population scale effect, or positive equilibrium relationship between the size of the labour force and aggregate productivity growth, can be used to explain the take-off phase that followed stagnation in the pre-industrial era (e.g. Boserup, 1965; Kremer, 1993). However, empirical evidence from growth in recent history is difficult to reconcile with the scale effect (e.g. Jones, 1995b; Laincz and Peretto, 2006). See Strulik et al. (2013) on how the transition between the two growth regimes can be explained endogenously through accumulation of human capital.

[2] In a product-line representation of technological progress, R&D takes place at the firm (or product) level, and new firms are allowed to enter the market. An implication is that the number of products grows over time, thereby diluting R&D inputs, so that growth does not necessarily rely on an increasing labour force assigned to R&D activities, but rather on the share of labour in the R&D sector (Laincz and Peretto, 2006).

agricultural output and its share of total output. We then employ the estimated model to jointly project outcomes up to 2100.

The key results are as follows. Trajectories from the estimated model suggest a population of 9.85 billion by 2050, further growing to 12.4 billion by 2100. Moreover, although the population growth rate declines over time, population does not reach a steady state over the period we consider. This is mainly due to the fact that the pace of technological progress, which is the main driver of the demographic transition in our model, declines over time, so that population growth remains positive over the horizon we consider. Despite a doubling of agricultural output associated with growth in population and per-capita income, however, agricultural land expansion stops by 2050 at around 1.8 billion hectares, a 10 percent increase on 2010.[3] A direct implication of our work is that the land constraint does not bind, even though (i) our population projections are higher than UN's latest (2012) estimates; and (ii) our projections are rather conservative in terms of technological progress (agricultural TFP growth in both sectors is below one percent per year and declining from 2010 onwards).

One important feature of these dynamics is that they derive entirely from the structure of the model, rather than changes in the underlying parameters. We also consider the sensitivity and robustness of our results to a number of assumptions, notably substitution possibilities in agriculture and the income elasticity of food demand. Overall we find that projections from the model are fairly robust to plausible changes in the structure of the model. Some variations suggest an optimal population path that is higher than our baseline case, although the evolution of agricultural land is only marginally affected. The robustness of our results essentially derives from estimating the model with 50 years of data, tying down trajectories over a long time horizon.

1.1 Related literature

Our work relates to at least three strands of economic research. First, there is unified growth theory, which studies economic development and population over the long run. Seminal contributions include Galor and Weil (2000) and Jones (2001) (see Galor, 2005, for a survey). Jones (2003) and Strulik (2005) analyse the joint development of population, technological progress

[3] This corresponds to the conversion of a further 150 million hectares of natural land into agriculture, roughly the area of Mongolia or three times that of Spain. Because developed countries will likely experience a decline in agricultural land area (Alexandratos and Bruinsma, 2012), land conversion in developing countries will need to be more than that.

and human capital (see also Tournemaine and Luangaram, 2012, for a recent investigation and comprehensive overview of the literature), while Hansen and Prescott (2002) and Strulik and Weisdorf (2008) consider the role of agriculture and manufacturing activities along the development path. The structure of our model, linking technology and economic growth with child rearing and education decisions, and the implied quality-quantity trade-off, is closely related to these papers.

In unified growth theory models, the initial phase of economic development relies on the scale effect to generate a take off. A key departure from these papers is that we focus on post-1960 growth and rule out the existence of a scale effect. Our work thus also relates to Schumpeterian growth theories that circumvent the scale effect with a product-line representation of R&D (Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998). These have been used to develop growth models with endogenous population and resource constraints, most notably Bretschger (2013) and Peretto and Valente (2015), and these theoretical contributions are thus close in spirit to our work. Our treatment of land as a scarce form of capital is, however, novel, and by taking our model to the data we also provide new evidence on the quantitative importance of resource constraints for global development.

A last set of papers has in common with us the use of quantitative macroeconomic models to study particular aspects of unified growth theory, especially economic development and the demographic transition. These include Mateos-Planas (2002), Doepke (2005), Strulik and Weisdorf (2008; 2014), Bar and Leukhina (2010), Jones and Schoonbroodt (2010), and Ashraf et al. (2013). These papers demonstrate that macroeconomic growth models are able to capture essential features of the demographic transition in countries where such a transition has already taken place. Our contribution is to show that models like these can not only closely replicate historical data, they can also be used to inform the future evolution of population, technology and land use, and in turn provide a tool to evaluate the potential role of Malthusian constraints for long-run development.

Finally, our approach also contributes to policy discussions and complements existing projections from different sources, most notably population projections by the United Nations (2013) and agricultural projections by the FAO (Alexandratos and Bruinsma, 2012). These projections are based on highly disaggregated, detailed approaches, but require exogenous assumptions about a number of quantities that are endogenous to the process of development (such as per-capita income or technological progress). By contrast, our approach lacks disaggregation and detail, but provides an empirical perspective in which the relevant quantities are jointly and endogenously determined based on different components from contemporary growth theory.

The remainder of the paper proceeds with a simple analytical model capturing the key features of our analysis (Section 2). The structure of our quantitative model and estimation strategy are presented in Section 3. Section 4 reports the results of the quantitative analysis, namely the estimation results, projections, and sensitivity analysis. We discuss some broader implications of our results and compare them with projections from other sources in Section 5. Some concluding comments are offered in Section 6.

2 Simple analytics of household fertility, technology and land

The objective of this section is to present the key elements of our model in a simplified set-up, thereby laying out the mechanisms driving the demographic transition and agricultural land expansion underlying our quantitative results. To do so, we study the optimal decisions of a representative household that faces exogenous technological progress, and where capital is omitted. The remaining state variables are the level of population and area of agricultural land. As we will show, this distills the problem into one of allocating labour between several competing uses. While the problem remains too complicated to yield analytical solutions for the whole development path, we can nonetheless obtain useful results relating to optimal fertility and agricultural land expansion between any two successive time periods. In turn, it allows us to identify the different components of the cost of effective labour units and incentives underlying land conversion decisions along optimal trajectories.

We consider a representative agent that lives for only one period (we introduce a probability of survival later) and has preferences over its own consumption of a homogeneous, aggregate manufactured good c_t, the number of children it produces n_t, and the utility that each of its children experiences in the future $U_{i,t+1}$. We use the class of preferences suggested by Barro and Becker (1989), defined recursively as:

$$U_t = u(c_t) + \beta b(n_t) \sum_{i=1}^{n_t} U_{i,t+1} \tag{1}$$

where $u(\cdot)$ is the per-period utility function and we assume that $u' > 0$, $u'' < 0$, and that $u(\cdot)$ also satisfies the Inada conditions such that $\lim_{c \to 0} u' = \infty$ and $\lim_{c \to \infty} u' = 0$. The function $b(\cdot)$ specifies preferences for fertility and is assumed to be isoelastic, an assumption made in the original Barro and Becker (1989) paper and that we will maintain throughout. $\beta \in (0, 1)$ is the discount factor.

We further assume that children are identical, so that $\sum_{i=1}^{n_t} U_{i,t+1} = n_t U_{t+1}$, and write the motion equation for population as $N_{t+1} = n_t N_t$.[4] Given these assumptions, the recursive nature of Barro-Becker preferences allows us to the define the utility function of the dynastic household head as:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t) b(N_t) N_t \tag{2}$$

The steps involved are described in Appendix A. Consistent with our quantitative analysis in which $U_t > 0$, a preference for fertility that is subject to diminishing returns, and in turn overall concavity of (2), requires that $\partial N b(N)/\partial N > 0$ and $\partial^2 N b(N)/\partial N^2 < 0$ (Jones and Schoonbroodt, 2010).[5] This also implies that fertility and the utility of children are complements in parents' utility (which is easiest to see in the context of (1), where our combination of assumptions yields $\partial^2 U_t / \partial n_t \partial U_{t+1} > 0$), and is consistent with empirical evidence reported in Brueckner and Schwandt (2015).

Each agent is endowed with one unit of time in each period, which can be spent working in a competitive market for manufacturing labour at wage w_t. We assume that identical, competitive manufacturing firms employ household labour and combine it with the exogenously given technology $A_{t,mn}$ to produce the composite good that households consume:

$$Y_{t,mn} = A_{t,mn} \cdot Y_{mn}(L_{t,mn}), \tag{3}$$

where $L_{t,mn}$ is time spent working in the manufacturing sector, $Y'_{mn} > 0$, $Y''_{mn} < 0$ and the Inada conditions hold. Given this, the household's budget constraint is $c_t N_t = w_t L_{t,mn}$, while profit maximisation requires $A_{t,mn} Y'_{mn}(L_{t,mn}) = w_t$.

Time spent rearing and educating children competes with labour-market activities as it does in the standard model of household fertility choice (Becker, 1960; Barro and Becker, 1989). In addition, we characterise a complementarity between technology and skills (Goldin and Katz, 1998)

[4] As discussed in Appendix A, introducing mortality in this context requires the further assumption that parents' welfare at $t + 1$ is identical to that of their children (see Jones and Schoonbroodt, 2010).

[5] We further assume that $\lim_{N \to 0} \partial N b(N)/\partial N = \infty$ and $\lim_{N \to \infty} \partial N b(N)/\partial N = 0$.

by postulating an increasing relationship between the time-cost of producing effective labour units and the level of technology in the economy (specifically in manufacturing), measured by $A_{t,mn}$.[6] Formally, the production of increments to the labour force is written as:

$$n_t N_t = \chi(L_{t,N}, A_{t,mn}) \,, \tag{4}$$

where $L_{t,N}$ denotes the labour time devoted to child-rearing and education. We will assume $\partial\chi(L_{t,N}, A_{t,mn})/\partial L_{t,N} > 0$, $\partial^2\chi(L_{t,N}, A_{t,mn})\Big/\partial L_{t,N}^2 < 0$, $\partial\chi(L_{t,N}, A_{t,mn})/\ \partial A_{t,mn} < 0$, $\partial^2\chi(L_{t,N}, A_{t,mn})/\partial A_{t,mn}^2 > 0$ and $\partial^2\chi(L_{t,N}, A_{t,mn})/\partial L_{t,N}\partial A_{t,mn} < 0$.

This formulation captures the essence of the more comprehensive model of Galor and Weil (2000), in which education decisions are reflected in a stock of human capital. In that framework, technological progress increases the demand for human capital, so that, as the technology improves, the return to education increases. Growing education requirements raise the cost of each individual child, which implies that technological progress induces a transition from a situation with a large number of children with low human capital (and low education cost), to one where households have a smaller number of children who possess higher human capital. Effectively, technological progress raises the cost of producing effective labour units, and human capital partially substitutes for the quantity of children in the labour force. Similarly, equation (4) imposes a positive relationship between technology and educational requirements for children to be productive in the labour market. However, by focusing on the direct impact of technology on the cost of effective labour units, we avoid the need to keep track of an additional state variable measuring the level of human capital. [7]

In our model there is an additional constraint bearing upon the household, which is that sufficient food must be available for it to eat at all times. The aggregate food requirement is the product of total population N_t and the per-capita food requirement $\overline{f_t}$:

$$\overline{f_t} N_t = A_{t,ag} Y_{ag}(L_{t,ag}, X_t) \tag{5}$$

[6] In our quantitative model, the cost of children is proportional to an output-weighted average of TFP in manufacturing and agriculture, although the consequent weight on the former is much larger.

[7] Since our model does not distinguish between time spent rearing children and the time spent educating them, parents' fertility decisions involve both the quantity and the education of children. Thus one implication of our approach is that we cannot consider the role of policies affecting education and human capital. For projections along a baseline trajectory, however, this is less of an issue.

where food is directly produced by the household by combining 'agricultural' labour $L_{t,ag}$ and land X_t with production function $Y_{ag}(\cdot)$, given agricultural TFP $A_{t,ag}$. We assume strictly positive and diminishing returns to labour and land, and that the Inada conditions also hold on both.

This treatment of the role of food via a constraint is similar to other papers that consider the role of agricultural production in long-run development, most notably Strulik and Weisdorf (2008), Vollrath (2011) and Sharp et al. (2012). The per-capita food requirement $\overline{f_t}$ could be given a physiological interpretation. That is, it could be the subsistence food requirement of each individual, as in the aforementioned papers. Alternatively it could be positively related to living standards, reflecting empirical evidence that food demand increases with income per capita, albeit at a decreasing rate (Subramanian and Deaton, 1996; Thomas and Strauss, 1997). We will allow this relation in our quantitative model.[8]

There is a finite supply of land $\overline{X}$ that is in full, private ownership of the household at all times. Land can be converted into agricultural land with the use of the household's labour $L_{t,X}$. The state equation for land is then:

$$X_{t+1} = \psi(L_{t,X}), \quad X_t \leq \overline{X} \tag{6}$$

where $\psi' > 0$, $\psi'' < 0$ and the Inada conditions again hold.[9] Land that is prepared for agricultural use thus acts as a productive stock of capital that is physically finite.

Collecting the budget constraint, the food constraint (5) and the feasibility condition on the household's allocation of labour $N_t = L_{t,mn} + L_{t,N} + L_{t,X} + L_{t,ag}$, the dynastic head's optimisation

[8] It is worth noting that, while food consumption does not directly enter the utility function of households, food availability will affect social welfare through its impact on population dynamics. Moreover, while introducing food in the utility function may be preferable from a theoretical standpoint, imposing food requirements as a side constraints permits a more transparent parametrisation of both income and substitution effects. In particular, this formulation limits substitution possibilities between food and manufacturing products, which potentially magnifies the role of Malthusian constraints in the analysis. In other words, if land is a limiting factor to development, the relative cost of food would increase, and allowing households to substitute more of the manufactured product for food would essentially make land constraints irrelevant. We return to the parametrisation of the income elasticity of food demand later in the paper.

[9] In this formulation agricultural land is "recolonised" by nature every period, i.e. the depreciation rate is 100 percent. This is obviously a simplification and we introduce a more realistic depreciation pattern in our quantitative analysis.

problem can be written as:

$$
\begin{aligned}
\max_{\{L_{t,j}\}} \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) b(N_t) N_t \\
\text{s.t.} \quad & N_{t+1} = \chi(L_{t,N}, A_{t,mn})\,; \quad X_{t+1} = \psi(L_{t,X})\,; \quad X_t \leq \overline{X} \\
& c_t N_t = w_t L_{t,mn}\,; \quad N_t = L_{t,mn} + L_{t,N} + L_{t,X} + L_{t,ag}\,; \quad \overline{f}_t N_t = A_{t,ag} Y_{ag}(L_{t,ag}, X_t) \\
& N_0,\, X_0 \quad \text{given}
\end{aligned}
$$

At the heart of the household's problem is therefore the allocation of labour between four competing uses: (i) supply of labour to the manufacturing sector, $L_{t,mn}$; (ii) spending time rearing and educating children, $L_{t,N}$; (iii) spending time producing food, $L_{t,ag}$; and (iv) spending time expanding the agricultural land area, $L_{t,X}$.

Necessary and sufficient conditions for a maximum imply that, along the optimal path, fertility is chosen so as to equate the marginal benefits and costs of increasing the population in the next period (see Appendix A):

$$
\underbrace{\beta u(c_{t+1}) \left[b'(N_{t+1}) N_{t+1} + b(N_{t+1}) \right]}_{A} + \underbrace{\beta u'(c_{t+1}) b(N_{t+1}) w_{t+1}}_{\text{B}} =
$$

$$
\underbrace{u'(c_t) b(N_t) w_t \bigg/ \frac{\partial \chi(L_{t,N}, A_{t,mn})}{\partial L_{t,N}}}_{\text{C}} + \underbrace{\beta u'(c_{t+1}) b(N_{t+1}) w_{t+1} \overline{f_{t+1}} \bigg/ \left[A_{t+1,ag} \frac{\partial Y_{ag}(L_{t+1,ag}; X_{t+1})}{\partial L_{t+1,ag}} \right]}_{\text{D}} \tag{7}
$$

Intuitively, the benefit of population increments in the next period comprises two terms. First, (A) represents the discounted value of the utility associated with an additional member of the dynasty in next period, which is positive by assumption. Notice that total rather than marginal utility enters (A), which highlights the assumption in our objective function that fertility and the utility of children are complements. Second, (B) is the discounted value of the additional output that will be produced in the next period, which is made possible by expanding the pool of labour that can be supplied to manufacturing and earn wage w_{t+1}. This term is proportional to the marginal product of labour in manufacturing, so that technological progress will generally raise the expected benefits of population increments.

Expression (7) shows that the cost of population increments also comprises two components. The first is the opportunity cost of present consumption foregone (C), as time is spent rearing and educating children rather than earning wage w_t. As mentioned previously, this implies that technological progress (by increasing labour productivity) indirectly makes children more expensive.[10] In addition, (C) is inversely proportional to the marginal product of labour devoted to child-rearing and education. Since we assume that $\partial^2\chi(L_{t,N}, A_{t,mn})/\partial L_{t,N}\partial A_{t,mn} < 0$, the technology index $A_{t,mn}$ reduces the marginal product of labour and thus increases the cost of population increments. As discussed in detail above, this represents the additional cost of education required to make labour units productive for a given level of technology and its associated demand for human capital.

The second component of the cost of population increments represents the discounted cost of food required to sustain the additional labour unit in the next period (D). While this can be interpreted as a goods cost, in the present setting it also represents an opportunity cost, since the representative household has to divert time away from the manufacturing sector to produce additional food in the next period. Term (D) is thus increasing in w_{t+1} and per-capita food demand $\overline{f_{t+1}}$. The food cost of population increments also depends on the marginal product of labour in agriculture, itself a function of agricultural technology. Thus improvements to agricultural technology that raise the marginal product of labour reduce the food component of the marginal cost of population increments. On the contrary, slowdown in the pace of technological progress in agriculture could put a brake on population growth.[11]

Similarly, the constraint on the expansion of agricultural land has the potential to affect population growth. In the extreme case where the constraint binds ($X_t = \overline{X}$), there are no improvements to agricultural TFP, and labour and land are perfect complements in food production, a fixed food constraint would imply that no further increase in the population can take place. More generally, the extent to which the population can grow despite the constraint binding mainly depends on technological improvements in agriculture and on the substitutability of labour and land in agricultural production.

It is in fact useful to briefly inspect the condition determining optimal expansion of agricultural

[10] Note that the effect of labour productivity on the overall fertility path occurs though several channels, which includes the effect on marginal benefits next period, as well as the marginal utility of consumption through the budget constraint. The latter effect in fact reduces the cost of children, as the marginal utility of consumption declines with c_t, so that the valuation of the opportunity cost of rearing and educating children declines with w_t.

[11] Over the period 1960-2005, agricultural productivity as measured by output per unit area – agricultural yield – increased by a factor of 2.4, although the growth rate declined from 2.03% per year in the period from 1960 to 1990 to 1.82% per year in the period from 1990 to 2005 (Alston et al., 2009).

land in period t (under the assumption that the land constraint does not bind):

$$u'(c_t)b(N_t)w_t/\psi'(L_{t,X}) = \beta u'(c_{t+1})b(N_{t+1})w_{t+1}\overline{f_{t+1}}\, \frac{\partial Y_{ag}(L_{t,+1ag}, X_{t+1})}{\partial X_{t+1}} \Bigg/ \frac{\partial Y_{ag}(L_{t+1,ag}, X_{t+1})}{\partial L_{t+1,ag}} \tag{8}$$

The term on the left-hand side is the marginal cost of land conversion, capturing present consumption foregone by diverting labour away from manufacturing. In the case where the land constraint binds, the shadow price of the constraint will appear as a cost in the form of a scarcity rent. The term on the right-hand side is the discounted marginal benefit of land conversion. Notice that the marginal benefit of land conversion is higher, the higher is the marginal productivity of land in agriculture relative to the marginal productivity of labour in the same sector.

One important counterfactual implication of (8) is associated with the fact that labour used to invest in the stock of agricultural land is subject to decreasing returns (see equation 6). Specifically, as the agricultural land area expands, investing in the stock of land becomes relatively more costly, which captures the fact that the most productive agricultural plots are converted first, whereas marginal land still available at a later stage of land conversion is less productive. Labour can be used to bring these marginal plots into agricultural production, although the cost of such endeavours increases together with the total land areap under agriculture use. Thus with substitutability and technological progress, land as a factor of production can be expected to become less important over time. In our simulation this will be the main driver of a slow-down in land conversion.

3 Quantitative model

In this section, we present the quantitative model and then describe how we estimate key parameters of the model for trajectories to match economic time series for 1960-2010. The model is an extension of the simple farming-household problem discussed above in which we add capital to the set of factor inputs. The problem is one of allocating labour and capital across sectors, as well as by selecting the savings/investment rate to build up the stock of capital. In addition, sectoral technological progress is endogenously determined by the allocation of labour to R&D activities. This implies that the change in the cost of children, and associated demographic transition, will occur endogenously.

Our empirical strategy relies on simulation methods, selecting the parameters of interest to minimise the distance between observed and simulated trajectories. The estimation procedure requires computing the model a very large number of times, and for that reason we consider only the optimal solution to the problem. Specifically a social planner maximises households' utility by selecting aggregate quantities subject to the technology that characterises the economy. First, this formulation of the problem makes transparent the conditions for the problem to be convex, so that a solution to the problem exists and is unique (see Alvarez, 1999). Second, the social planner formulation affords a number of simplifications, and permits the use of efficient solvers for constrained non-linear optimisation problems, making simulation-based estimation practical.

One apparent issue associated with a social planner representation is that, in a Schumpeterian model of growth, the socially optimal growth rate can be expected to differ from the one prevailing under a decentralised allocation. In particular, the presence of externalities in R&D activities (e.g. Romer, 1994) implies that market-driven technological progress is suboptimal (in the setting we consider, it is likely to be too low, see Tournemaine and Luangaram, 2012). We stress, however, that even though we solve the model as a social planner problem, our work does not take a normative view of global development. By estimating the model, we rationalise observed outcomes 'as if' these resulted from the decisions of a social planner, so that market imperfections and externalities that have affected observed economic outcomes over the past 50 years are included in the quantitative analysis. In other words, because the estimated model matches observed historical growth rates, our analysis captures market imperfections originating from a decentralised allocation. An implication is that externalities and market imperfections prevailing over the estimation period will be reflected in our estimates rather than in the simulated trajectories. We come back to this below.

3.1 The economy

3.1.1 Production

In agriculture and manufacturing aggregate output is represented by a constant-returns-to-scale production function with endogenous, Hicks-neutral technological change.[12] In manufacturing,

[12] Assuming technological change is Hicks-neutral, so that improvements to production efficiency do not affect the relative marginal productivity of input factors, considerably simplifies the analysis at the cost of abstracting from a number of interesting issues related to the direction or bias of technical change (see Acemoglu, 2002).

aggregate output in period t is given by a standard Cobb-Douglas production function:

$$Y_{t,mn} = A_{t,mn} K_{t,mn}^{\vartheta} L_{t,mn}^{1-\vartheta}, \tag{9}$$

where $K_{t,mn}$ is capital allocated to manufacturing and $\vartheta \in (0,1)$ is a share parameter. Conditional on technical change being Hicks-neutral, the assumption that output is Cobb-Douglas is consistent with long-term empirical evidence (Antràs, 2004).

In agriculture, we posit a two-stage constant-elasticity-of-substitution (CES) functional form (e.g. Kawagoe et al., 1986; Ashraf et al., 2008):

$$Y_{t,ag} = A_{t,ag} \left[(1-\theta_X) \left(K_{t,ag}^{\theta_K} L_{t,ag}^{1-\theta_K} \right)^{\frac{\sigma-1}{\sigma}} + \theta_X X_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{10}$$

where $\theta_{X,K} \in (0,1)$, and σ is the elasticity of substitution between a capital-labour composite factor and agricultural land. This specification provides flexibility in how capital and labour can be substituted for land, and it nests the Cobb-Douglas specification as a special case ($\sigma = 1$). While a Cobb-Douglas function is often used to characterise aggregate agricultural output (e.g. Mundlak, 2000; Hansen and Prescott, 2002), it is quite optimistic in that, in the limit, land is not required for agricultural production. Long-run empirical evidence reported in Wilde (2013) indeed suggests that $\sigma < 1$.

3.1.2 Innovations and technological progress

The evolution of sectoral TFP is based on a discrete-time version of the Schumpeterian model by Aghion and Howitt (1992). In this framework innovations are drastic, so that a firm holding the patent for the most productive technology temporarily dominates the industry until the arrival of the next innovation. The step size of productivity improvements associated with an innovation is denoted $s > 0$, and we assume that it is the same in both sectors.[13] Without loss of generality, we assume that there can be at most $I > 0$ innovations over the length of a time period, so that the maximum growth rate of TFP each period is $S = (1+s)^I$. For each sector $j \in \{mn, ag\}$, the growth rate of TFP is then determined by the number of innovations arriving within each time-period, and

[13] In general, the "size" of an innovation in the Aghion and Howitt (1992) framework is taken to be the step size necessary to procure a right over the proposed innovation. For the purposes of patent law, an innovation must represent a substantial improvement over existing technologies (not a marginal change), which is usually represented as a minimum one-time shift.

this rate can be specified in relation to maximum feasible TFP growth S:[14]

$$A_{t+1,j} = A_{t,j} \cdot (1 + \rho_{t,j} S) \ , \quad j \in \{mn, ag\} \, . \tag{11}$$

where $\rho_{t,j}$ is the arrival *rate* of innovations each period, in other words how many innovations are achieved compared to the maximum number of innovations.

Arrival of innovations in each sector is a function of labour hired for R&D activities:

$$\rho_{t,j} = \overline{\lambda}_{t,j} \cdot L_{t,A_j} \ , \quad j \in \{mn, ag\} \, ,$$

where L_{t,A_j} is labour employed in R&D for sector j and $\overline{\lambda}_{t,j}$ measures labour productivity. As mentioned in the introduction, the standard Aghion and Howitt (1992) framework implies a scale effect by virtue of which a larger population implies a large equilibrium growth rate of the economy, which is at odds with empirical evidence on modern growth. Instead we work with the scale-invariant formulation proposed by Chu et al. (2013), where $\overline{\lambda}_{t,j}$ is specified as a decreasing function of the scale of the economy (here population N_t). In particular, we define

$$\overline{\lambda}_{t,j} = \lambda_j L_{t,A_j}^{\mu_j - 1} / N_t^{\mu_j}$$

where $\lambda_j > 0$ is a productivity parameter and $\mu_j \in (0, 1)$ is an elasticity.

Several comments are in order. First, in this model labour allocated to R&D drives TFP growth, and the growth rate is proportional to the *share* of labour allocated to R&D (see Chu et al., 2013). In steady state the share of labour allocated to each sector is constant, so that the growth rate of the economy is independent of the size of population. Therefore, as discussed previously, the scale effect is absent in the model. Second, a potential critique of this specific representation is that a model in which TFP growth is driven by the share of R&D employment is inconsistent with the microfoundations of Schumpeterian growth models, as innovation is generated by individuals (see Jones, 1995a). However, as shown by Dinopoulos and Thompson (1999), a model in which *aggregate* TFP growth is driven by the share of labour is equivalent to Schumpeterian models in which innovation results from R&D workers hired by firms and entry of new firms is allowed. Thus this

[14] The arrival of innovations is a stochastic process, and we implicitly make use of the law of large numbers to integrate out the random nature of growth over discrete time-intervals. Our representation is qualitatively equivalent, but somewhat simpler, to the continuous time version of the model where the arrival of innovations is described by a Poisson process.

representation is consistent with microfoundations of more recent Schumpeterian growth models such as Dinopoulos and Thompson (1998), Peretto (1998) and Young (1998). Intuitively, as Laincz and Peretto (2006) put it, the share of employment in R&D can be seen as a proxy for average employment hired to improve the quality of a growing number of product varieties.[15] Finally, our representation of R&D implies decreasing returns to labour in R&D through the parameter μ_j, which captures the duplication of ideas among researchers (Jones and Williams, 2000).

3.1.3 Land

As in the simple analytical model above, land used for agriculture has to be converted from a finite stock of reserve land $\overline{X}$. Converting land from the available stock requires labour, therefore there is a cost in bringing new land into the agricultural system. Once converted, agricultural land gradually depreciates back to the stock of natural land in a linear fashion. Thus the allocation of labour to convert land determines agricultural land available each period, and over time the stock of land used in agriculture develops according to:

$$X_{t+1} = X_t(1-\delta_X) + \psi \cdot L_{t,X}^{\varepsilon}, \quad X_0 \text{ given}, \quad X_t \leq \overline{X}, \tag{12}$$

where $\psi > 0$ measures labour productivity in land clearing activities, $\varepsilon \in (0,1)$ is an elasticity, and the depreciation rate δ_X measures how fast converted land reverts back to natural land.

As discussed in Section 2, decreasing returns to labour in land-clearing activities imply that the marginal cost of land clearing increases with the amount of land already converted. More specifically, as the amount of land used in agriculture increases, labour requirements to avoid it depreciating back to its natural state increase more than proportionally.

3.1.4 Preferences and population dynamics

We now further specialise households' preferences described in Section 2. We again use the dynastic representation that is associated with Barro and Becker (1989) preferences, so that the size of the dynasty coincides with the total population N_t (see Appendix A). We use the standard constant elasticity function $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$, where $1/\gamma$ is the intertemporal elasticity of substitution,

[15] Note that another strategy to address the scale effect involves postulating a negative relationship between labour productivity in R&D and the existing level of technology, giving rise to "semi-endogenous" growth models (Jones, 1995a). In this setup, however, long-run growth is driven by the population growth rate, which is also at odds with empirical evidence (Ha and Howitt, 2007).

and specify $b(n_t) = n_t^{-\eta}$, where η is an elasticity determining how the utility of parents changes with n_t. The utility of the dynasty head is then:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \, N_t^{1-\eta} \, \frac{c_t^{1-\gamma} - 1}{1-\gamma} \,, \tag{13}$$

Parametric restrictions ensuring overall concavity of the objective and in turn existence and uniqueness of the solution are easy to impose. For $\gamma > 1$, which is consistent with macro-level evidence on the intertemporal elasticity of substitution (Guvenen, 2006), concavity of Equation (13) in (c_t, N_t) requires $\eta \in (0, 1)$.[16] This implies that, depending on η, preferences of the dynastic head correspond with both classical and average utilitarian objectives, in terms of social planning, as limiting cases.[17]

Aggregate consumption $C_t = c_t N_t$ is derived from the manufacturing sector. Given a social planner representation, manufacturing output can either be consumed by households or invested into a stock of capital:

$$Y_{t,mn} \; = \; C_t + I_t \,, \tag{14}$$

The accumulation of capital is then given by:

$$K_{t+1} \; = \; K_t(1 - \delta_K) + I_t \,, \quad K_0 \text{ given} \,, \tag{15}$$

where δ_K is a per-period depreciation rate. In this formulation investment decisions mirror those of a one-sector economy (see Ngai and Pissarides, 2007, for a similar treatment of savings in a multi-sector growth model).

In each period, fertility n_t determines the change in population together with mortality d_t:

$$N_{t+1} \; = \; N_t + n_t N_t - d_t \,, \quad N_0 \text{ given} \,. \tag{16}$$

[16] Note that in this formulation per-period utility can in principle be both positive (if $c_t > 1$) and negative (if $c_t < 1$), so that equation (13) is not concave over the whole domain. In our simulations, however, per-capita consumption is initialised above one and grows thereafter. Thus without affecting our results we impose a feasibility lower bound at 1, which ensures that the objective function is numerically well behaved over the relevant domain of the variables.

[17] See Baudin (2011) for a discussion of the relationship between dynastic preferences and different classes of social welfare functions.

We make the simplifying assumption that population equals the total labour force, so that $n_t N_t$ and d_t represent an increment and decrement to the stock of effective labour units, respectively. The mortality rate is assumed to be constant, so that $d_t = N_t \delta_N$, where $1/\delta_N$ captures the expected working lifetime.

We specify the production function for effective labour units as:

$$n_t N_t \;=\; \overline{\chi}_t \cdot L_{t,N}\,,$$

where $\overline{\chi}_t$ is an inverse measure of the cost of producing effective labour units. Consistent with equation (4), education requirements are assumed to be proportional to the technological advancement of the economy, so that the cost of children increases with the amount of human capital that is required to be productive in a given stage of development. Formally, we assume that labour productivity in the production of effective labour units declines with technology, so that the time cost of rearing and educating children increases:

$$\overline{\chi}_t \;=\; \chi L_{t,N}^{\zeta-1} / A_t^{\omega}$$

where $\chi > 0$ is a productivity parameter, $\zeta \in (0,1)$ is an elasticity representing scarce factors required in child-rearing and education,[18] A_t is an economy-wide index of technology (a weighted average of sectoral TFP where the weights are the relative shares of sectors' output in GDP), and $\omega > 0$ measures how the cost of children increases with the level of technology. As discussed in Section 2, this formulation captures the more detailed mechanism in Galor and Weil (2000) whereby education decisions respond to the demand for human capital, itself derived from the prevailing level of technology.

Population dynamics are further constrained by food availability, as measured by agricultural output. Per-capita demand for food $\overline{f_t}$ determines the quantity of food required for maintaining an individual in a given society, such as a fixed physiological requirements or a minimum per-capita caloric intake (Strulik and Weisdorf, 2008; Vollrath, 2011; Sharp et al., 2012). In line with these

[18] More specifically, ζ captures the fact that the costs of child-rearing over a period of time may increase more than linearly with the number of children (see Barro and Sala-i Martin, 2004, p.412, Moav, 2005, and Bretschger, 2013).

studies, we impose a market clearing constraint for agricultural outputs:

$$\overline{f}_t N_t \;=\; Y_{t,ag}\,.$$

Empirical evidence on the income elasticity of the demand for calories, however, suggests an increasing and concave relationship between food demand and per-capita income (e.g. Subramanian and Deaton, 1996; Thomas and Strauss, 1997). We thus specify per-capita food demand as: $\overline{f}_t \;=\; \xi \cdot \left(\frac{Y_{t,mn}}{N_t}\right)^{\kappa}$, where ξ is a scale parameter and $\kappa > 0$ is the income elasticity of food consumption. This formulation nests the simple food constraint and allows us to flexibly integrate empirical evidence about income elasticity of food demand into our quantitative simulations.

3.2 Optimal control problem and empirical strategy

We consider a social planner choosing paths for C_t, $K_{t,j}$ and $L_{t,j}$ by maximising the utility of the dynastic head (13) subject to technological constraints (9), (10), (11), (12), (14), (15), (16) and feasibility conditions for capital and labour:

$$K_t \;=\; K_{t,mn} + K_{t,ag}\,, \quad N_t \;=\; L_{t,mn} + L_{t,ag} + L_{t,A_{mn}} + L_{t,A_{ag}} + L_{t,N} + L_{t,X}\,.$$

Aggregate consumption results from allocating capital and labour to the manufacturing sector, as well as labour to manufacturing R&D. Increases in the population require time to be spent rearing and educating children. In addition, sufficient food must be provided at all times to feed the population, by allocating capital, labour and land to agriculture, as well as labour to agricultural R&D. Insofar as increasing agricultural production requires inputs of land, labour must also be allocated to convert or maintain natural land into agricultural land.

Since consumption grows over time and since fertility and the welfare of children are complements in parents' utility, the main driver of any slowdown in fertility will be the cost of fertility itself and how it evolves over time. Building on Section 2, we can identify several components to this evolution. First, technological progress increases human-capital requirements and in turn lowers the marginal productivity of labour in the production of children, because more time is required for their education. Second, the marginal productivity of labour in rearing and educating children changes relative to the marginal productivity of labour in other activities. Among other things, technological progress raises labour productivity in the two production sectors, which will tend to increase the opportunity cost of labour in child-rearing and education. Third, there are diminishing returns to labour in the production of children, implying that the marginal cost of fertility with respect to labour is an increasing and convex function. This is the usual assumption for the cost of education (Moav, 2005), and can also represent a form of congestion (see Bretschger, 2013). Fourth, a cost of fertility is meeting food requirements, and the demand for food increases with per-capita income (albeit at a decreasing rate). Thus growth in population and per-capita income are associated with an increasing demand for agricultural output. This can be achieved either through technological progress, or by allocating primary factors, i.e. labour, capital and land, to agriculture. However, agricultural land is ultimately fixed, either because it is constrained by physical availability or because its conversion cost increases with the area already converted. Thus over time the cost of agricultural output will increase, adding a further break to population growth.

3.2.1 Numerical solution concept

The optimisation problem is an infinite horizon optimal control problem, and we use mathematical programming techniques to solve for optimal trajectories. In particular, we employ a solver for constrained non-linear optimisation problems, which directly mimics the welfare maximisation program: the algorithm searches for a local maximum of the concave objective function (the discounted sum of utility), starting from a candidate solution and improving the objective subject to maintaining feasibility as defined by the technological constraints.[19]

A potential shortcoming of direct optimisation methods, as compared to dynamic programming for example, is that they cannot explicitly accommodate an infinite horizon.[20] As long as $\beta < 1$, however, only a finite number of terms matter for the solution, and instead we solve the associated finite-horizon problem truncated to the first T periods. The truncation may induce differences between the solution to the infinite-horizon problem and its finite-horizon counterpart because the shadow values of the stock variables are optimally zero in the terminal period

[19] The program is implemented in GAMS and solved with KNITRO (Byrd et al., 1999, 2006), which alternates between interior-point and active-set methods.

[20] By definition, the objective function is a sum with an infinite number of terms, and the set of constraints includes an infinite number of elements, which is incompatible with finite computer memory. The main alternative class of numerical solution methods is dynamic programming (see Judd, 1998), and exploiting a recursive formulation could accommodate an infinite horizon. Because dynamic programming is subject to the curse of dimensionality with respect to the number of continuous state variables, the computational burden associated with recursive methods would make simulation-based estimation impractical.

T, whereas they will be so only asymptotically if the planning horizon is infinite. Since we are interested in trajectories over the period from 2010 to 2100 (1960 to 2010 for the estimation of the model), we check that the solution over the first $T' = 90$ periods are not affected by the choice of T, finding that $T = 300$ is sufficient to make computed trajectories over the first T' periods independent of T.[21] Similarly, re-initialising the model in $T' = 90$ and solving the problem onwards, we remain on the same optimal path with a precision of 0.1 percent for all the variables in the model. Given the truncation over 300 periods and appropriate scaling of variables, the model solves in a matter of seconds.

3.2.2 Empirical strategy

Having defined the numerical optimisation problem, our empirical strategy proceeds in three steps. First, a number of parameters are determined exogenously. Second, we calibrate some of the parameters to match observed quantities, mainly to initialise the model based on 1960 data. Third, we estimate the remaining parameters with simulation methods. These are the crucial parameters determining the cost of fertility (χ, ζ, ω), technological progress (μ_{mn}, μ_{ag}) and land conversion (ψ, ε). We now discuss each step in turn. The full set of parameters of the model is listed in Table 1.

Exogenous parameters

Starting with production technology, we need to select values for the share parameters ϑ, θ_K and θ_X, and for the elasticity of substitution σ. In manufacturing, the Cobb-Douglas functional form implies that the output factor shares (or cost components of GDP) are constant over time, and we use a standard value of 0.3 for the share of capital (see for example Gollin, 2002). In agriculture, the CES functional form implies that the factor shares are not constant, and we choose θ_X to approximate a value for the share of land in global agricultural output of 0.25 in 1960. For the capital-labour composite, we follow Ashraf et al. (2008) and also use a standard value of 0.3 for the share of capital. Taken together, these estimates of the output value shares in agriculture are broadly in agreement with factor shares for developing countries reported in Hertel et al.

[21] For the estimation the model is initialised in 1960 and solved up to 2260. For projection the model is initialised in 2010 and solved up to 2310.

Table 1: List of parameters of the model and associated numerical values

Imposed parameters			
ϑ	Share of capital in manufacturing	0.3	
θ_K	Share of capital in capital-labour composite for agriculture	0.3	
θ_X	Share of land in agriculture	0.25	
σ	Elasticity of substitution between land and the capital-labour composite	0.6	
δ_K	Yearly rate of capital depreciation	0.1	
S	Maximum increase in TFP each year	0.05	
$\lambda_{mn,ag}$	Labour productivity parameter in R&D	1	
γ	Inverse of the intertemporal elasticity of substitution	2	
η	Elasticity of altruism towards future members of the dynasty	0.001	
κ	Income elasticity of food demand	0.25	
β	Discount factor	0.99	
Initial values for the stock variables and calibrated parameters			
N_0	Initial value for population	3.03	
X_0	Initial the stock of converted land	1.35	
$A_{0,mn}$	Initial value for TFP in manufacturing	4.7	
$A_{0,ag}$	Initial value for TFP in agriculture	1.3	
K_0	Initial value for capital stock	20.5	
ξ	Food consumption for unitary income	0.4	
δ_N	Exogenous mortality rate	0.022	
δ_X	Rate of natural land reconversion	0.02	
Estimated parameters (range of estimates for relaxed goodness-of-fit objective in parenthesis)			
χ	Labour productivity parameter in child-rearing	0.153	(0.146 – 0.154)
ζ	Elasticity of labour in child-rearing	0.427	(0.416 – 0.448)
ω	Elasticity of labour productivity in child-rearing w.r.t. technology	0.089	(0.082 – 0.106)
μ_{mn}	Elasticity of labour in manufacturing R&D	0.581	(0.509 – 0.585)
μ_{ag}	Elasticity of labour in agricultural R&D	0.537	(0.468 – 0.545)
ψ	Labour productivity in land conversion	0.079	(0.078 – 0.083)
ε	Elasticity of labour in land conversion	0.251	(0.238 – 0.262)

(2012).[22]

As mentioned previously, the elasticity of substitution between land and the capital-labour composite input is a crucial parameter for long-run growth. If land is an essential input into agriculture it is expected to be less than one (Cobb-Douglas being the limiting case where it is essential only asymptotically), which is confirmed by empirical evidence reported in Wilde (2013). Using long-run data on land and other inputs in pre-industrial England, he finds robust evidence that $\sigma \simeq 0.6$. While external validity of these estimates may be an issue (in particular for the

[22] For 2007, the factor shares for the global agricultural sector reported in Hertel et al. (2012) are 0.15 for land, 0.47 for labour and 0.37 for capital. While there are no data on the global land factor share in 1960, it has been shown to be negatively correlated with income (Caselli and Feyrer, 2007), so that factor shares for developing countries are probably better estimates of the value shares prevailing at the global level in 1960. That said, our results are not significantly affected by variations in the estimated value shares within a plausible range.

currently developing countries with rapidly growing population), it reflects long-run substitution possibilities that are consistent with our CES functional form (10). We consider $\sigma = 0.6$ to be the best estimate available, and derive trajectories assuming $\sigma = 0.2$ and $\sigma = 1$ in the sensitivity analysis.

The yearly rate of capital depreciation δ_K is set to 0.1 (see Schündeln, 2013, for a survey and evidence for developing countries), and maximum TFP growth per year S is set to 5 percent. The latter number is consistent with evidence on yearly country-level TFP growth rates from Fuglie (2012), which do not exceed 3.5 percent. The labour productivity parameter in R&D λ_j is not separately identified from S, and we set it to 1 without affecting our results.

The next set of imposed parameters determines preferences over consumption and fertility. First, our central estimate for the income elasticity of food demand κ is 0.25, which is in line with estimates reported in Thomas and Strauss (1997) and Beatty and LaFrance (2005). Given the importance of this parameter and uncertainty about its value, in the sensitivity analysis we consider two alternative values: (i) $\kappa = 0.5$, which is towards the high end of plausible estimates for the poorest households (Banerjee and Duflo, 2007), and consistent with results in Subramanian and Deaton (1996) and Logan (2009); and (ii) $\kappa = 0$, which makes our representation equivalent to the constant per-capita food demand of Strulik and Weisdorf (2008), Vollrath (2011) and Sharp et al. (2012). Second, the elasticity of intertemporal substitution is set to 0.5 in line with estimates from Guvenen (2006), which corresponds with $\gamma = 2$. Given the constraint on η to maintain concavity of the objective function, we initially set it to 0.01 so that the planner effectively has a classical utilitarian objective. Intuitively, this implies that parents' marginal utility of fertility is almost constant, or that altruism towards the welfare of children remains constant as the number of children increases. Correspondingly, we also assume a high degree of altruism by setting the discount factor to 0.99, which implies a pure rate of time preference of 1 percent per year. We report sensitivity analysis for the case where altruism declines with n_t, in particular $\eta = 0.5$, and for a discount factor of 0.97.[23]

Initial values and external calibration

Starting values for the state variables are calibrated to observed quantities in 1960. Initial population N_0 is set to an estimate of the world population in 1960 of 3.03 billion (United Nations,

[23] In fact, as we show below, the estimation error is significantly higher if we assume $\eta = 0.5$, and only slightly lower for $\beta = 0.97$.

1999). Initial crop land area X_0 is set to 1.348 billion hectares (Goldewijk, 2001) and the total stock of natural land reserves that can be converted for agriculture is 3 billion hectares (see Alexandratos and Bruinsma, 2012). For the remaining state variables, sectoral TFP $A_{0,ag}, A_{0,mn}$ and the stock of capital K_0, there are no available estimates, and we target three moments. First, we use an estimate of world GDP in 1960 of 8.79 trillion 1990 international dollars (Maddison, 1995; Bolt and van Zanden, 2013). Second, we obtain an estimate of world agricultural production by assuming that the share of agriculture in total GDP in 1960 is 15% (see Echevarria, 1997). Third, we assume that the marginal product of capital in 1960 is 15 percent. While this may appear relatively high, it is not implausible for developing economies (see Caselli and Feyrer, 2007). Solving for the targeted moments as a system of three equations with three unknowns gives initial values of 4.7 and 1.3 for TFP in manufacturing and agriculture respectively, and a stock of capital of 20.5.

Three other parameters of the model are calibrated to observed quantities. First, the parameter measuring food consumption for unitary income (ξ) is calibrated such that the demand for food in 1960 represents about 15% of world GDP, which is consistent with the calibration targets for initial TFP and capital stock. This implies $\xi = 0.4$, for the base case of $\kappa = 0.25$. Second, the mortality rate δ_N is calibrated by assuming an average adult working life of 45 years (United Nations, 2013), which implies $\delta_N = 0.022$. We vary that assumption in the sensitivity analysis, using $\delta_N = 0.015$ instead, in other words a 65 year working life. Finally we set the period of regeneration of natural land to 50 years so that $\delta_X = 0.02$.

Estimation of the remaining parameters

The seven remaining parameters $\{\mu^{mn,ag}, \chi, \zeta, \omega, \psi, \varepsilon\}$ are conceptually more difficult to tie down using external sources, and we therefore estimate them using simulation-based structural methods. The moments we target are taken from observed trajectories over the period 1960 to 2010 for world GDP (Maddison, 1995; Bolt and van Zanden, 2013), world population (United Nations, 1999, 2013), crop land area (Goldewijk, 2001; Alexandratos and Bruinsma, 2012) and sectoral TFP (Martin and Mitra, 2001; Fuglie, 2012; Alston and Pardey, 2014).[24] For each time series, we target one data point for each five-year interval, denoted τ, yielding 11 data points for each

[24] Data on TFP is derived from TFP growth estimates and are thus more uncertain than other trajectories. Nevertheless, a robust finding of the literature is that the growth rate of economy-wide TFP and agricultural TFP is on average around 1.5-2% per year. To remain conservative about the pace of future technological progress, we assume TFP growth was at 1.5 percent between 1960 and 1980, declined to 1.2 percent from 1980 to 2000, and was equal to 1 percent over the last decade of the estimation period.

targeted quantity (55 points in total).[25] The data are reported in Appendix B.

The targeted quantities in the model are respectively $Y_{t,mn} + Y_{t,ag}$,[26] N_t, X_t, $A_{t,mn}$ and $A_{t,ag}$, and we formulate a minimum distance estimator as follows. For a given vector of parameters υ, we solve the model and obtain the values for each targeted quantity, which we denote $Z^*_{\upsilon,k,\tau}$, where k indexes targeted quantities. We then compute the squared deviations between the solution of the model and observed data points $Z_{k,\tau}$, and sum these both over k and τ to obtain a measure of the estimation error over time and across targeted variables. Formally the error for a vector of parameters υ is given by:

$$error_\upsilon = \sum_k \left[\sum_\tau \left(Z^*_{k,\tau} - Z_{k,\tau}\right)^2 \Big/ \sum_\tau Z_{k,\tau} \right], \tag{17}$$

where the error for each variable is scaled to make these comparable. Therefore, our estimation procedure is essentially non-linear least squares defined over several jointly determined model outcomes. Importantly the error for each vector of parameters is computed for all targeted variables in one run of the model, so that all the parameters are jointly rather than sequentially estimated.

In order to select the vector of parameters that minimises the goodness-of-fit objective (17), we simulate the model over the domain of plausible parameter values, starting with bounds of a uniform distribution, which is our initial 'prior' for the parameters. For elasticity parameters, these bounds are 0.1 and 0.9 and for the labour productivity parameters we use 0.03 and 0.3. We then solve the model for 10,000 randomly drawn vectors of parameters and evaluate the error between the simulated trajectories and those observed. Having identified a narrower range of parameters for which the model approximates observed data relatively well, we reduce the range of values considered for each parameter and draw another 10,000 vectors to solve the model.

[25] Considering five-year intervals smooths year-on-year variations and allows us to focus on the long-run trends in the data. Using yearly data would not change our results. Similarly, we use the level of TFP rather than its growth rate to mitigate the impact of discontinuities implied by the TFP growth rates.

[26] In the model investments in sectoral TFP $I_{t,A_j} = \lambda_j (L_{t,A_j}/N_t)^{\mu_j}$ and in land conversion $I_{t,X} = \psi L^{\varepsilon}_{t,X}$ are not intermediate goods (they are not used in period t production) and hence could be included in our simulated measure of GDP. In practice, however, these activities represent a very small share of total production, and their exclusion does not affect our quantitative results.

This algorithm gradually converges to the estimates reported in Table 1.[27]

4 Quantitative results

This section provides the quantitative results of the analysis. We start by reporting targeted and non-targeted trajectories over the estimation period, and discuss the goodness-of-fit of the model and associated parameter estimates. We then present implications of the model up to 2100. Finally we present sensitivity of our results to a number of assumptions underpinning our approach.

4.1 Estimation results: 1960-2010

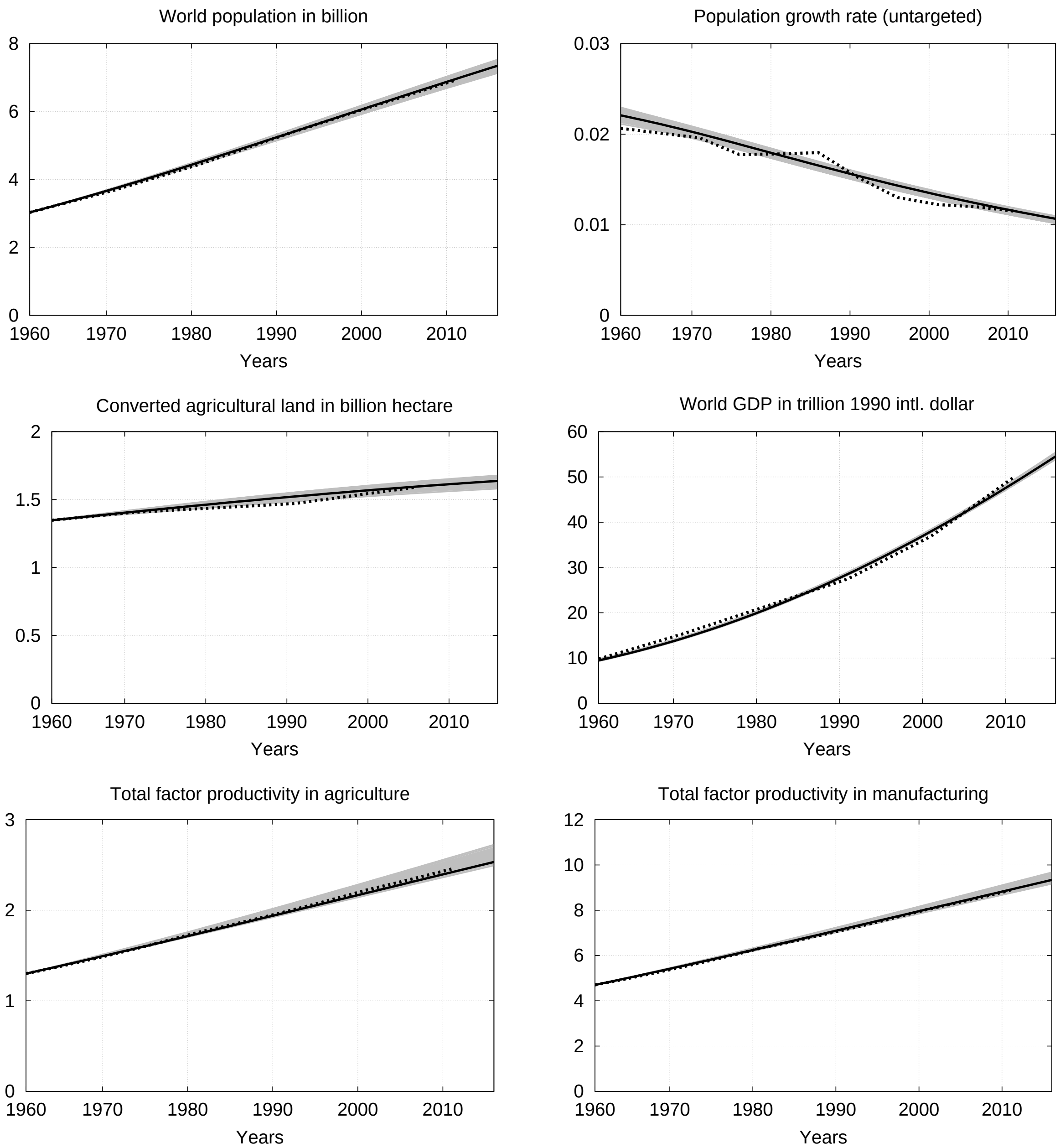
Trajectories for the targeted quantities over the period 1960 to 2010 are reported in Figure 1. More specifically, we compare the observed trajectories for world GDP, world population, crop land area and sectoral TFP against simulated trajectories obtained from the estimated model. By definition the estimated parameters are selected to minimise the distance between observed and simulated trajectories through equation (17), and they are reported in Table 1.

The model is able to closely replicate observed trajectories, with a relative squared error of 3.52 percent across all variables. The difference between the model and observed trajectories is mainly driven by the error on output (3.3 percent), followed by land (0.1 percent) and population (0.03 percent). In Figure 1 we also report runs for which the goodness-of-fit objective is relaxed by 10% relative to the best fit achieved, as represented by the shaded area. In other words, the shaded area reports the set of simulated trajectories with an error of 3.9 percent at most. The associated range of parameters is reported in Table 1.

Having considered the fit of the model to targeted trajectories we now consider non-targeted trajectories. First, because fertility and population growth are central determinants of potential Malthusian constraints, the model should also closely match changes in the population growth rate even though it is not directly targeted by the estimation. Indeed, because observed population growth rates are more volatile than the level of population, providing a good fit in terms of the

[27] As for other simulation-based estimation procedures involving highly non-linear models, the uniqueness of the solution to the estimation of the parameters cannot be formally proved (see Gourieroux and Monfort, 1996). Our experience with the model suggests however that the solution is unique, with no significantly different vector of parameters providing a comparable goodness-of-fit objective. In other words, estimates reported in Table 1 provide a global solution to the estimation problem. The fact that we simultaneously estimate the whole vector of parameters makes the criteria highly demanding, as changing one parameter will impact trajectories across all variables in the model.

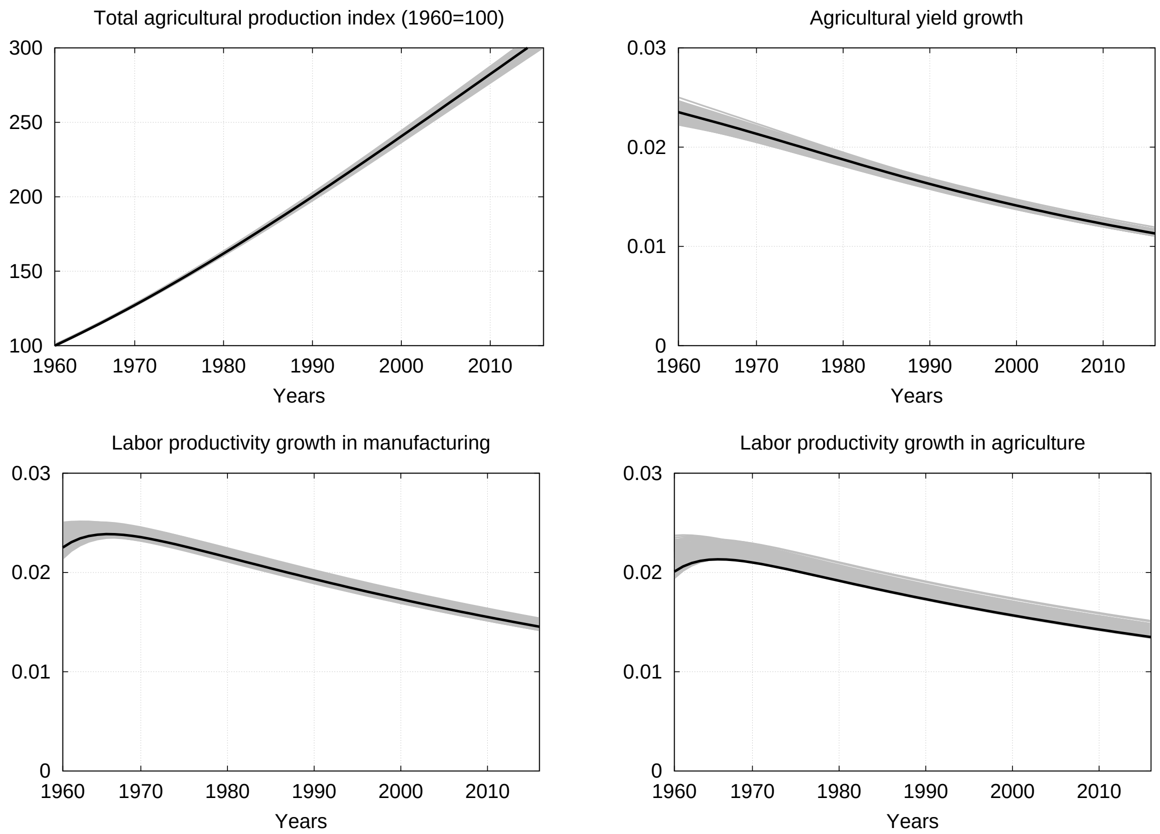
Figure 1: Estimation of the model 1960-2010



population level does not necessarily imply that the model provides a good representation of the decline in population growth. As shown in the top right panel of Figure 1 the model closely replicates the decline of population growth observed in the past fifty years.

A second measure not directly targeted in the estimation that is important for the analysis is the evolution of agricultural output over time. According to FAO, global agricultural output has grown by 2 percent per year on average from 1960 to 2010, or an equivalent of 269 percent over

Figure 2: Non-targeted trajectories 1960-2010



that period (Alexandratos and Bruinsma, 2012). As shown in Figure 2 agricultural output in our model increased by 279 percent over the same period. An implication is that the model provides a good account of the industrialisation process as measured by the size of the agricultural sector relative to total GDP. Similarly, the model provides a good account of growth in agricultural yields, shown in Figure 2, as compared to figures reported in Alston and Pardey (2014), 2% per year from 1961 to 1990 and 1.8% from 1990 to 2005, and Alexandratos and Bruinsma (2012), 1.9% per year from 1960 to 1985 and 1.4% from 1985 to 2007.

The model does less well regarding the control variables, namely the allocation of capital and labour (aside from fertility which provides plausible figures for the cost of children, discussed below). In particular, the share of labour allocated to agriculture relative to the manufacturing sector declines from around 40% in 1960 to 27% in 2010, which is lower than observations (in

2010 around 40% of the world labour force was employed in agriculture).[28] Nevertheless, Figure 2 shows that labour productivity growth (in terms of output per worker) in both manufacturing and in agriculture are in line with expectations.

Another approach to evaluate the goodness-of-fit of the quantitative model is to assess whether the estimated parameters are in a plausible range of values. We emphasise, however, that comparison with external sources is not straightforward because the estimated parameters are conditional on the model from which they are estimated. It is also important to bear in mind that the estimates we report cannot be interpreted as the technology parameters of a representative firm: we solve the model as a central planner problem, and externalities driving a difference between the social optimum and decentralised solution will be reflected in the estimates (since simulated trajectories fit historical data and thus factor in externalities and market imperfections).[29] That being said, quantitative evidence reported in Tournemaine and Luangaram (2012) suggests that any difference is likely to be small. Specifically, using a model with endogenous R&D and fertility, they show that differences in equilibrium growth rates prevailing under centralised and decentralised allocations occur in the third or fourth decimal places. Illustrating this in the context of our model, we find that increasing average manufacturing TFP growth by 0.0005 (0.05 percentage points) over the estimation period is equivalent to a change in the R&D parameters μ_{mn} from 0.581 to 0.543. This suggests that the technology parameter of a representative firm is likely to fall in the range of values reported in Table 1, and that the choice of a solution concept is not of critical importance for our results.

We start by discussing our estimates for the production function of effective labour units, which capture the costs of child rearing and education. For example, Jones and Schoonbroodt (2010) report calibrated value for the cost of children in terms of years of output for the U.S. around 1970, which ranges from 4.5 to 15.4. Jones and Schoonbroodt (2014) further estimate the cost of children in terms of both time and goods. The time cost amounts to 15 percent of work time, while the goods cost amounts to around 20 percent of household income. In our model the *implied* time cost of children increases from 7.5 years ($\overline{\chi}_t$ = 0.133) in 1960 to 17.9 years in 2010 ($\overline{\chi}_t$ = 0.056). While our 2010 estimate then appears to be high, remember that it combines the

[28] The share of capital allocated also declines from around 40% in 1960 to 30% in 2010, although the stock of capital used in agriculture increases over time.

[29] Importantly, we stress that our objective is not to obtain estimates for the structural parameters of a representative firm. Rather, we want the model to rationalise observed trajectories in order to study the joint determination of outcomes, and the estimated parameters provide the flexibility for doing this.

time and goods costs of children.

A key component of the cost of fertility is the advancement of technology, and the elasticity of fertility with respect to technology (ω) can also be compared to the empirical evidence derived from Herzer et al. (2012). In particular, they estimate that the long-run elasticity of fertility with respect GDP growth is around -0.0018.[30] In our model, a one percent increase in TFP (and hence GDP) reduces fertility by -0.00089 *in the same period*, or about half of the long-term impact. Our elasticity estimate is hence in the same ballpark.

The elasticity of labour in R&D activities (μ_j) is also discussed in the literature. However, there is disagreement on what this parameter should be. In particular, Jones and Williams (2000) argue that it is around 0.75, while Chu et al. (2013) use a value of 0.2. These two papers however rely on thought experiments to justify their choices. According to our results, a doubling of the share of labour allocated to R&D would increase TFP growth by around 50%. We are also not aware of comparable evidence for our estimates related to land clearing. Note however that that these estimates rationalise the relatively slow development of agricultural land area as compared to agricultural output and thus reflect forces determining the allocation of land, such as the demand for pastures and urban areas.

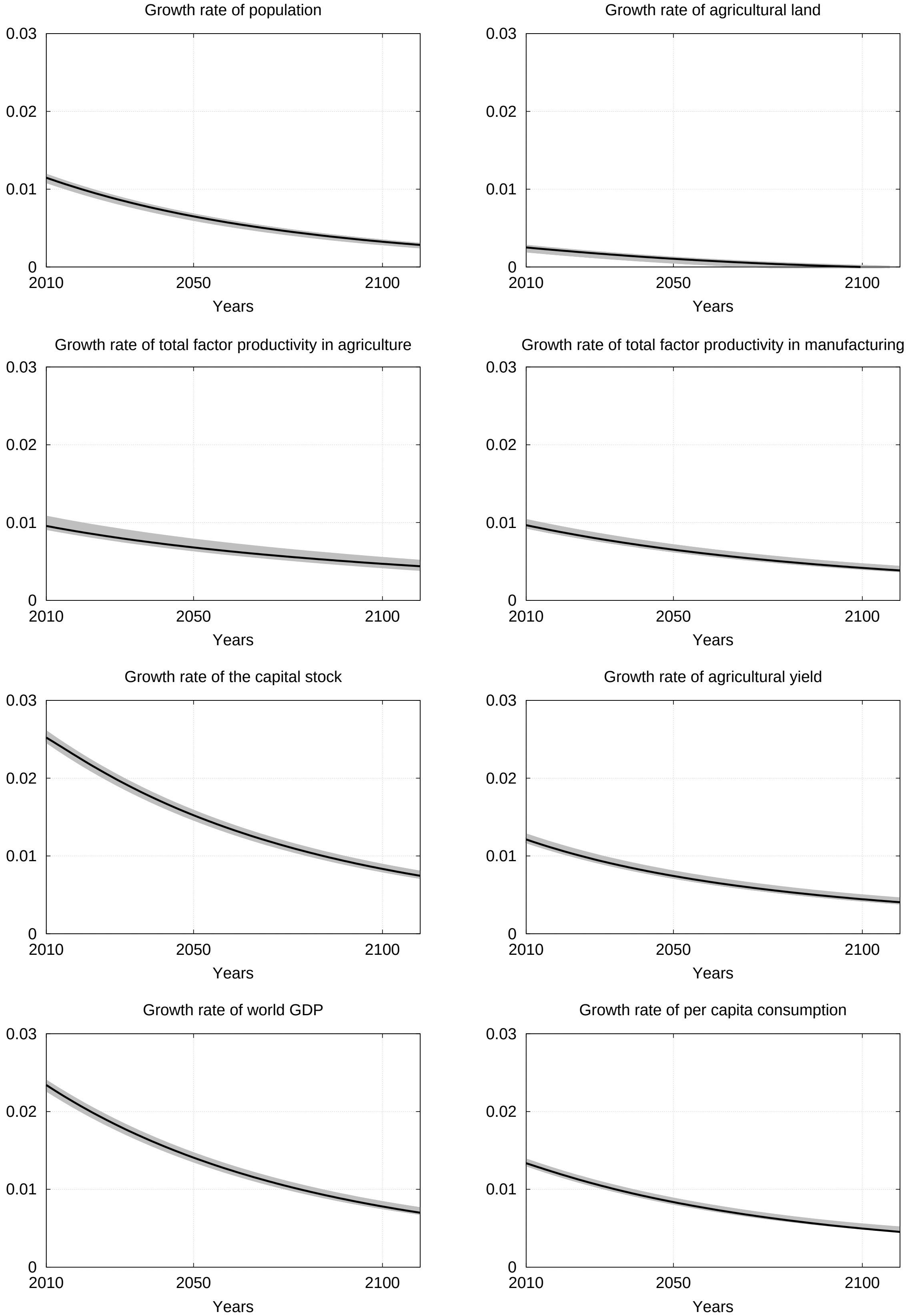
Despite the difficulties in assessing the magnitude of estimates, estimation results suggest that the implications of the quantitative model are broadly in agreement with global development trends observed over the past 50 years. In fact, given that the model is based on several components whose empirical relevance have been demonstrated in the literature, the finding that it can rationalise several key features of global development dynamics is not a surprise. Nevertheless, it provides confidence that the model can be used to study implications for the future evolution of the system.

4.2 Global projections: 2010 – 2100

We next describe projections implied by the estimated model. Figure 3 displays the growth rate of key variables from 2010 to 2100. The main feature of these paths is that they all decline towards a balanced growth trajectory where population, land and capital reach a steady state. Agricultural

[30] More specifically they estimate a long-run cointegrating relationship between the crude birth rate and the log of GDP, with their central estimate being -5.83. For a one percent increase in GDP, this implies a reduction of the crude birth rate of -0.058, or -0.0018 percent at their mean fertility level of 33. In a model with country-specific time trends, they report an elasticity of -3.036, which is associated with an elasticity of -0.0009 and almost identical to our own estimate.

Figure 3: Growth rate of selected variables 2010 – 2100



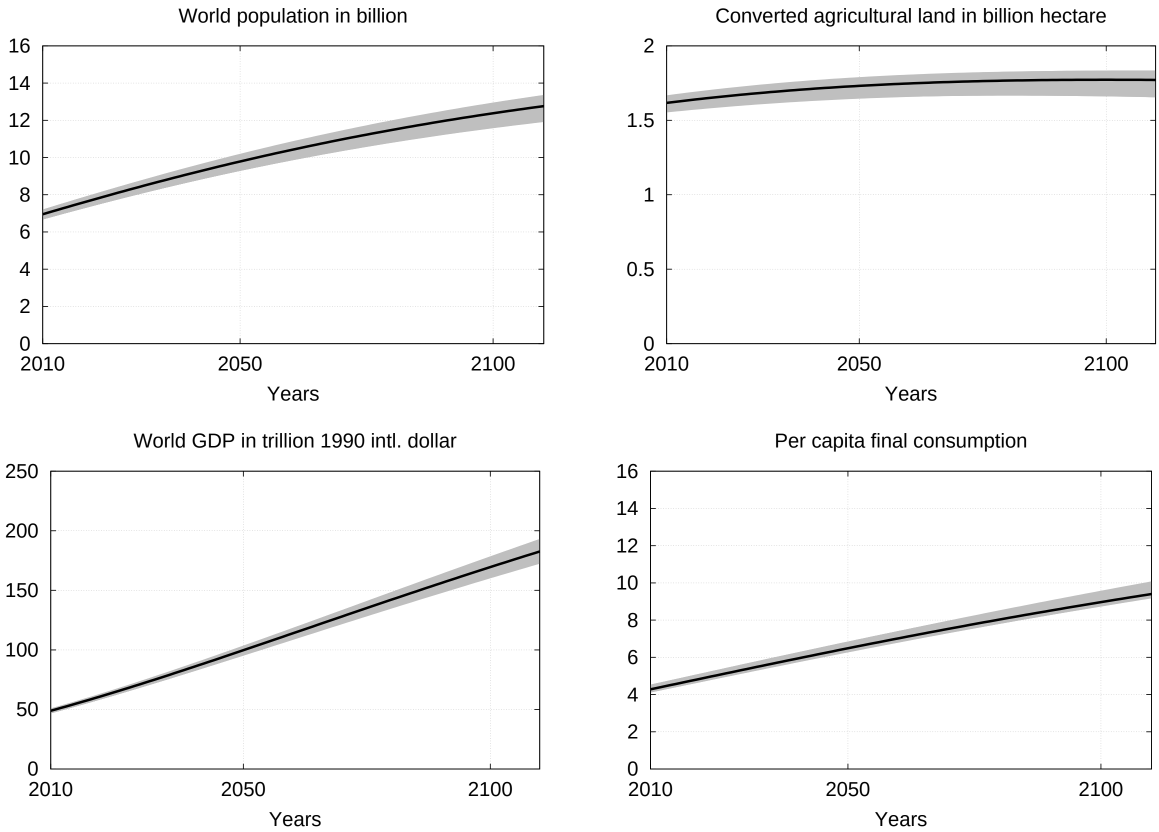
land area is the first state variable to reach a steady state as its growth rate becomes negligible by 2050. Thus the total amount of land that can be used for agriculture is never exhausted. Population growth on the other hand remains significantly above zero over the whole century, being around 0.3 percent by 2100. Thus the model is far from predicting a complete collapse of population growth over the coming fifty years. Nevertheless population growth continues to decline after that, being around 0.1 percent in 2150.

The pace of technological progress also declines over time, starting at around one percent per year and reaching about half of one percent by the end of the century. This has the consequence that, over time, labour productivity and the educational costs of children grow less quickly than in the period 1960-2010. This is the main explanation for why population growth does not fall more quickly, which in turn implies a relatively high population *level* reported in Figure 4 (see also Appendix B). In particular, world population is around 9.85 billion by 2050, and with continued population growth over the entire century, global population reaches 12.4 billion by 2100. The shaded band for the population growth rate, which represents a range of alternative pathways for vectors of parameters with a slightly lower fit, implies a range of possible 2100 population levels between 11 and 13 billion. We discuss how our results compare to projections from other sources in Section 5.

Our model indicates that a significant increase of population over the century is compatible with food production possibilities. Between 1960 and 2010, agricultural output in the model increased by 279 percent, and projections from the fit indicate an increase by a further 67 percent between 2010 and 2050. After 2050, our model suggests a further increase in agricultural output of 31 percent by 2100, so that by the end of the century agricultural output roughly doubles relative to the current level. This can be compared to 80 percent growth in population and a 95 percent increase in per-capita income.

In light of these results, the fact that agricultural land area stabilises at around 1.78 billion hectares is an important finding. First, as with population growth, land conversion will mostly occur in developing countries, as agricultural area in developed countries has declined and presumably will continue to do so. A net increase in global agricultural land thus implies that developing countries are likely to experience a proportionally larger amount of land being brought into the agricultural system. Second, TFP growth in agriculture remains below 1 percent, which is a fairly conservative trajectory, and indicates that the pace of technological progress does not

Figure 4: Projections for selected variables 2010 – 2100



need to be very high to allow for sustained growth in agricultural output. Third, the halt of agricultural land expansion suggests that the elasticity of substitution (σ) is high enough to allow agricultural output to grow from the accumulation of capital (we return to the role of σ in the sensitivity analysis). Indeed, although the share of capital allocated to agriculture declines over time, the stock of capital in agriculture almost doubles between 2010 and 2050.[31] This would mainly represent improvements to irrigation facilities. Both technology improvement and capital accumulation are reflected in the growth rate of agricultural yield (Figure 3), measuring growth in agricultural output per hectare used in agricultural production.

Finally, the growth rate of GDP falls from more than two percent in 2010 to less than one percent in 2100, which implies that world GDP doubles by 2050 and more than triples by 2100. Similarly, per-capita consumption more than doubles by 2100 relative to 2010.

[31] As expected, both the share and the quantity of labour allocated to agriculture decline over time.

4.3 Sensitivity analysis

We now report the results of sensitivity analysis with respect to a number of assumptions we have made: substitution possibilities in agriculture (σ), income elasticity of food demand (κ), the elasticity of utility with respect to fertility (η), the discount factor (β) and the expected working lifetime ($1/\delta_N$). For each change in the value of a parameter, it is necessary to re-estimate the vector of parameters to match observed data over the period 1960-2010. Here we focus on trajectories for two of the main variables of interest, population and agricultural land, against our baseline results discussed above. We report the vector of estimates associated with each sensitivity run in Table 2.

The parameter σ determines the elasticity of substitution between land and the capital-labour composite input in the agricultural production function. Our baseline case is obtained under the assumption that $\sigma = 0.6$, which follows from empirical work by Wilde (2013). However, evidence with regard to this parameter remains scarce, and it is one of the important determinants of the demand for agricultural land (and in turn the ability to produce food and sustain the population), so that considering alternative assumptions seems in order.

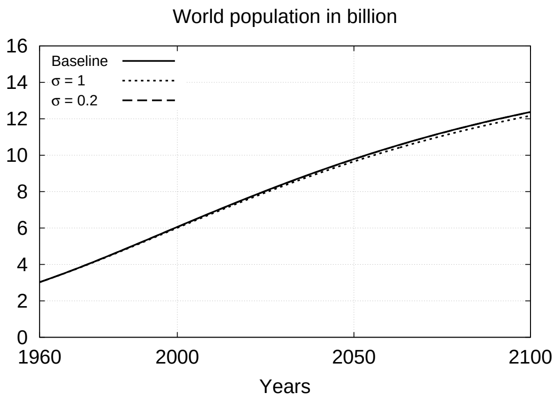
We therefore re-estimate the model assuming alternatively that $\sigma = 1$, so that agricultural production is Cobb-Douglas, and $\sigma = 0.2$, which we interpret as a lower bound on substitution possibilities in agriculture. The results reported in Figure 5 demonstrate that the choice of σ has a small impact on land conversion and virtually no impact on population. As expected, a high value of σ implies less land conversion, since other factors can be more easily substituted when the marginal cost of land conversion increases. Conversely, a lower σ makes land more important in agriculture, so that the overall area of agricultural land is higher. However, estimating the model over 50 years of data largely ties down the trajectory for land use in a robust manner, irrespective of the choice of σ. Estimates of labour productivity in land conversion imply a higher (lower) conversion cost under $\sigma = 0.2$ ($\sigma = 1$). Estimates of the marginal productivity of labour in agricultural R&D also adjust, implying lower productivity for $\sigma = 0.2$, exemplifying interdependencies in our estimation procedure. The fit of the model remains very similar.

A second important driver of the food market equilibrium is the way in which the demand for food evolves over time. In our baseline set of assumptions, we have assumed that the income elasticity of food demand is $\kappa = 0.25$, which is consistent with evidence from Thomas and Strauss (1997) and Beatty and LaFrance (2005). However there is again uncertainty around the econo-

Table 2: Estimates supporting the sensitivity analysis

Parameter	Baseline	$\sigma = 0.2$	$\sigma = 1$	$\kappa = 0.5$	$\kappa = 0$	$\eta = 0.5$	$\beta = 0.97$	$\delta_N = 0.015$
χ	0.153	0.155	0.151	0.155	0.138	0.205	0.155	0.104
ζ	0.427	0.417	0.427	0.402	0.509	0.399	0.460	0.516
ω	0.089	0.085	0.089	0.078	0.091	0.161	0.087	0.091
μ_{mn}	0.581	0.575	0.581	0.591	0.581	0.751	0.523	0.525
μ_{ag}	0.537	0.549	0.509	0.591	0.426	0.482	0.383	0.512
ψ	0.079	0.063	0.083	0.071	0.081	0.078	0.083	0.077
ε	0.251	0.174	0.256	0.216	0.218	0.239	0.243	0.186
Estimation error	0.035	0.033	0.035	0.108	0.036	0.189	0.029	0.045

Figure 5: Sensitivity analysis on substitution possibilities in agriculture



metric evidence, and Subramanian and Deaton (1996) and Logan (2009) report estimates closer to $\kappa = 0.5$. While such estimates can be considered to be on the high end (Banerjee and Duflo, 2007), it is nevertheless interesting to see what it implies in terms of aggregate development trajectories. At the other extreme, our second assumption is in line with other papers in the growth literature cited previously and uses $\kappa = 0$ (i.e. food production is proportional to population).

Results from the model re-estimated under alternatives values of κ are reported in Figure 6.[32] The main difference relative to the baseline occurs for the case where per-capita food demand is not related to income, as it induces the population trajectory to be significantly above the baseline, reaching more than 14 billion by 2100. This suggests that per-capita income, through its effect on diets and increasing food demand, plays an important role in constraining population growth

[32] Note that under each assumption the parameter ξ is re-calibrated to ensure that aggregate food production in 1960 remains approximately equivalent to 15% of world GDP.

Figure 6: Sensitivity analysis on the demand for food

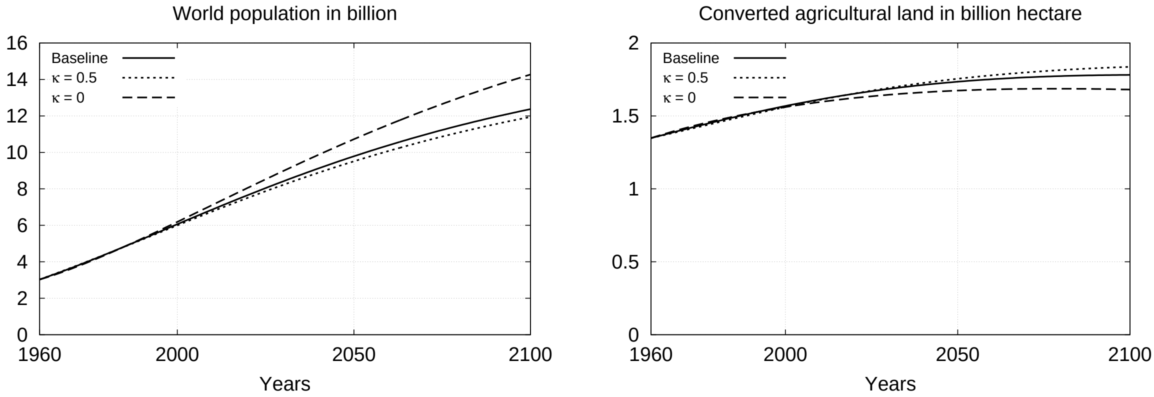
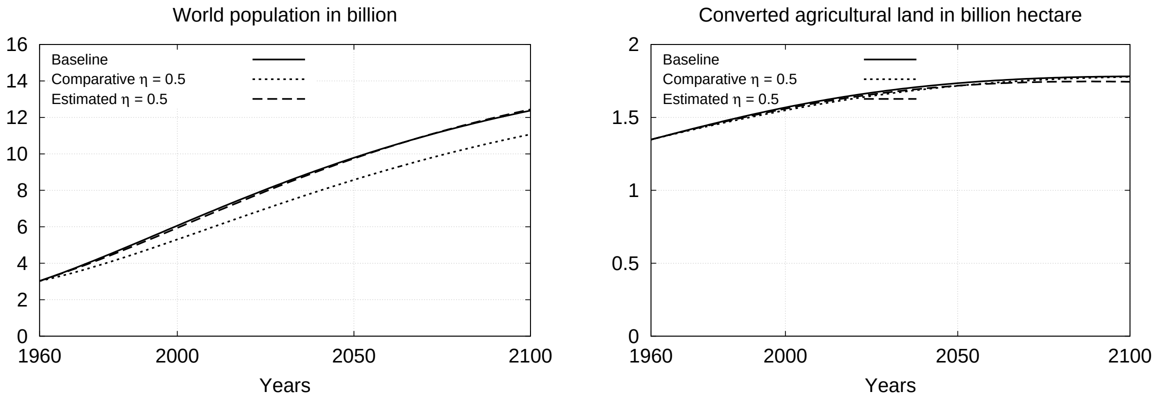


Figure 7: Sensitivity analysis on altruism towards children



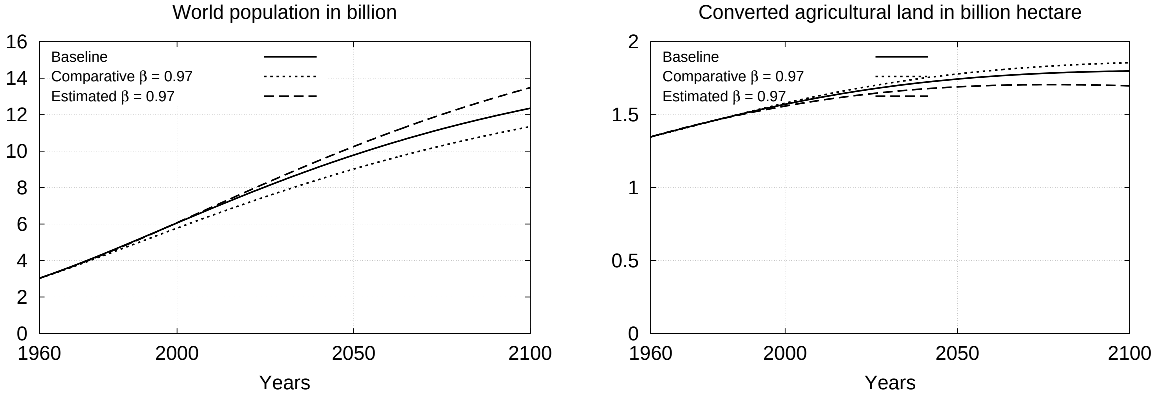
over the long run. The trajectory for land, however, remains close to its baseline counterpart.

The third sensitivity test we conduct targets η, the elasticity of utility with respect to fertility. We consider the case of $\eta = 0.5$, so that the marginal utility of fertility (and population) declines more rapidly than under our baseline assumption of $\eta = 0.01$.[33] We re-estimate the parameters of the model so that the model fits observed trajectories given $\eta = 0.5$, and report the resulting trajectories in Figure 7. In addition, we also report trajectories obtained with $\eta = 0.5$ but where the baseline parameter estimates are retained. This can be thought of as a comparative-static experiment (we label these trajectories "comparative").

As Figure 7 shows, when the model is not re-estimated, reducing η while keeping the estimated parameters to their baseline values implies lower population growth. This results from putting

[33] Note that in our setting an average utilitarian objective corresponds to $\eta = 0$, but it implies that the objective function is not globally concave.

Figure 8: Sensitivity analysis for the discount factor

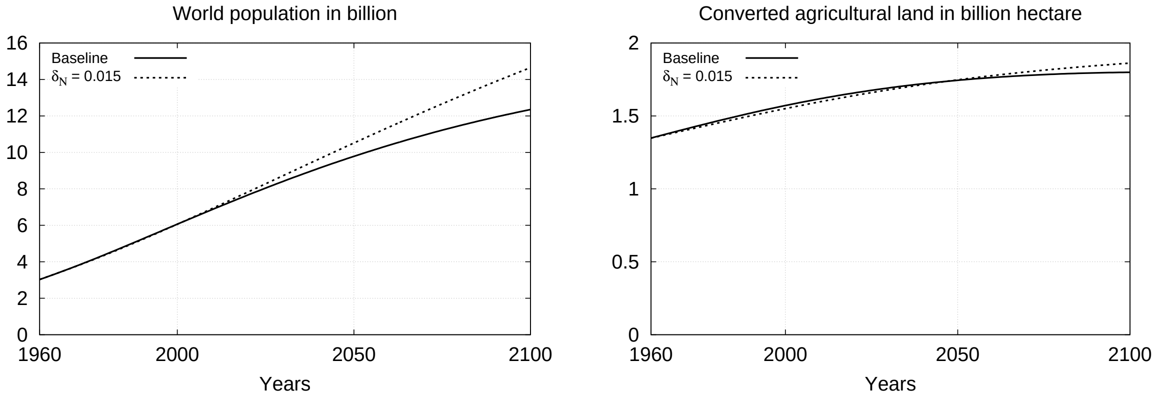


less weight on the welfare of future members of the dynasty, so that the dynastic head reallocates resources to increase its own consumption at the expense of population growth. However, once we re-estimate the model to observed trajectories over 1960 to 2010, the population path is virtually identical to the baseline trajectory. Note that the estimated parameters under $\eta = 0.5$ are very different from those in the baseline case, and the estimation error is significantly higher (see Table 2).

The fourth parameter we vary is the discount factor. The baseline value of $\beta = 0.99$ implies a relatively low discount rate, and we instead use $\beta = 0.97$. More specifically, Figure 8 shows trajectories associated with re-estimating the model to 1960-2010 data under the assumption that $\beta = 0.97$, as well as a 'comparative-static' exercise in which we set $\beta = 0.97$ while keeping other parameters to their baseline values. Intuitively, reducing β gives less weight to the welfare of future members of the dynasty, thus reducing the demand for children and lowering population growth. This implies that the population trajectory associated with a lower discount factor but retaining the baseline estimates is lower than the baseline trajectory. Moreover reducing the discount factor implies a lower saving rate, so that there is less capital available for agricultural production, and more land is needed to compensate.

However, by re-estimating the model to 1960-2010 data under the assumption $\beta = 0.97$, we find that the opposite is true. As compared to the baseline, a lower discount factor implies a higher long-run population, while the agricultural land area is smaller. As Table 2 shows, estimates of the cost of fertility imply higher labour productivity and more weakly decreasing returns to labour, and hence a lower marginal cost of fertility both within and across periods. In turn, the accumulation of labour becomes cheap relative to capital and land, incentivising the accumulation of population

Figure 9: Sensitivity analysis on the expected working lifetime



as a substitute for the accumulation of capital and land. This result contrasts with changes in η, which did not directly affect incentives to accumulate capital and land.

The final sensitivity test is on the death rate δ_N, or equivalently the expected working lifetime $1/\delta_N$. We illustrate the effect of this parameter by using a somewhat extreme value of 65 years, corresponding to $\delta_N = 0.015$. Trajectories are reported in Figure 9. As expected this implies a larger long-run population, reaching more than 10 billon in 2050 and around 15 billion by 2100. The impact of this parameter is mostly felt in the long run, as it implies that the growth rate of population declines less rapidly over time, on account of the larger expected benefits associated with effective labour units. This result confirms the importance of δ_N as a driver of population dynamics, as demonstrated by Jones and Schoonbroodt (2010) and Strulik and Weisdorf (2014). In practice however a change of this magnitude is unlikely, as future increases in life expectancy will be at least partly compensated by an increase of mortality associated with an ageing population.

Overall, the sensitivity analysis illustrates how our estimates are affected by structural assumptions, but at the same time it shows that the resulting projections do not change significantly. This can also be interpreted as further evidence that the choice of a particular solution concept is unlikely to alter our main conclusions. If we solved for a competitive equilibrium instead of a social planner's allocation, while retaining the *baseline* vector of parameter estimates, externalities would imply that fewer resources are allocated to R&D. In turn, economic growth would be lower. However, if the model could be re-estimated estimated by solving for a decentralised allocation, the resulting estimates that would be consistent with observations over the last fifty years would imply higher labour productivity in R&D activities, and in turn very similar growth trajectories.[34]

5 Discussion

The central feature of our work is to endogenise the evolution of quantities that are jointly determined along the development path, integrating plausible components from growth theory into an empirical framework. The dynamic relationship between these variables is informed by structurally estimating the model, selecting the parameters to minimise the distance between observed and simulated trajectories. Our model thus treats the representation of preferences and technology as fixed, with the dynamics being driven exclusively by structural assumptions.

By contrast, existing projections by others rely on exogenously determined drivers as the main source of variation. For example, UN population projections are based on an assumption that all countries around the world converge towards a replacement fertility rate of 2.1 over the next century, irrespective of their starting point.[35] The resulting fertility and population trajectories, shown in Figure 10, imply a global population of 9.6 billion in 2050 and 10.9 billion in 2100, by which time the population growth rate is close to zero. The assumption that fertility converges to a replacement level mainly derives from extrapolating observed convergence of *developed* countries. Note, however, that empirical evidence in developing countries suggests no clear pattern of convergence towards a low fertility regime (Strulik and Vollmer, 2015).

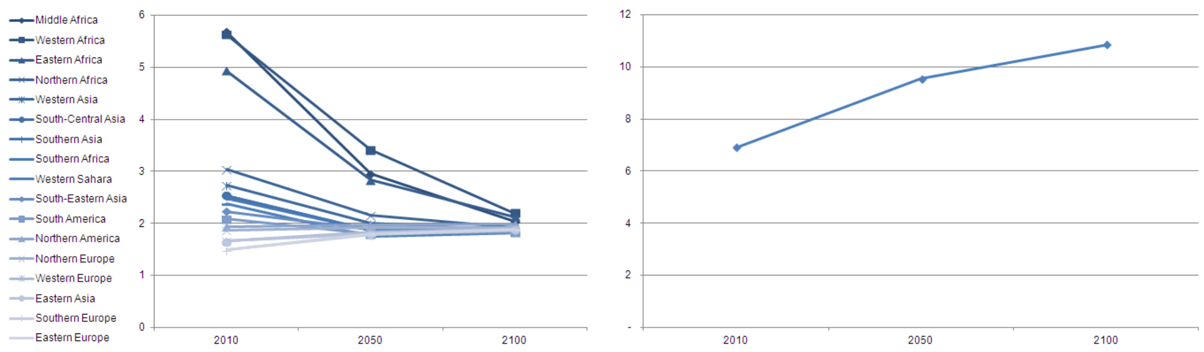
Thus one implication of our work is a novel perspective on population dynamics. Specifically, in our projections population growth declines over time but remains positive (and significantly so) in 2100. While uncertainty over such a time horizon cannot be overstated, a key finding of our analysis is therefore that population does not reach a steady state in the foreseeable future. In our model, this essentially derives from the inertia in the system, and because better economic prospects will sustain the demand for children despite an increasing cost associated with child-rearing and education. In fact, the slowdown of technology accumulation is reflected in a slowdown in the decline in fertility, so to speak, so that the decline in population growth

[34] Note that an important assumption here is the absence of a scale effect. If the model featured a scale effect, so that technological progress were a function of population, the planner could exploit it by generating higher population growth and in turn higher economic growth. Because the long-run properties of the model would differ, an equilibrium with higher population would presumably prevail.

[35] The UN uses a so-called 'cohort-component projection method', i.e. it works from the basic demographic identity that the number of people in a country at a particular moment in time is equal to the number of people at the last moment in time, plus the number of births, minus the number of deaths, plus net migration, all of this done for different age groups. This requires assumptions about fertility, mortality and international migration rates.

Figure 10: United Nations population projections 2010 – 2100 (United Nations, 2013)

(a) Regional fertility (b) World population



itself slows down. Because population growth falls more slowly than in the existing population projections of the United Nations (2013), our model produces higher levels of global population. In particular, while United Nations (2013) projects a 57% increase in 2100 population relative to 2010, our results suggest a 78% increase, or a 1.5 billion difference.[36]

Our work also integrates population development in the wider debate about food production and the evolution of agricultural productivity, where key contributions from natural sciences include Godfray et al. (2010) and Tilman et al. (2011). Our qualitative results can also be compared to agricultural and land use projections by FAO reported in Alexandratos and Bruinsma (2012). As noted in the introduction, the FAO projections rely on a number of exogenous factors, such as per-capita income from the World bank, population growth (Alexandratos and Bruinsma, 2012, employ the 2008 UN projections) and growth in agricultural yields (assumed to be 0.8 per year from 2010 to 2050). Given this, Alexandratos and Bruinsma (2012) forecast that global agricultural output will grow by 58 percent between 2010 and 2050, while cropland area is expected to increase by 2.5 percent to 1.66 billion hectare.

Conceptually, our results confirm the widespread expectation that the long-standing processes of growth in population and land conversion are in decline, and imply a "smooth landing". This stems from a quality-quantity trade-off: shifting from a quantity-based economy with rapid population growth and associated land conversion, towards a quality-based economy with investments

[36] Our results are also significantly higher than results of previous UN projections, as their figures have been revised upwards systematically in recent years. For example, in the 2008 projections global population would peak at 9.4 billion, an assumption still implicit in many policy discussions. The view that population growth will come to a halt over the coming century is also present in alternatives sources of demographic projections (see e.g. Lutz et al., 2014). Probabilistic projections using the UN's latest (2012) revision suggest that there is a 95 percent chance that in 2100 the population will lie between 9 and 13 billion (Gerland et al., 2014).

in technology and education, and lower levels of fertility. Land is the first quantity to *endogenously* reach a steady state, with agricultural land area being around 10% larger than in 2010. We find, however, that a halt in land conversion is consistent with sustained growth in food demand and agricultural output (67% increase between 2010 and 2050) as well as mildly optimistic trajectories of technological progress in the future (less than 1% per year).

Structural estimation of the model across several interlinked outcomes and over a relatively long period of time implies that our quantitative results are quite robust to different assumptions. This is notably the case for the land constraint, which is unlikely to bind in most configurations. This result is consistent with the past fifty years, during which agricultural production almost tripled, while growth in agricultural land was below twenty percent. However, this does not imply that food will not remain a problem for many areas of the world. We take a highly aggregated view of the problem, and food security is very likely to remain of concern at the regional level.

6 Concluding comments

One of the key challenges associated with global population growth is the ability of the economy to produce food. In this paper we have proposed a model in which population, technology and land use are jointly determined. Being based on plausible ingredients from the growth literature, we have shown that the model can match quite well the evolution of key economic time series over recent history. Our results suggest that sustained population growth over the coming century is compatible with an evolution of agricultural output close to what has been observed in the past, mainly on account of technological change and capital accumulation. Furthermore, estimating the model over fifty years implies that our conclusions are fairly robust in their account of development in the long run.

While this work provides a first attempt to see future population development, technology and potential Malthusian constraints from the perspective of economic growth theory, our approach necessitated a number of simplifications and opens a number of avenues for future research. First, declining fertility implies population ageing, which may affect both the mortality rate and labour productivity, and in turn economic growth. For example, Mierau and Turnovsky (2014) include an age-structured population in a general equilibrium growth model, although they treat the demographic structure as exogenous for the model to remain tractable. Integrating a richer representation of population heterogeneity into a model with endogenous fertility remains an important research topic. Second, we have abstracted from uneven economic development across regions, whereas fundamental drivers of fertility and growth will differ across the globe. Regional heterogeneity also raises interesting questions related to international trade, migration, and technology diffusion. Third, we have focused on baseline trajectories consistent with recent history, and our framework also provides a rich empirical framework to study policies affecting key drivers of long-run growth. Finally, there may be factors (such as water) affecting the ability to produce food, which are not included in the model and whose scarcity may increase in the future. Incorporating such constraints would constitute another interesting area for future work.

Appendix A

Derivation of the objective function (equation 2)

This section details the derivations necessary to obtain the dynastic (social) planner's utility, equations (2) and (13). Most of the steps involve standard assumptions and we closely follow Jones and Schoonbroodt (2010) in their treatment of a positive survival probability.

Starting from the recursively-defined utility function in equation (1):

$$U_t = u(c_t) + \beta b(n_t) \sum_{i=1}^{n_t} U_{i,t+1} \,,$$

we assume that (i) parents survive with probability $1 - \delta_N$, (ii) children are identical, and (iii) parents care about their (surviving) selves as much as they care about their children. This implies:

$$U_t = u(c_t) + \beta b((1 - \delta_N) + n_t)[(1 - \delta_N) + n_t]U_{t+1} \,.$$

Note that assuming $\delta_N = 1$ (agents live only one period) brings us back to the original Barro-Becker preferences considered in Section 2. Denoting $\tilde{n}_t = (1 - \delta_N) + n_t$, the utility of the dynastic head is obtained by sequential substitution starting from $t = 0$:

$$\begin{aligned}
U_0 &= u(c_0) + \beta b(\tilde{n}_0)\tilde{n}_0 U_1 \\
&= u(c_0) + \beta b(\tilde{n}_0)\tilde{n}_0[u(c_1) + \beta b(\tilde{n}_1)\tilde{n}_1 U_2] \\
&= u(c_0) + \beta b(\tilde{n}_0)\tilde{n}_0 u(c_1) + \beta^2 b(\tilde{n}_0) b(\tilde{n}_1)\tilde{n}_0\tilde{n}_1[u(c_2) + \beta b(\tilde{n}_2)\tilde{n}_2 U_3] \\
= \cdots &= \sum_{t=0}^{\infty} \beta^t u(c_t) \left(\prod_{\tau=0}^{t} b(\tilde{n}_\tau)\tilde{n}_\tau \right) + \lim_{t\to\infty} \beta^{t+1} \left(\prod_{\tau=0}^{t} b(\tilde{n}_\tau)\tilde{n}_\tau \right) U_{t+1}
\end{aligned}$$

where the limit term is assumed to be zero. We will further assume that the function $b(\cdot)$ has a standard constant elasticity form, $b(\tilde{n}) = \tilde{n}^{-\eta}$, and write population dynamics (16) as:

$$N_{t+1} = N_t + n_t N_t - \delta_N N_t = N_t[(1 - \delta_N) + n_t] = N_t \tilde{n}_t$$

and hence we have that

$$\begin{aligned}\prod_{\tau=0}^{t} b(\tilde{n}_\tau)n_\tau &= \tilde{n}_0^{1-\eta}\cdot\tilde{n}_1^{1-\eta}\cdot\tilde{n}_2^{1-\eta}\cdot\ \ldots\ \cdot\tilde{n}_t^{1-\eta} \\ &= \left(\frac{N_1}{N_0}\right)^{1-\eta}\cdot\left(\frac{N_2}{N_1}\right)^{1-\eta}\cdot\left(\frac{N_3}{N_2}\right)^{1-\eta}\cdot\ \ldots\ \cdot\left(\frac{N_t}{N_{t-1}}\right)^{1-\eta}=\left(\frac{N_t}{N_0}\right)^{1-\eta}.\end{aligned}$$

This gives the following expression for the time zero utility function:

$$U_0=\left(\frac{1}{N_0}\right)^{1-\eta}\sum_{t=0}^{\infty}\beta^t u(c_t)N_t^{1-\eta}$$

where N_0 is a constant and does not affect choices. This is equation (13), while equation (2) can be obtained by recalling that $N_t^{1-\eta}=b(N_t)N$.

Derivation of equation 7

Write the dynastic household's optimisation problem as

$$\mathcal{L}=\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{c} u(c_t)b(N_t)N_t+\mu_{t,N}\left[N_{t+1}-\chi(L_{t,N},A_{t,mn})\right]+\mu_{t,X}\left[X_{t+1}-\psi(L_{t,X})\right] \\ +\theta_{t,X}[\overline{X}-\psi(L_{t,X})]+\theta_{t,N}\left[N_t-L_{t,mn}-L_{t,N}-L_{t,X}-L_{t,ag}\right] \\ +\theta_{t,ag}\left[A_{t,ag}Y_{ag}(L_{t,ag},X_t)-\overline{f_t}N_t\right]\end{array}\right\}$$

Substituting in the budget constraint, $c_t = 1/N_t\ w_t L_{t,mn}$, the necessary first-order conditions for a maximum include that

$$\frac{\partial\mathcal{L}}{\partial L_{t,mn}}=u'(c_t)b(N_t)w_t-\theta_{t,N}=0$$

$$\frac{\partial\mathcal{L}}{\partial L_{t,N}}=-\mu_{t,N}\frac{\partial\chi(L_{t,N},A_{t.mn})}{\partial L_{t,N}}-\theta_{t,N}=0$$

$$\frac{\partial\mathcal{L}}{\partial L_{t,ag}}=\theta_{t,ag}A_{t,ag}\frac{\partial Y_{ag}(L_{t,ag},X_t)}{\partial L_{t,ag}}-\theta_{t,N}=0$$

$$\frac{\partial\mathcal{L}}{\partial L_{t,X}}=(-\mu_{t,X}-\theta_{t,X})\psi'(L_{t,X})-\theta_{t,N}\leq 0$$

The marginal effect on household welfare of fertility in period t, at the optimum, can be characterised as

$$\frac{\partial\mathcal{L}}{\partial N_{t+1}}=\beta u(c_{t+1})\left[b'(N_{t+1})N_{t+1}+b(N_{t+1})\right]+\mu_{t,N}+\beta\theta_{t+1,N}-\beta\theta_{t+1,ag}\overline{f_{t+1}}=0$$

We now proceed by using the first-order conditions on the controls to eliminate the shadow prices. It is straightforward to verify that –

$$\mu_{t,N} = \left[-u'(c_t)b(N_t)w_t\right] \Big/ \frac{\partial\chi(L_{t,N}, A_{t,mn})}{\partial L_{t,N}} \ ,$$

$$\theta_{t+1,N} = u'(c_{t+1})b(N_{t+1})w_{t+1} \text{ and}$$

$$\theta_{t+1,ag} = \left[u'(c_{t+1})b(N_{t+1})w_{t+1}\right] \Big/ \left[A_{t+1,ag}\frac{\partial Y_{ag}(L_{t+1,ag}, X_{t+1})}{\partial L_{t+1,ag}}\right]$$

Equation 7 follows immediately.

Appendix B Observed and simulated data

The table below reports both observed and simulated data from 1960 to 2100, by 10-year intervals. Note that agricultural area is only available for 2005.

Year	Population (billion)		Population growth (%)		Crop land area (billion ha)		GDP (trillions 1990 intl. $)	
	Observed	Simulated	Observed	Simulated	Observed	Simulated	Observed	Simulated
1960	3.03	3.03	0.021	0.022	1.37	1.35	8.79	9.5
1970	3.69	3.74	0.020	0.020	1.41	1.41	14.46	14.3
1980	4.45	4.51	0.018	0.018	1.43	1.47	19.98	20.6
1990	5.32	5.32	0.015	0.015	1.47	1.52	26.88	28.5
2000	6.13	6.14	0.012	0.013			36.93	38.0
2005					1.59	1.60		
2010	6.92	6.95	0.011	0.011		1.62	49.97	48.9
2020		7.73		0.010		1.66		60.9
2030		8.47		0.009		1.69		73.7
2040		9.17		0.007		1.71		87.2
2050		9.82		0.006		1.73		101.2
2060		10.43		0.006		1.74		115.3
2070		10.99		0.005		1.76		129.5
2080		11.50		0.004		1.77		143.6
2090		11.96		0.004		1.78		157.4
2100		12.38		0.003		1.78		170.9

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