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## Abstract

Offshoring has gained a significant momentum in recent years. Firm size appears to be the leading factor differentiating firms that offshore from those that do not. We present a model that blends offshoring, or trade in tasks, with a Melitz-style model of monopolistic competition with heterogeneous firms and show that this is indeed the case. Accounting for firm heterogeneity offers new tools for analyzing the effects of offshoring on the employment dynamics within an individual firm and at the aggregate sector level. We show that offshoring unambiguously reduces per-firm labour demand in smaller firms, but has ambiguous effects in larger firms. As a result, irrespective of whether or not the number of firms operating in the offshoring nation increases, the overall sector employment may increase or decrease. Policies promoting free trade have a significant role to play in job creation. The model also permits a straightforward derivation of positive and normative effects of offshoring: trade in tasks increases productivity of active firms and improves welfare in the offshoring nation.

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Offshoring has gained a significant momentum in recent years. Firm size appears to be the leading factor differentiating firms that offshore from those that do not. We present a model that blends offshoring, or trade in tasks, with a Melitz-style model of monopolistic competition with heterogeneous firms and show that this is indeed the case. Accounting for firm heterogeneity offers new tools for analyzing the effects of offshoring on the employment dynamics within an individual firm and at the aggregate sector level. We show that offshoring unambiguously reduces per-firm labour demand in smaller firms, but has ambiguous effects in larger firms. As a result, irrespective of whether or not the number of firms operating in the offshoring nation increases, the overall sector employment may increase or decrease. Policies promoting free trade have a significant role to play in job creation. The model also permits a straightforward derivation of positive and normative effects of offshoring: trade in tasks increases productivity of active firms and improves welfare in the offshoring nation.

**Keywords:** Trade in tasks, offshoring, heterogeneous firms, employment, productivity, welfare
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# 1. Introduction

The phenomenon of offshoring emerged a couple of decades ago. It began with the movement of manufacturing jobs abroad, when firms started to take advantage of the growing productivity-adjusted wage gap between developed and developing nations and lower telecommunication and transportation costs. More recently, the advancement in communication and information technologies has made it increasingly possible to offshore parts of production processes in service sectors as well (call centers, back office, accounting and computer programming operations are only a few examples). This trend has generated mounting concerns among politicians and media reporters in the developed nations about the consequences of offshoring. The most prominent fears relate to job losses, which reached their height during the 2004 Presidential election campaign in the United States[1].

These developments fostered a rich body of literature, particularly empirical, investigating the causes and effects of the international fragmentation of production. One of the most active lines of empirical research is to determine how many jobs are transferred out of the nation country and the effects on the total labour demand as a result of offshoring. Yet, a few important aspects of the offshoring phenomenon have been thus far ignored. We argue that firm heterogeneity is one of the omitted elements. For example, only by introducing firm heterogeneity it is possible to explain the anecdotal evidence that offshoring is done predominantly by large firms[2]. Furthermore, accounting for firm heterogeneity allows us to trace the employment decisions taken at the individual firm level which we show are crucial in analysing the aggregate labour effects of offshoring.

In the present paper we blend GRH-type offshoring[3] with a Melitz (2003) style model of monopolistic competition with heterogeneous firms and two nations of which one has access to better technology in producing all goods, and trade is costly[4]. The advantage of this is evident. In Melitz-type models firms' heterogeneity materializes in the form of productivity differences, which, unsurprisingly, may lead individual firms to react differently to the possibility of going offshore. The primary factors

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[1] See Mankiw and Swagel (2006) and Amiti and Wei (2005).

[2] See, for example, Gentle (2004) for offshoring trends in the financial services industry.

[3] We follow Grossman and Rossi-Hansberg (2006a,b) in defining offshoring as 'trade in tasks'. More specifically, the term 'offshoring' implies that "tasks formerly undertaken in one country are now being performed abroad". Stated differently, "offshoring includes not only foreign sourcing from unrelated suppliers, but also the migration abroad of some of the activities conducted by a multinational firm" (see Grossman and Rossi-Hansberg, 2006b, p.1). Note, in the literature the former is usually referred to as 'international outsourcing' (Helpman, 2006), whereas the latter as 'vertical' FDI.

[4] According to Baldwin and Robert-Nicoud (2007a), most of the theoretical work can be broken into two categories, by relating offshoring either to certain sectors or to certain tasks. One of them – the 'JK offshoring' – views offshoring as a technical progress story, whereby nations specialize in producing those intermediate goods and services in which they have comparative advantage. Contributions in this area include Deardorff (1998a, b), Findlay and Jones (2000), Jones and Kierzkowski (1990, 1998). In the other – Grossman-Rossi Hansberg (GRH) offshoring – technologically superior firms seek to reduce production costs by relocating the performance of some tasks into lower wage countries. Grossman-Rossi Hansberg (2006a,b), Baldwin and Robert-Nicoud (2007a,b) and Beverelli (2007) belong to this strand of literature.

determining which firms offshore in the equilibrium are related to the level of offshoring costs, which are modelled as fixed, and the magnitude of gains arising from the factor returns differential between two countries[5]. More precisely, trade in tasks prompts four distinct long-run equilibrium outcomes: (i) all firms offshore; (ii) most efficient of locally selling firms as well as all firms selling in both markets offshore; (iii) all locally selling firms keep production processes onshore, while all firms that sell locally and export offshore and (iv) only the most efficient firms selling in both markets offshore.

Besides showing how offshoring can be introduced into a heterogeneous firms' model and what the potential long-run equilibria are, the contribution of the paper is twofold. First we analyse the positive and normative effects of offshoring. Although our main focus is on the case where only the largest firms offshore, we show that the incidence of trade in tasks is qualitatively identical in all equilibrium outcomes. Offshoring reduces overall prices and improves welfare of the offshoring nation with opposite effects on the recipient country. In addition, while offshoring has an unambiguously adverse impact on the mass of firms operating in the low-technology nation, the mass of varieties produced in the high-technology nation and the mass of varieties consumed in both nations may increase or decrease. Importantly, the offshoring country gets to host more productive firms, as firms with high unit labour requirements are forced out of the industry – the so-called 'selection' effect.

Secondly, we address the above-mentioned worries that equate the number of jobs offshored to the number of jobs lost in the country and we argue that they are at best ungrounded. The model shows that offshoring may lead to job creation in the offshoring sector and this without accounting for the employment in the fixed costs sector. This increase in labour demand may come either as a result of the increase in the number of firms operating in the offshoring country or as a result of the increase in per-firm employment levels of larger firms. Note that one of the most interesting findings in the model is that offshoring causes an unambiguous decrease in per-firm employment in firms selling in the domestic market only. However, it may create jobs in firms serving both markets and this effect may be so strong as to increase the overall employment in the sector even if the number of active firms decreases. Crucially, we find that offshoring boosts overall labour demand provided that trade barriers are not too high.

Our findings are consistent with the empirical literature. The positive productivity effects of offshoring are documented by numerous studies at various levels of aggregation (see Olsen, 2006, for a survey). The most relevant for our purpose are Amity and Wei (2006a,b), Mann (2004) and Egger *et al* (2001), which focus on the general labour productivity effects (and not the productivity of low-skilled workers). Amity and Wei (2006a,b) estimate the effects of material and service inputs offshoring on productivity in 96 US manufacturing industries (2-digit data from Bureau of Labour Statistics). They

[5] Note offshoring in this model happens in one direction, as firms of the technologically advanced nation combine their own technology with cheaper labour abroad.

find that over the period 1992 to 2000 both types of offshoring had a significant positive impact on the productivity levels. The authors calculate that offshoring of service inputs accounted for around 10 percent of the average growth in labour productivity while offshoring of material inputs for 5 percent. Mann (2004) estimates that service offshoring in the IT industry in the US led to an annual increase in productivity of 0.3 percentage points between 1995 and 2002. Using a sample of 18 manufacturing industries (2-digit NACE) over 1990-1998, Egger *et al* (2001) find that offshoring to Eastern European countries improved significantly the productivity levels in Austria.

Consistent with our conclusions, empirical studies estimating the effects of offshoring on labour demand at sectoral level detect an array of relationships ranging from weak negative to positive (see Crinò, 2007, and Kirkeggard, 2007, for a survey). Amiti and Wei (2006a) examine the impact of offshoring on the US manufacturing employment at various levels of industry aggregation over 1992-2000. Using a panel of 450 industries (4-digit SIC), they identify a statistically significant negative effect of service offshoring on employment of a magnitude of 0.4 of a percent. However, this effect disappears in a more aggregated sample consisting of 96 industries (2-digit data from Bureau of Labour Statistics); notably, in some specifications it becomes significantly positive. According to the authors, there is "sufficient growth in demand in other industries within these broadly defined classifications to offset any negative effects [of service offshoring]" (Amiti and Wei, 2006a, p.29). Regarding material offshoring, although the study fails to detect any significant relationship in disaggregated data, it finds a significant positive employment effect at the aggregate level. In their companion study on 69 manufacturing industries in the UK between 1995 and 2001, Amiti and Wei (2005) find no evidence that offshoring exerts a negative effect on labour demand at the sectoral level. Rather, the service offshoring coefficient is positive and statistically significant in some specifications and the coefficient on material offshoring, although sometimes negative, is insignificant in all specifications. Similarly, the authors were unable to identify a robust negative relationship between service and material offshoring and employment in 9 services industries. In line with the message of our theoretical model the authors note that the insignificant results "may be explained by the level of industry aggregation. For example, a worker may lose her job due to outsourcing but then find a job in another firm within the same industry classification. So if there is sufficient job creation within the broadly defined sectors to offset any job loss, then the job loss effect of outsourcing would not show up in aggregate data" (Amiti and Wei, 2005, p.337)[6]. This strand of literature also includes a study by Gorg and Hanley (2005), who find that service offshoring has a small negative effect on labour demand in a panel of Irish electronics firms over the period 1990 to 1995. Finally, Ando and Kimura

[6] Note that Amiti and Wei (2005) refer to 'outsourcing', but by this term they mean 'international outsourcing': "we focus on international outsourcing, defined as the procuring of service or material inputs by a firm from a source in a foreign country" (see Amiti and Wei, 2005, p. 313). Therefore their terminology is identical to ours when we use the term 'offshoring'.

(2007a) analyze the effects of reallocation of production facilities by Japanese firms into East Asia[7]. They find that larger manufacturing firms (as measured by their employment levels) are more likely to reduce the number of domestic establishments and affiliates. In addition, firms that globalize their activities lead to domestic job creation at the individual firm level.

The paper is organized as follows. The next section presents a slightly modified version of Melitz (2003) model that is in the spirit of Helpman, Melitz and Yeaple (2004) which we augment by assuming technological differences between countries. In the same section we provide long run solutions for the pre-offshoring equilibrium and briefly describe positive and normative effects of differences in production technology. In section 3, we extend our model to offshoring. After introducing the assumptions, we analyze four types of long-run equilibria that may arise under different sets of model parameters. In section 5, we formally derive the long-run equilibrium for the case where only the most productive firms offshore and contrast it against the pre-offshoring equilibrium. The same section looks at the implications of trade in tasks on the per-firm employment and the total labour demand in the differentiated good sector. Section 6 concludes.

## 2. The model

### *2.1 Assumptions*

The model assumes two countries - Home and Foreign[8] - that use a single production factor, labour (*L*), to produce goods in two sectors, agriculture (*A*) and manufacturing (*M*). Labour is internationally immobile and each consumer supplies inelastically one unit of labour, meaning that each nation's endowment of labour matches its population, *L*.

All consumers share the same preferences, which are defined over a continuum of differentiated varieties indexed by $i \in \Theta$ and a homogenous good. The utility of the representative consumer is described by a two-tier Cobb-Douglas utility function:

$$U = C_M{}^{m} C_A{}^{1-m}; \quad C_M \equiv \left( \int_{i \in \Theta} c_i^{1-1/s} \, di \right)^{1/(1-1/s)} \tag{1}$$

where $C_A$ and $C_M$ are, respectively, the consumed quantity of the *A*-sector good and the composite of *M*-sector varieties; $c_i$ is the consumption of each available variety $i$ with $\Theta$ representing the set of all

[7] Note that the authors do not explicitly state whether 'globalizing corporate activities' (or FDI to which they alternatively refer) is meant to be offshoring or 'horizontal' FDI (where firms engaged in horizontal FDI replicate abroad the same activities as those performed domestically with the aim of avoiding trade costs). However, one of the findings of the paper is that reallocation of production into East Asia does not substitute for domestic operations, from which we infer that 'globalizing corporate activities' primarily means offshoring.
[8] Foreign variables are indicated by an asterisk '*'

potential varieties in sector *M*; $m$ is the expenditure share on the industrial good ($0 < m < 1$) and $s$ is the constant elasticity of substitution between each pair of *M*-sector varieties ($s > 1$).

Sector *A* is perfectly competitive and produces homogeneous good under constant returns to scale. The good is traded costlessly. The manufacturing sector *M* is a Dixit-Stiglitz monopolistic competition sector that produces a differentiated good consisting of a continuum number of varieties subject to increasing returns to scale at the level of individual firm. Marginal costs are constant, but increasing returns arise from a set of fixed costs incurred prior the production. International trade in this good is marked by 'iceberg' trade costs and selling one unit in the export market requires $t \geq 1$ units to be shipped[9].

Following Melitz (2003), the model allows for heterogeneity in marginal costs of manufacturing firms which arises due to differences in productivity levels. To enter the industry a potential entrant *j* bears the irreversible fixed variety-development cost, $F_I$, measured in labour units. The firm then draws a labour unit input coefficient $a_j$ generated from a density function *G(a)* with support on $0 \leq a \leq a_0$. As it is common in the Melitz model literature[10], we assume a Pareto distribution to have explicit solutions. Denoting by $a_0$ and *k* the 'scale' and the 'shape' parameters, the cumulative density function is given by:

$$G(a) = \frac{a^k}{a_0^k},\ 0 \leq a \leq a_0\ ,\ k \geq 1 \tag{2}$$

Upon observing the productivity draw, a firm may decide to exit the industry and not produce. If it chooses to produce, however, it has to pay additional fixed market-entry costs of $F_D$ and $F_X$ units of labour to enter the local and export markets, respectively[11]. The presence of these costs ensures that only firms with sufficiently high productivity levels produce; firms with large unit labour requirements have marginal costs inconsistent with breaking even in the local market and therefore exit. Under the regularity condition introduced below, firms with intermediate input coefficients sell only locally, while the most efficient ones are present in both markets[12].

[9] Trade is free when $t$ equals one; trade costs are prohibitive when $t \to \infty$

[10] See for example Helpman *et al* (2004), Baldwin and Forslid (2004) and Baldwin and Okubo (2005).

[11] The fixed market-entry costs reflect the costs of introducing a new variety into a market. These costs may include, among others, the costs of meeting market-specific regulations, forming a distribution and servicing network, establishing a brand name etc. These costs may be sunk or reoccurring each period; given that Melitz (2003) type of models, including the present one, ignore transitional dynamics, we assume that these costs are one-time sunk costs.

[12] In our analysis we view $a^{1-s}$ as a productivity index based on the fact that $a^{1-s}$ changes monotonically with labour productivity, $1/a$, since $s > 1$.

We assume that Foreign is technologically inferior in producing all goods. Technology is different in a Hicks-neutral sense (see Davis, 1995,) with all labour coefficients $1/g > 1$ times higher in F-country than in H-country. Thus production in the homogenous good sector requires $a_A$ units of labour per unit of output at Home and $a_A^* = a_A / g$ in Foreign. Similarly, if labour coefficient is $a_j$ for a differentiated variety *j* at Home, it is $a_j^* = a_j / g$ in Foreign. As it will be clear below, this is the key assumption to make offshoring feasible in the current set up.

We follow Melitz (2003) in focusing only on steady state equilibria.

### *2.2 Equilibrium analysis*

Consumer's problem is to maximize utility described in (1) subject to the budget constraint, $p_A C_A + \int_{i\in\Theta} p_i c_i di = E$ and $p_A C_A + \int_{i\in\Theta^*} p_i c_i di = E^*$, for Home and Foreign, respectively, with $p_A$ representing the price of one unit of agricultural good, $p_i$ the delivered price of one unit of variety *i* and *E* ($E^*$) the expenditure at Home (Foreign). The CES demand function facing the typical firm *j* takes the following form:

$$
\begin{aligned}
c_j &= \frac{p_j^{-s} mE}{P^{1-s}}, \quad P \equiv \left(\int_{i\in\Theta} p_i^{1-s}\, di\right)^{1/(1-s)} \\
c_j^* &= \frac{p_j^{-s} mE^*}{P^{*1-s}}, \quad P^* \equiv \left(\int_{i\in\Theta^*} p_i^{1-s}\, di\right)^{1/(1-s)}
\end{aligned}
\tag{3}
$$

where $P^{1-s}$ and $P^{*1-s}$ are the CES price indices for H and F. The solution also yields the well-known result of a constant division of expenditure between the homogenous and the manufacturing goods with the following demand functions:

$$
\begin{aligned}
C_M &= \frac{mE}{P} \quad \text{and} \quad C_A = \frac{(1-m)E}{p_A} \\
C_M^* &= \frac{mE^*}{P^*} \quad \text{and} \quad C_A^* = \frac{(1-m)E^*}{p_A}
\end{aligned}
$$

Perfect competition in the homogenous good sector yields marginal cost pricing. As long as the good is produced in both nations, free trade leads to factor price equalization. Taking good *A* as a numéraire and choosing units such that $a_A = 1$, sets its price to unity in both countries. The productivity adjusted labour wages are also equalized (the so-called effective factor price equalization), with nominal wage equal to unity in Home and $g$ in Foreign:

$$p_A = p_A^* = w = w^* / g = 1 \tag{4}$$

In the manufacturing sector, the expenditure and the price index $P$ ($P^*$) are exogenous from the standpoint of an individual firm due to the Dixit-Stiglitz monopolistic setting. Each firm takes the prices of other firms as given as there are many varieties produced in the equilibrium and the impact of its price on the economy-wide price index is negligible. In this case, the mark-up is constant and mill-pricing is optimal. The equilibrium prices for a typical home firm *j* in its local and export markets are given by the equality of the marginal cost, $mc_j$, and marginal revenue:

$$p_j(1-1/s) = mc_j \tag{5}$$

Therefore, the pricing rules for a typical *j* firm at Home and Foreign in local and export markets must satisfy[13]:

$$p_j^{HH} = \frac{wa_j}{1-1/s},\ p_j^{HF} = \frac{wa_j t}{1-1/s},\ p_j^{FF} = \frac{w^*(a_j/g)}{1-1/s},\ p_j^{FH} = \frac{w^*(a_j/g)t}{1-1/s}$$

Taking into account (4), we can refer to unit input coefficients $a_j$ as a measure of firm's specific marginal cost[14]. The pricing conditions in this case can be re-written as:

$$p_j^{HH} = p_j^{FF} = \frac{a_j}{1-1/s}, \qquad p_j^{HF} = p_j^{FH} = \frac{a_j t}{1-1/s} \tag{6}$$

Operating profits ignoring all fixed costs, $op_j$, of a typical firm *j* are defined as $(p_j - mc_j)c_j$. Manipulation of the pricing rule (5) yields $op_j = p_j c_j / s$. Using (3) together with (6), operating profits earned in the local and export markets by a Home and Foreign firm *j* are respectively given by:

$$op_j^{HH} = \left(\frac{a_j}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) \tag{7}$$

$$op_j^{HF} = f\left(\frac{a_j}{1-1/s}\right)^{1-s}\left(\frac{mE^*}{sP^{*1-s}}\right) \tag{8}$$

$$op_j^{FF} = \left(\frac{a_j}{1-1/s}\right)^{1-s}\left(\frac{mE^*}{sP^{*1-s}}\right) \tag{9}$$

$$op_j^{FH} = f\left(\frac{a_j}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) \tag{10}$$

[13] The superscripts *H* and *F* denote the direction of the sales.

[14] Note that while $a_j$ is a measure of firm *j*'s marginal cost in both countries, only for H-firms it also represents the unit labour requirement; for a *j*-firm in country F, the unit labour requirement is given by $a_j / g$.

It is interesting to note that lower cost firms set lower prices, are bigger in size (as measured by their sales) and employ more labour (see equations (3) and (6)). They also earn higher revenues and profits than less productive firms (see equations (7)-(10)).

The operating profits net of $F_I$ for an H-firm serving domestic and export markets, denoted by $p_{local}$ and $p_{exp}$, are respectively:

$$p_{local} = \left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) - F_D \tag{11}$$

$$p_{exp} = f\left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mE^*}{sP^{*1-s}}\right) - F_X \tag{12}$$

The existence of the market-entry costs, $F_D$ and $F_X$, ensures that not all the firms that paid variety development costs, $F_I$, will actually produce. Depending on the level of the marginal cost attributed in the productivity draw, the firm will belong to one of the three types: firms that do not produce (N-types), firms that produce but sell only in the domestic market (D-types) and firms that produce and sell in both domestic and export markets (X-types). In this case, the D-type and X-type firms earn[15]:

$$p_D = \left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) - F_D \tag{13}$$

$$p_X = \left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) + f\left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mE^*}{sP^{*1-s}}\right) - (F_D + F_X) \tag{14}$$

where $p_D$ and $p_X$ stand for the operating profits earned by D-types and X-types respectively. Analogous expressions hold for F-firms.

The cut-off levels of marginal costs, defining the thresholds for each group, are determined by the break even conditions for the sales in domestic and export markets[16]. At the cut-off level $a_D$ the operating profits are just enough to cover the fixed costs for entering the domestic market, $F_D$. Thus, the least productive firms, with labour coefficients in excess of $a_D$, expect negative profits and exit the industry. The cut-off $a_X$ is the productivity level which is consistent with breaking even in the exports market. The regularity condition, typically put in place in Melitz (2003) type of models, requires that $a_X < a_D$. This condition is necessary to ensure that only a fraction of producing firms export. As a result, firms with the highest productivity levels, i.e. an $a$ lower than $a_X$, find it

[15] It is straightforward that $p_{local} = p_D$ and $p_X = p_D + p_{exp}$.

[16] Notice that this part of the analysis takes as given that the firm has already paid the variety-development costs $F_I$.

worthwhile paying both $F_D$ and $F_X$ to serve local and export markets. Firms with unit labour requirements between $a_X$ and $a_D$ find it profitable to sell in the domestic market, but expect to lose money from exports. Using (11) and (12), the cut-off coefficients are determined by[17]:

$$F_D = \left(\frac{a_D}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) \tag{15}$$

$$F_X = f\left(\frac{a_X}{1-1/s}\right)^{1-s}\left(\frac{mE^*}{sP^{*1-s}}\right) \tag{16}$$

Analogous conditions hold for the F firms:

$$F_D = \left(\frac{a_D^*}{1-1/s}\right)^{1-s}\left(\frac{mE^*}{sP^{*1-s}}\right) \tag{17}$$

$$F_X = f\left(\frac{a_X^*}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) \tag{18}$$

Having identified the cut-off levels of marginal costs, the Home and Foreign manufacturing price indices are then:

$$P^{1-s} = \int_0^{a_D}\left(\frac{a}{1-1/s}\right)^{1-s} ng\left[a \middle| a_D\right]da + \int_0^{a_X^*} f\left(\frac{a}{1-1/s}\right)^{1-s} n^* g\left[a \middle| a_D^*\right]da \tag{19}$$

$$P^{*1-s} = \int_0^{a_D^*}\left(\frac{a}{1-1/s}\right)^{1-s} n^* g\left[a \middle| a_D^*\right]da + \int_0^{a_X} f\left(\frac{a}{1-1/s}\right)^{1-s} ng\left[a \middle| a_D\right]da \tag{20}$$

where $n$ and $n^*$ are the overall mass of varieties produced in each nation[18].

In the long run, free entry ensures that the total number of firms operating in each country adjusts to the point where a potential entrant expects to earn zero pure profits. The logic behind this is as follows. Bearing sunk innovation costs is risky, because the entrant may end up exiting the industry. So firms will be willing to pay $F_I$ only if the expected reward from introducing a new variety is at least as great as the cost of doing so. The free entry conditions are then expressed as equality between the expected operating profits of a potential entrant and the $F_I$:

[17] Note that given the way we set up the model, the cut off levels $a_D$ and $a_X$ represent both the marginal costs and the unit labour requirements; $a_D^*$ and $a_X^*$ are the cut-off levels of marginal costs with the respective unit labour coefficients equal to $a_D^*/g$ and $a_X^*/g$ .

[18] Notice that $n = mG(a_D)$ , where $m$ is the mass of firms with each level of unit labour requirement, $a$ . This means that $mdG(a) = ndG\left[a \middle| a_D\right]$. Analogous definitions hold for F country.

$$\int_0^{a_D}\left(\frac{a^{1-s}}{(1-1/s)^{1-s}}\frac{mE}{sP^{1-s}}-F_D\right)dG(a)+\int_0^{a_X}\left(\frac{fa^{1-s}}{(1-1/s)^{1-s}}\frac{mE^*}{sP^{*1-s}}-F_X\right)dG(a)=F_I \tag{21}$$

$$\int_0^{a_D^*}\left(\frac{a^{1-s}}{(1-1/s)^{1-s}}\frac{mE^*}{sP^{*1-s}}-F_D\right)dG(a)+\int_0^{a_X^*}\left(\frac{fa^{1-s}}{(1-1/s)^{1-s}}\frac{mE}{sP^{1-s}}-F_X\right)dG(a)=F_I \tag{22}$$

We focus on the case where both countries produce the differentiated varieties. This implies that there is a positive mass of entrants in each country, ensuring that free entry conditions hold.

Country's expenditure includes only labour payments, because pure profits in the economy at the aggregate level are driven to zero by the free entry[19]:

$$E = wL = L \text{ and } E^* = w^*L = gL = gE \tag{23}$$

The long-run equilibrium solutions for the cut-off levels of unit input coefficients and the number of active firms are simultaneously determined by the cut-off and free entry conditions. Using (23), the equations (15) through (22) provide explicit closed form solutions for the eight unknowns $a_D$, $a_X$, $a_D^*$, $a_X^*$, $n$, $n^*$, $P$ and $P^*$[20]. Thus, the equilibrium cut-offs are:

$$a_D^k = a_D^{*\,k} = \frac{F_I(1+k-s)a_0^k}{F_D(s-1)}\frac{1}{1+\Omega}, \qquad a_X^k = a_X^{*\,k} = \frac{F_I(1+k-s)a_0^k}{F_X(s-1)}\frac{\Omega}{1+\Omega}$$

where we defined $\Omega \equiv \left(\frac{1}{f}\right)^{\frac{k}{1-s}}\left(\frac{F_X}{F_D}\right)^{\frac{1+k-s}{1-s}}$, as it is common in the literature.

The ratio of the cut-off conditions suggests that the necessary and sufficient condition for the regularity condition $a_X < a_D$ to hold is:

$$\frac{F_X/f}{F_D} > 1 \tag{24}$$

Since $f \le 1$ and $s > 1$, (24) also implies that the openness parameter $\Omega$ is bound between zero and unity[21]. The variable $\Omega$ combines two types of trade barriers effects: fixed and variable costs of entering into the foreign market. It tends to zero when iceberg trade costs are prohibitive, $t \to \infty$, or when it becomes prohibitively costly to enter the foreign market relative to domestic, i.e. $F_X/F_D \to \infty$. With zero iceberg costs and $F_X = F_D$, $\Omega$ equals unity.

[19] Some active firms do earn pure profits, but these are balanced by the pure losses encountered by the firms whose marginal costs were too high inducing them to abandon production.

[20] A detailed description of the way to solve the model is presented in the Appendix 1.

[21] This can be easily seen by noting that $\Omega$ can be alternatively expressed as $f(F_X/fF_D)^{(1+k-s)/(1-s)}$.

Having solved for the cut-off levels of unit labour requirements, it is straightforward to solve for the remaining variables. The mass of varieties produced at Home and Foreign are respectively given by:

$$n = \frac{mE(1+k-s)}{F_D sk}\frac{1-g\Omega}{1-\Omega^2}$$
$$n^* = \frac{mE(1+k-s)}{F_D sk}\frac{g-\Omega}{1-\Omega^2}$$

The mass of firms operating at Home is larger than at Foreign ($n > n^*$) as $1-g\Omega > g-\Omega$[22]. This is expected as Home firms are more efficient than Foreign firms, due to superior technology. Notice, that as technological differences become less pronounced ($g$ increases), the mass of firms at Home decreases while at Foreign increases; as $g$ gets closer to 1 the mass of firms converges in both countries. Given that by regularity condition $\Omega < 1$, the mass of firms $n$ and $n^*$ is positive if and only if $\Omega < g < 1/\Omega$. Since by our assumption $g < 1$, the requirement $g < 1/\Omega$ is not binding, as $\Omega < 1$. So overall, the constraint for $g$ is: $\Omega < g < 1$.

As some varieties are sold only locally, not all varieties are consumed by all consumers. The mass of varieties available for consumption at Home, $n_C$, is a sum of varieties produced domestically and varieties exported from the other nation, i.e. $n_C = n + n_X^*$. Using the fact that $n_X^* = m^* G(a_X^*)$ and $n = mG(a_D)$, we have $n_C = n(1+(m^* {a_X^*}^k)/(m a_D^k))$. Since $n^* = m^* G(a_D^*)$, the mass of varieties available to Home consumers can be alternatively expressed as $n_C = n(1+(n^* {a_X^*}^k)/(n {a_D^*}^k))$. Analogous expressions hold for the mass of varieties consumed in country F. Substituting the solutions for the cut-off unit labour coefficients and the mass of firms we get:

$$n_C = \frac{mE(1+k-s)}{F_D sk}\left(\frac{1-g\Omega}{1-\Omega^2}\right)\left(1+\Omega\frac{F_D}{F_X}\frac{g-\Omega}{1-g\Omega}\right)$$
$$n_C^* = \frac{mE(1+k-s)}{F_D sk}\left(\frac{g-\Omega}{1-\Omega^2}\right)\left(1+\Omega\frac{F_D}{F_X}\frac{1-g\Omega}{g-\Omega}\right)$$

Contrasting the equilibrium values for $n_C$ and $n_C^*$, it follows that a larger mass of varieties is consumed at Home.

With regard to the long-run differentiated good price indices, these are given by:

[22] This can be seen by re-writing the inequality as $1+\Omega > g(1+\Omega)$ and noting that $g < 1$.

$$P^{1-s} = \frac{mEa_0^{1-s}}{(1-1/s)^{1-s}\, sF_D}\left(\frac{F_I(1+k-s)}{F_D(s-1)}\frac{1}{1+\Omega}\right)^{\frac{1-s}{k}}$$

$$P^{*1-s} = \frac{mgEa_0^{1-s}}{(1-1/s)^{1-s}\, sF_D}\left(\frac{F_I(1+k-s)}{F_D(s-1)}\frac{1}{1+\Omega}\right)^{\frac{1-s}{k}}$$

It follows directly that $P^{*1-s} = gP^{1-s}$, suggesting that Home nation enjoys a lower manufacturing price index ($P < P^{*}$).

Welfare can be evaluated using the indirect utility function for the preferences in (1), which is $V = E/\widetilde{P}$, where $\widetilde{P}$ is the perfect price index, defined as $\widetilde{P} \equiv p_A^{1-m} P^m = P^m$. Therefore, welfare rises when the manufacturing varieties price index decreases. As a result, Home's superior technology brings the country superior welfare gains comparing to F.

We complete our analysis of the pre-offshoring equilibrium by determining the aggregate labour demand in the differentiated good sector, excluding its usage in the fixed costs. The part of the manufacturing production process associated with variable costs requires an individual firm $j$ of country H to employ $l_j^{HH} = a_j c_j^{HH}$ units of labour for its local sales and $l_j^{HF} = a_j c_j^{HF} t$ for its export sales. Then the total labour used in the variable costs sector of manufacturing production, denoted by $L_{VC}$, is $L_{VC} = \int_0^{a_D} l^{HH} mdG(a) + \int_0^{a_X} l^{HF} mdG(a)$. This, along with (3), (6) and the fact that $mdG(a) = ndG[a|a_D]$, implies that:

$$L_{VC} = \int_0^{a_D} \frac{na^{1-s}}{(1-1/s)^{-s}}\frac{mE}{P^{1-s}}dG[a|a_D] + \int_o^{a_X} \frac{nfa^{1-s}}{(1-1/s)^{-s}}\frac{mgE}{P^{*1-s}}dG[a|a_D]$$

Using the cut-off conditions to substitute for the price indices and the model solutions for the equilibrium cut-off productivity levels, the total employment in the manufacturing sector, excluding the fixed costs, amounts to:

$$L_{VC} = \frac{nk(s-1)}{1+k-s}F_D(1+\Omega)$$

or

$$L_{VC} = \frac{mE(s-1)}{s}\frac{1-g\Omega}{1-\Omega}$$

if we substitute for the equilibrium mass of firms, $n$.

## 3. Offshoring: Implications for Home firms

### *3.1 Assumptions*

We now modify the basic model to allow manufacturing firms perform a part of their production process abroad. Following Baldwin and Robert-Nicoud (2007b), we assume that the production of differentiated varieties involves two 'tasks', indexed by 1 and 2, and firms are able to offshore the second task. The natural way to think of the tasks is to consider them as service tasks or segments of production where task's output is an intermediate input. We decompose a typical *j* firm's unit labour requirements into task-by-task coefficients as:

$$a_j \equiv a_{j1} + a_{j2}$$

We also assume that the unit input coefficients of the two tasks are proportional to each other and the factor of proportionality, $d$ , is constant across all industrial firms in both countries, i.e.:

$$a_{j2} = da_{j1},\ d > 0$$

We assume that the offshore production unit can only supply the home firm and that the 'output' of the offshored task is repatriated back to the country of origin at zero variable cost. However, performing a task abroad is costly and firms that choose to offshore incur fixed offshoring costs, $F_{OS}$, measured in labour units. These costs may include setting up a subsidiary to perform the offshored task, coordinating spatially the two separated tasks, communication between two units, etc. We model $F_{OS}$ as a sunk cost paid only once, independently of whether the firm sells locally or in both markets. This effectively implies that the total fixed costs paid by an offshoring firm are $F_D$+$F_{OS}$ if it is a D-type and $F_D$+$F_X$+$F_{OS}$ if it is an X-type.

Although performing a task abroad involves additional fixed costs, it is cost-saving at the level of firm's variable costs. This is because offshoring is about combining home nation's production technology with foreign labour. As such, offshoring firms get to pay the wage rate of the recipient country rather than factor's marginal product. With this in mind, offshoring in this model happens only in one direction and only by Home firms.

It follows then that offshoring reduces Home firms' marginal costs and rises their operating profits, *ceteris paribus*. Formally, since the second task is a constant proportion $d$ of the first and the foreign labour performing it is paid its local wage, $g$ , marginal cost function for an offshoring firm *j*, $mc_j^{OS}$ , takes the following form[23]:

[23] This is easy to see by noting that with offshoring $mc_j^{OS} = wa_{j1} + w^* a_{j2} = a_{j1} + ga_{j2} = a_{j1}(1+gd) = a_j(1+gd)/(1+d)$.

$$mc_j^{OS} = ba_j\,,\; b \equiv \frac{1+dg}{1+d} < 1$$

Therefore, pricing conditions for a typical offshoring firm *j* in the local and export markets are given by:

$$p_j^{HH\,OS} = \frac{ba_j}{1-1/s}\,,\quad p_j^{HF\,OS} = \frac{ba_j t}{1-1/s} \tag{25}$$

Clearly, an offshoring firm charges lower prices than a non-offshoring firm with the same unit labour requirement $a_j$; the difference in prices becomes more substantial as technological gap between two countries widens. Pricing rules in (25) immediately suggest that offshoring firm *j*'s operating profits in the local and export markets become:

$$op_j^{HH\,OS} = b^{1-s}\left(\frac{a_j}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) \tag{26}$$

$$op_j^{HF\,OS} = fb^{1-s}\left(\frac{a_j}{1-1/s}\right)^{1-s}\left(\frac{mgE}{sP^{*1-s}}\right) \tag{27}$$

Combining this with our assumption on the fixed costs yields per-firm operating profits net of $F_I$ for offshoring D- and X-types as:

$$p_D^{OS} = b^{1-s}\left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) - (F_D + F_{OS}) \tag{28}$$

$$p_X^{OS} = b^{1-s}\left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) + fb^{1-s}\left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mgE}{sP^{*1-s}}\right) - (F_D + F_X + F_{OS}) \tag{29}$$

The corresponding operating profits earned for local sales and exports are:

$$p_{local}^{OS} = b^{1-s}\left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) - F_D - F_{OS} \tag{30}$$

$$p_{exp}^{OS} = fb^{1-s}\left(\frac{a}{1-1/s}\right)^{1-s}\left(\frac{mgE}{sP^{*1-s}}\right) - F_X \tag{31}$$

Note that the expression for operating profits in the export market (31) does not include the fixed offshoring costs, $F_{OS}$. This is an implication of our assumption above regarding the cost structure by which export sales are free of $F_{OS}$ charge within an X-type firm.

Setting the operating profit expressions in (30) and (31) to zero, we obtain the cut-off conditions for the offshoring firms:

$$F_D + F_{OS} = \left( \frac{ba_D^{OS}}{1-1/s} \right)^{1-s} \left( \frac{mE}{sP^{1-s}} \right) \quad (32)$$

$$F_X = f\left( \frac{ba_X^{OS}}{1-1/s} \right)^{1-s} \left( \frac{mgE}{sP^{*1-s}} \right) \quad (33)$$

where $a_D^{OS}$ and $a_X^{OS}$ denote the cut-off labour requirements at which offshoring firms break even in the domestic and exports markets, respectively.

As $b^{1-s} > 1$, equations (26) and (27) show that the more efficient the firm, the larger are the gains from going offshore. Stated differently, a decrease in unit labour requirement causes operating profits of an offshoring firm to increase faster vis-à-vis a non-offshoring firm, *ceteris paribus*. Consequently, despite the existence of $F_{OS}$, there will always be some bigger firms for which gains from offshoring are substantial enough to make offshoring advantageous[24]. Smaller less productive firms may be discouraged by the fixed offshoring costs as the increase in their operating profits may not be sufficient enough to cover these additional costs.

To which types then the offshoring and non-offshoring firms belong? The answer to this question depends on the level of fixed offshoring costs and the wage gap between countries relative to other model parameters. In general, one can distinguish four equilibrium outcomes. To get intuition, start with zero $F_{OS}$ and a non-zero wage differential. In this situation all Home firms offshore, simply because offshoring brings gains at no additional cost. The same result occurs in the case when offshoring costs are very small and/or wage differential is very large (low $g$ ), such that operating profits under offshoring are in excess of operating profits under non-offshoring for all active firms. However, as offshoring costs increase and/or wage differential decreases less productive firms no longer find offshoring the most profitable thing to do. The first to renounce are the least efficient D-types. For even higher $F_{OS}$ and/or higher $g$ , all D-types choose not to offshore the second task. At last, offshoring becomes less profitable for higher cost X-types and only most productive X-types offshore.

To make our distinction among possible equilibria more explicit, we resort to a graphical representation, using the approach suggested in Helpman *et al* (2004). We plot the operating profit functions net of variety-development costs as a function of $a^{1-s}$ for both offshoring and non-offshoring Home firms. The profit functions are linear and upward-sloping in $a^{1-s}$ and given that $b^{1-s} > 1$, the they are steeper for offshoring firms. Additionally, the operating profit function for export sales is steeper for offshoring firms than for non-offshoring firms, but the intercepts are the

[24] Even if the offshoring fixed costs $F_{OS}$ are infinite, firms with labour requirements close to 0 will be indifferent between offshoring and staying onshore. We assume that in this kind of situations, firms always choose to offshore.

same (see equations (31) and (12)). Thus, offshoring firms that export break even at lower levels of productivity than their non-offshoring counterparts.

In all cases we assume that the regularity condition holds for offshoring firms, i.e. in each equilibrium outcome the fixed offshoring costs are not too low and/or wage difference is not too high to eliminate all D-type firms under offshoring scheme. We continue focusing on steady states and our comparisons should be viewed as capturing the long run consequences of offshoring.

## 4. Long run equilibria under offshoring: Four cases

### *4.1 Case 1*

We begin by considering equilibrium where all firms active at Home find it more advantageous to perform the second task abroad. Intuitively, this outcome requires low fixed offshoring costs combined with large gains from offshoring (low $b$). More formally, the combination has to be such as to ensure that operating profits from local sales are larger for offshoring firms than for non-offshoring ones for the entire range of unit labour coefficients under which it is profitable to produce. The case is depicted in Figure 1[25].

**Figure 1 about here**

In this figure, the operating profit function for local sales of offshoring firms, $p^{OS}_{local} = p^{OS}_{D}$, lies above the corresponding profit function for non-offshoring firms, $p_{local} = p_{D}$, for productivity levels $a^{1-s} > a_{D}^{1-s}$, meaning that all locally selling firms choose to offshore[26]. The most productive ones find it even more profitable to incur $F_X$ and produce for the foreign market, becoming offshoring X-types. The overall profit functions for D- and X-types firms of are shown by the solid kinked lines. The thresholds $a_{D}^{OS}$ and $a_{X}^{OS}$ are respectively the cut-off labour requirements at which offshoring locally selling firms and offshoring exporters just break even, defined by (32) and (33). Firms with $a > a_{D}^{OS}$ can not break even and exit the industry. The luckier ones with $a$'s between $a_{X}^{OS}$ and $a_{D}^{OS}$ produce for domestic markets while the most efficient ones with $a < a_{X}^{OS}$ also engage in exporting. Given that profit functions increase faster for the offshoring firms, the necessary and sufficient

[25] In the figures 1-4, $F_X > F_D$, which is the sufficient condition for the regularity condition $a_X < a_D$ to hold (see inequality (24)). Plainly, the regularity condition may still be satisfied if $F_X < F_D$, provided (24) holds.

[26] In this and the following graphs, the superscript 'OS' shows the profit lines for offshoring firms; the lack of it refers to non-offshoring firms.

condition for the Case 1 to apply boils down to the requirement that D-type offshoring firms break even at lower productivity levels, implying $a_D^{OS^{1-s}} \leq a_D^{1-s}$[27].

### *4.2 Case 2*

We now look at the case where fixed offshoring costs are higher and/or wage differential between countries is lower. In this equilibrium, the most efficient D-types and all X-types choose to offshore a part of their production process. On the contrary, the least productive D-type firms stay onshore as their larger labour requirements mean that gains from offshoring are not strong enough to make the overall profits net of fixed offshoring costs larger than the non-offshoring profits. Figure 2 lays out the set up.

**Figure 2 about here**

Overall there are four categories of firms. The least productive firms, with unit labour requirements in excess of $a_D$, become N-types because they can not cover the market-entry costs. Firms with productivity levels above $a_D{}^{1-s}$, but below $\tilde{a}_D^{OS^{1-s}}$, which is he threshold level of productivity at which operating profits of locally selling offshoring and non-offshoring firms are equalized, produce and sell locally. It is evident from the figure that in this range of input coefficients it is more profitable to keep production facilities at home than offshoring. This is however not the case for more efficient firms. If a firm draws $a$ such that $\tilde{a}_D^{OS^{1-s}} < a^{1-s} < a_X^{OS^{1-s}}$, it serves domestic market only but chooses to shift the performance of the second task abroad. Firms with productivity levels above the cut-off $a_X^{OS^{1-s}}$ find it more profitable to be offshoring X-types as they are able to increase their gains further, despite having to pay $F_X$.

As before, we impose a set of restrictions on the model parameters to ensure the validity of Case 2. First, not all locally selling firms should find offshoring the most profitable, which entails $a_D^{OS^{1-s}} > a_D^{1-s}$. At the same time, the fixed costs should not be too high and/or gains from offshoring should not be too low to prevent more efficient D-types from offshoring. Thus our second requirement is expressed as $a_X^{OS^{1-s}} > \tilde{a}_D^{OS^{1-s}}$.

### *4.3 Case 3*

The third case explores a combination of model parameters such that in the equilibrium no D-type firm chooses to offshore while all those that offshore are X-types. This outcome naturally extends our analysis of Case 2 to even larger offshoring costs and/or even lower difference in wages between the

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[27] At the point where $a_D^{1-s^{OS}} = a_D^{1-s}$ firms are indifferent whether or not to offshore; it is our assumption that they all choose to offshore.

two countries. Under these circumstances, firms never find it more profitable to become D-type offshoring over D-type non-offshoring or X-type offshoring. This is demonstrated in Figure 3.

**Figure 3 about here**

The threshold $\tilde{a}_D^{OS^{1-s}}$ continues to define the level of productivity beyond which it is more profitable for the firms to become a D-type offshoring as opposed to a D-type non-offshoring. However, as the figure reveals, offshoring exporters break even at the productivity level lower than this threshold ($a_X^{OS^{1-s}} < \tilde{a}_D^{OS^{1-s}}$). This, coupled with the fact that operating profits for offshoring X-types are larger than for offshoring D-types for all productivity levels above $a_X^{OS^{1-s}}$, rules out the existence of offshoring firms selling into the local market only. This is shown by the function $p_X^{OS}$ which lies above $p_{local}^{OS} = p_D^{OS}$ for $a^{1-s} > a_X^{OS^{1-s}}$ which also includes points above $\tilde{a}_D^{OS^{1-s}}$. Further, $p_X^{OS}$ lies above $p_{local} = p_D$ for unit input coefficients $a^{1-s} > \tilde{a}^{OS^{1-s}}$, with $\tilde{a}^{OS}$ determined by the intersection point of the two functions. Therefore, only the most efficient of X-type firms drawing $a < \tilde{a}^{OS}$ find it more profitable to offshore the second task. Smaller firms, with productivity levels between $\tilde{a}^{OS}$ and $a_D$, benefit the most from producing locally and serving the domestic market only. The very inefficient ones with $a > a_D$ expect negative operating profits and abandon production. The validity of Case 3 is ensured by the requirement that $a_D^{OS^{1-s}} > a_D^{1-s}$, $a_X^{OS^{1-s}} \leq \tilde{a}^{OS^{1-s}}$ and $\tilde{a}^{OS^{1-s}} \leq a_X^{1-s}$ (excluding the case where $a_X^{OS} = \tilde{a}^{OS} = a_X$ since $g \neq 1$).

### *4.4 Case 4*

The final possible case arises in a situation where offshoring costs are so high and/or gains from offshoring are so low that only a fraction of X-type firms offshores. In this set up, offshoring becomes less profitable for D-types and low-productivity X-types. Solely high-productivity X-type firms gain more from moving a part of production facilities abroad. Figure 4 portrays this equilibrium.

**Figure 4 about here**

Firms drawing labour coefficients above $a_D$ exit production. More fortunate entrants with labour requirements between $a_D$ and $a_X$ stay in the industry, but profit the most if they produce and sell locally. Firms with productivity levels in excess of $a_X^{1-s}$ become X-types. However, only for the most efficient of them, offshoring boosts profits strongly enough to make it the most profitable activity, despite having to pay $F_{OS}$. The threshold $\tilde{a}_X^{OS^{1-s}}$ corresponds to the intersection point of the operating profit functions for offshoring and non-offshoring X-type firms ($p_X^{OS}$ and $p_X$, respectively). So firms which are at least as productive as $\tilde{a}_X^{OS^{1-s}}$ are offshoring X-types. The necessary and sufficient condition for the Case 4 to arise, is $a_X^{1-s} < \tilde{a}_X^{OS^{1-s}}$. This requirement ensures that model parameters are such that only relatively more productive X-type firms find it advantageous to offshore; the rest stay onshore.

## 5. Equilibrium analysis: Case 4

Offshoring is a fast growing phenomenon, but it appears that it is predominantly larger firms that choose to operate offshore. In light of this, we choose to focus on analyzing the effects of offshoring for the equilibrium of Case 4. While we address other cases in Appendices 2-4, it is worth mentioning here that qualitative predictions are identical in all four cases.

### *5.1 Case 4 equilibrium*

The set up of the Case 4 leads to similar cut-off rules as in the pre-offshoring equilibrium, with the cut-off labour requirements for Home and Foreign nations $a_D$, $a_X$, $a_D^*$ and $a_X^*$ determined by the zero-profit conditions (15)-(18). The threshold labour coefficient $\tilde{a}_X^{OS}$ is defined by the equality of operating profits for offshoring and non-offshoring X-type firms, specified by equations (14) and (29). After re-arranging the terms the condition reduces to:

$$F_{OS} = \frac{(b^{1-s}-1)\tilde{a}_X^{OS^{1-s}}}{(1-1/s)^{1-s}}\frac{mE}{sP^{1-s}} + \frac{f(b^{1-s}-1)\tilde{a}_X^{OS^{1-s}}}{(1-1/s)^{1-s}}\frac{mgE}{sP^{*^{1-s}}} \tag{34}$$

The equilibrium is characterised by a new set of differentiated good price indices. These indices reflect prices for varieties produced by offshoring and non-offshoring firms. As highlighted previously, a subset of H-produced low cost varieties is now supplied by offshoring X-type firms. Given the cut-offs, the CES price indices at H and F take the following form:

$$\begin{aligned} P^{1-s} &= \int_o^{\tilde{a}_X^{OS}} \frac{(ba)^{1-s}}{(1-1/s)^{1-s}} ng\left[a|a_D\right]da + \int_{\tilde{a}_X^{OS}}^{a_D} \frac{a^{1-s}}{(1-1/s)^{1-s}} ng\left[a|a_D\right]da + \int_o^{a_X^*} \frac{fa^{1-s}}{(1-1/s)^{1-s}} n^* g\left[a|a_D^*\right]da \\ P^{*^{1-s}} &= \int_o^{a_D^*} \frac{a^{1-s}}{(1-1/s)^{1-s}} n^* g\left[a|a_D^*\right]da + \int_o^{\tilde{a}_X^{OS}} \frac{f(ba)^{1-s}}{(1-1/s)^{1-s}} ng\left[a|a_D\right]da + \int_{\tilde{a}_X^{OS}}^{a_X} \frac{fa^{1-s}}{(1-1/s)^{1-s}} ng\left[a|a_D\right]da \end{aligned} \tag{35}$$

Prior to entry, the expected H-firm profits include profits of becoming a non-offshoring D- or X-type or an offshoring X-type. Hence the equilibrium free entry condition that a potential entrant at Home faces is now expressed as:

$$\int_0^{\tilde{a}_X^{OS}}\left(\frac{(ba)^{1-s}}{(1-1/s)^{1-s}}\frac{mE}{sP^{1-s}}+\frac{f(ba)^{1-s}}{(1-1/s)^{1-s}}\frac{mgE}{sP^{*1-s}}-(F_D+F_X+F_{OS})\right)dG(a)$$
$$+\int_{\tilde{a}_X^{OS}}^{a_X}\left(\frac{fa^{1-s}}{(1-1/s)^{1-s}}\frac{mgE}{sP^{*1-s}}-F_X\right)dG(a)+\int_{\tilde{a}_X^{OS}}^{a_D}\left(\frac{a^{1-s}}{(1-1/s)^{1-s}}\frac{mE}{sP^{1-s}}-F_D\right)dG(a)=F_I \tag{36}$$

Unrestricted entry of new firms in country F also implies zero expected profits with the free entry condition identical to (22).

To solve the model we follow the procedure outlined in Appendix 1 for the pre-offshoring case. The unknowns include four cut-off unit labour coefficients, $a_D$, $a_X$, $a_D^*$, $a_X^*$, the threshold $\tilde{a}_X^{OS}$, as well as $n$, $n^*$, $P$ and $P^*$, determined by (15)-(18), (22), (34)-(36). One complication here is that condition (34) does not allow us to express the cut-offs and the threshold $\tilde{a}_X^{OS}$ in equivalent powers because it involves a summation sign. As a result no closed form solutions can be obtained[28]. We overcome this difficulty by introducing a coefficient $y$ such that $\tilde{a}_X^{OS}=y a_X$ and we impose $y<1$ as required by the Case 4[29]. Then the free entry conditions along with (15)-(18), yield implicit solutions for the cut-off unit labour coefficients and the threshold $\tilde{a}_X^{OS}$[30]:

[28] More precisely, substituting price indices $P$ and $P^*$ from (15)-(18) into (36), (22) and (34), yields: $a_D^k=\frac{F_I(1+k-s)a_0^k}{-F_D(1-s)}+\frac{F_X a_X^k}{-F_D}+\frac{F_{OS}\tilde{a}_X^{OS\,k}}{-F_D}$, $\quad a_D^{*\,k}=\frac{F_I(1+k-s)a_0^k}{-F_D(1-s)}+\frac{F_X a_X^{*\,k}}{-F_D}$ and $F_{OS}=(b^{1-s}-1)\tilde{a}_X^{OS\,1-s}\left(\frac{F_D}{a_D^{1-s}}+\frac{F_X}{a_X^{1-s}}\right)$. These, together with the ratios $\frac{a_D^{1-s}}{a_X^{*\,1-s}}=\frac{fF_D}{F_X}$ and $\frac{a_X^{1-s}}{a_D^{*\,1-s}}=\frac{F_X}{fF_D}$ have to be solved for the unknown cut-offs and $\tilde{a}_X^{OS}$. However, it is impossible to obtain explicit solutions given that we can not express all the unknowns in equivalent powers.

[29] Note that an analogous simplification is in order for the solution of Case 3.

[30] Setting $\tilde{a}_X^{OS}=y a_X$ allows us to re-write the free entry condition (36) as $a_D^k=\frac{F_I(1+k-s)a_0^k}{-F_D(1-s)}+\frac{(F_X+F_{OS}y^{\,k})a_X^k}{-F_D}$. This expression together with $a_D^{*\,k}=\frac{F_I(1+k-s)a_0^k}{-F_D(1-s)}+\frac{F_X a_X^{*\,k}}{-F_D}$, $\frac{a_D^{1-s}}{a_X^{*\,1-s}}=\frac{fF_D}{F_X}$ and $\frac{a_X^{1-s}}{a_D^{*\,1-s}}=\frac{F_X}{fF_D}$ gives implicit solutions for $a_D$, $a_X$, $a_D^*$, $a_X^*$ and $\tilde{a}_X^{OS}$ with the requirement that $y$ conforms with $F_{OS}=(b^{1-s}-1)y^{\,1-s}\left(F_D\frac{a_X^{1-s}}{a_D^{1-s}}+F_X\right)$ (which is just the expression (34) with substituted price indices $P$ and $P^*$ from (15) and (16) and $\tilde{a}_X^{OS}=y a_X$) and $y<1$. Note that although we are unable to obtain closed form solutions, the requirement that $y<1$ is sufficient to pursue the comparative statics analysis below.

$$
\begin{aligned}
a_D^k &= \frac{F_I(1+k-s)a_0^k}{F_D(s-1)}\frac{1-a\Omega}{1-a\Omega^2} \quad a_X^k = \frac{F_I(1+k-s)a_0^k}{F_X(s-1)}\frac{\Omega(1-\Omega)}{1-a\Omega^2} \\
a_D^{*\,k} &= \frac{F_I(1+k-s)a_0^k}{F_D(s-1)}\frac{1-\Omega}{1-a\Omega^2} \quad a_X^{*\,k} = \frac{F_I(1+k-s)a_0^k}{F_X(s-1)}\frac{\Omega(1-a\Omega)}{1-a\Omega^2} \\
\tilde{a}_X^{OS\,k} &= \frac{F_I(1+k-s)a_0^k}{F_X(s-1)}\frac{y^k\Omega(1-\Omega)}{1-a\Omega^2}
\end{aligned}
\tag{37}
$$

where for notational convenience we set $a \equiv 1+y^k F_{OS}/F_X$. Since $\Omega<1$, the cut-offs are positive provided $a<1/\Omega$[31]. Given the equilibrium solution for $\tilde{a}_X^{OS}$, the relationship between $F_{OS}$ and $y$ has to conform with (34) for any combination of other parameters values.

Manufactured varieties price indices change monotonically with the cut-off labour requirements. Using (15), (16) and (37), the equilibrium price indices are:

$$
P^{1-s} = \frac{mEa_0^{1-s}}{(1-1/s)^{1-s}sF_D}\left(\frac{F_I(1+k-s)}{F_D(s-1)}\frac{1-a\Omega}{1-a\Omega^2}\right)^{\frac{1-s}{k}}
$$

$$
P^{*1-s} = \frac{mgEa_0^{1-s}}{(1-1/s)^{1-s}sF_D}\left(\frac{F_I(1+k-s)}{F_D(s-1)}\frac{1-\Omega}{1-a\Omega^2}\right)^{\frac{1-s}{k}}
$$

The mass of firms active in each country can be obtained by substituting the expressions for price indices (35) together with the solutions of the cut-offs and threshold labour requirements into the cut-off conditions. After some manipulation, we get:

$$
\begin{aligned}
n &= \frac{mE(1+k-s)}{F_D sk}\frac{(1-g\Omega(1-a\Omega)/(1-\Omega))(1-a\Omega)}{(1-a\Omega^2)(1-\Omega(1+y^{1+k-s}(b^{1-s}-1)))} \\
n^* &= \frac{mE(1+k-s)}{F_D sk}\frac{(g-\Omega)(1-\Omega)-g\Omega^2(a-1)-\Omega(1-\Omega)y^{1+k-s}(b^{1-s}-1)(1+g)}{(1-a\Omega^2)(1-\Omega(1+y^{1+k-s}(b^{1-s}-1)))}
\end{aligned}
$$

Since $1<a<1/\Omega$, $g<1$ and $\Omega<1$, the numerator of the second term in the expression for $n$ is positive. Therefore, the equilibrium values for the mass of firms in H and F are positive if:

$$
1-\Omega-\Omega y^{1+k-s}(b^{1-s}-1)>0 \tag{38}
$$

$$
(g-\Omega)(1-\Omega)-g\Omega^2(a-1)-\Omega(1-\Omega)y^{1+k-s}(b^{1-s}-1)(1+g)>0 \tag{39}
$$

Additional results used in the subsequent analysis include the mass of varieties available to a typical consumer at Home and Foreign and the amount of labour employed by Home's manufacturing sector

[31] $a<1/\Omega$ ensures that $1-a\Omega^2$ is also positive.

to perform non-offshored tasks. Using the equilibrium solutions for the cut-off labour coefficients and the mass of firms operating in each country, it is straightforward to solve for the mass of varieties consumed in each country:

$$n_C = n\left(1+\Omega\frac{F_D}{F_X}\frac{(g-\Omega)(1-\Omega)-g\Omega^2(a-1)-\Omega(1-\Omega)y^{1+k-s}(b^{1-s}-1)(1+g)}{1-\Omega-g\Omega+ag\Omega^2}\right)$$
$$n_C^* = n^*\left(1+\Omega\frac{F_D}{F_X}\frac{1-\Omega-g\Omega+ag\Omega^2}{(g-\Omega)(1-\Omega)-g\Omega^2(a-1)-\Omega(1-\Omega)y^{1+k-s}(b^{1-s}-1)(1+g)}\right)$$

Turning to employment, observe that offshoring X-type firms perform only the first task locally. This means that, excluding the fixed costs, only a fraction $1/(1+d)$ of the amount of labour used by these firms is local workers. As before, a typical H-firm employs $l_j^{HH} = a_j c_j^{HH}$ and $l_j^{HF} = a_j c_j^{HF} t$ units of labour in the variable costs sector for its domestically sold and exported varieties, respectively. Given the demand functions in (3) and the offshored and non-offshored variety *j* price levels in (6) and (25), the total employment of the manufacturing sector at Home, excluding the fixed costs, becomes:

$$L_{VC} = \int_0^{\tilde{a}_X^{OS}} \frac{nb^{-s}a^{1-s}}{(1+d)(1-1/s)^{-s}}\frac{mE}{P^{1-s}}dG[a|a_D] + \int_{\tilde{a}_X^{OS}}^{a_D} \frac{na^{1-s}}{(1-1/s)^{-s}}\frac{mE}{P^{1-s}}dG[a|a_D]$$
$$+\int_0^{\tilde{a}_X^{OS}} \frac{nfb^{-s}a^{1-s}}{(1+d)(1-1/s)^{-s}}\frac{mgE}{P^{*1-s}}dG[a|a_D] + \int_{\tilde{a}_X^{OS}}^{a_X} \frac{nfa^{1-s}}{(1-1/s)^{-s}}\frac{mgE}{P^{*1-s}}dG[a|a_D]$$

Solving the integral and plugging in the equilibrium values of the cut-off coefficients, the expression for labour employment reduces to:

$$L_{VC} = \frac{nk(s-1)F_D}{1+k-s}\left\{1+\frac{\Omega(1-\Omega)}{1-a\Omega}\left(1+\frac{(b^{-s}/(1+d)-1)(a-1)}{b^{1-s}-1}\right)\right\}$$

where $n$ is the equilibrium mass of firms operating at Home.

Lastly, note that the above long-run equilibrium solutions are valid only if a number of parameter restrictions are met. First, as mentioned in the previous section, the validity of Case 4 entails $a_X^{1-s} < \tilde{a}_X^{OS\,1-s}$, or equivalently, $y<1$. Second, we impose regularity constraints for non-offshoring and offshoring firms, which suggest setting $a_D/a_X > 1$ and $a_D^{OS}/a_X^{OS} > 1$, with the cut-offs $a_D^{OS}$ and $a_X^{OS}$ determined by (32) and (33). It can be easily shown that $a_D/a_X = (a_D^{OS}/a_X^{OS})(F_D/(F_D+F_{OS}))^{1/(1-s)}$. Since $(F_D/(F_D+F_{OS}))^{1/(1-s)} < 1$, the inequality $a_D/a_X > 1$ necessarily holds whenever $a_D^{OS}/a_X^{OS} > 1$ is satisfied. Hence the condition

$a_D^{OS} / a_X^{OS} > 1$ is more stringent than $a_D / a_X > 1$[32]. Using the equilibrium solutions, the regularity condition for offshoring firms implies[33]:

$$\left( \frac{F_{OS} + F_D}{F_D} \right)^{\frac{k}{1-s}} \frac{F_X (1 - a\Omega)}{F_D \Omega (1 - \Omega)} > 1$$

Similarly, only a subset of relatively more efficient F-firms exports if $a_D^* / a_X^* > 1$. Substituting the equilibrium cut-offs, we get:

$$\frac{F_X}{F_D \Omega} \frac{1 - \Omega}{1 - a\Omega} > 1$$

Note that both ratios of this inequality are larger than unity by our assumption in (24) and $a > 1$. It follows then that the regularity condition for F-firms is always satisfied.

To summarize, the most binding inequalities that have to be satisfied for the Case 4 to apply include (38), (39), $y < 1$ and $a_D^{OS} / a_X^{OS} > 1$[34]. These are particularly important for the numerical simulations.

### *5.3 Comparison of the long-run equilibria: offshoring versus non-offshoring*

This section analyzes the effects of offshoring by comparing the pre- and post-offshoring equilibria. In particular, we examine the impact of offshoring on the cut-off productivity levels determining which firms stay in production and export, the mass of firms operating in each economy and the mass of varieties consumed, manufactured good price levels as well as welfare. To this end, we evaluate the changes by taking ratios of respective variables before and after offshoring.

---

[32] This is a general result which holds in all four cases; as such we only focus on the regularity condition for offshoring firms, $a_D^{OS} / a_X^{OS} > 1$.

[33] Notice that re-writing the regularity condition for offshoring firms in terms of $a$ gives $a < \frac{1}{\Omega} - (1 - \Omega) \frac{F_D}{F_X} \left( \frac{F_D}{F_D + F_{OS}} \right)^{\frac{k}{1-s}}$. This means that when $a_D^{OS} / a_X^{OS} > 1$ is satisfied, $a$ is necessarily lower than $1/\Omega$ and the cut-off coefficients are positive.

[34] Specifying the regularity condition for offshoring firms in terms of $y$ results in $y < \left[ (1 - \Omega) \left( \frac{F_X}{F_{OS}} \frac{1}{\Omega} - \frac{F_D}{F_{OS}} \left( \frac{F_D}{F_{OS} + F_D} \right)^{\frac{k}{1-s}} \right) \right]^{1/k}$. Comparing this to $y < 1$, it can be shown that each of these two conditions may be the most binding one.

We start by contrasting the lower bound efficiency levels of the operating firms. The relative magnitude of the cut-off labour requirements for domestic and export sales of Home and Foreign firms are given by[35]:

$$\frac{a^{k}_{D(OS)}}{a^{k}_{D(NOS)}} = \frac{a^{*\,k}_{X\,(OS)}}{a^{*\,k}_{X\,(NOS)}} = \frac{(1-a\Omega)(1+\Omega)}{1-a\Omega^{2}}$$

$$\frac{a^{*\,k}_{D(OS)}}{a^{*\,k}_{D(NOS)}} = \frac{a^{k}_{X\,(OS)}}{a^{k}_{X\,(NOS)}} = \frac{1-\Omega^{2}}{1-a\Omega^{2}}$$

Re-arranging the terms of the RHS in the first expression we get $\frac{1-a\Omega^{2}-\Omega(a-1)}{1-a\Omega^{2}}$, which is clearly less than unity, since $\Omega<1$ and $a>1$. Thus, $a_{D(OS)}<a_{D(NOS)}$ and $a^{*}_{X\,(OS)}<a^{*}_{X\,(NOS)}$. In words, offshoring triggers a selection effect at Home as the least productive firms are forced to exit. The basic intuition for this result lies in the fact that offshoring reduces the differentiated varieties price index in H whose change is given by:

$$\frac{P^{k}_{(OS)}}{P^{k}_{(NOS)}} = \frac{(1-a\Omega)(1+\Omega)}{1-a\Omega^{2}} < 1$$

Lower price index unambiguously lowers operating profits of an individual non-offshoring firm meaning that the least productive firms can no longer break even, i.e. $a_{D(OS)}<a_{D(NOS)}$. Similarly, lower price index at Home, means that only more productive foreign firms can afford to continue exporting, explaining the $a^{*}_{X\,(OS)}<a^{*}_{X\,(NOS)}$ outcome.

On contrary, manufacturing price index increases in F[36]:

$$\frac{P^{*\,k}_{(OS)}}{P^{*\,k}_{(NOS)}} = \frac{1-\Omega^{2}}{1-a\Omega^{2}} > 1$$

This boosts per-firm operating profits and allows less productive firms to break even in this country. Offshoring thus gives an opportunity to higher cost locally selling F-firms to be active, $a^{*}_{D(OS)}>a^{*}_{D(NOS)}$. At the same time, increased manufacturing price index at Foreign brings lower-productivity H-firms into the export market, such that $a_{X\,(OS)}>a_{X\,(NOS)}$.

---

[35] The subscripts '(OS)' and '(NOS)' indicate the pre- and post- offshoring variables and functions.

[36] $(1-\Omega^{2})/(1-a\Omega^{2})$ is larger than unity, because $a>1$.

Turning to the mass of varieties produced at Home, the magnitude of a relative change is given by:

$$\frac{n_{(OS)}}{n_{(NOS)}}=\frac{1-g\Omega(1-a\Omega)/(1-\Omega)}{1-g\Omega}\frac{1-\Omega^2}{1-\Omega(1+y^{1+k-s}(b^{1-s}-1))}\frac{1-a\Omega}{1-a\Omega^2} \tag{40}$$

As can be easily seen, the first two terms of the RHS are larger than unity, while the last one is less than unity, indicating that offshoring has an ambiguous effect on the mass of firms operating at Home. Two opposite forces are at work here. The possibility to offshore a part of the production process brings new profit opportunities, because marginal costs fall for offshoring firms. Although the existence of fixed offshoring costs means that only most productive firms will be able to cover these costs, potential entrants are encouraged by the higher potential returns associated with a good productivity draw. This increases the mass of firms operating at each productivity level. The negative effect is due to the selection effect, where the least productive firms, which were active in the pre-offshoring equilibrium, exit the industry. To illustrate this, we decompose (40) into two parts as:

$$\frac{n_{(OS)}}{n_{(NOS)}}=\frac{1-\Omega-g\Omega(1-a\Omega)}{(1-g\Omega)(1-\Omega(1+y^{1+k-s}(b^{1-s}-1)))}\frac{(1+\Omega)(1-a\Omega)}{1-a\Omega^2}$$

where the first term reflects the increase in the mass of operating firms at each unit labour coefficient and the second is the decrease in the cut-off labour requirement for domestic sales.

Offshoring unambiguously lowers the mass of varieties produced in F-country. Specifically, the relative change in $n^*$ is described by:

$$\frac{n^*_{(OS)}}{n^*_{(NOS)}}=\frac{1-\Omega^2}{g-\Omega}\frac{(g-\Omega)(1-\Omega)-g\Omega^2(a-1)-\Omega(1-\Omega)y^{1+k-s}(b^{1-s}-1)(1+g)}{(1-a\Omega^2)(1-\Omega(1+y^{1+k-s}(b^{1-s}-1)))}$$

which is less than unity for any acceptable levels of model parameters[37]. In this case, it is obvious that the drop in the number of new entrants more than offsets the positive effect caused by entry of less productive firms in the industry and overall mass of active F-firms actually decreases.

In terms of the mass of varieties bought by a typical consumer in each country, the effects of offshoring are likewise uncertain. The magnitudes of the changes at Home and Foreign nations are equal to:

[37] The easiest way to prove this is to evaluate whether the difference between the numerator and the denominator of the ratio is greater or less than zero. After a great deal of manipulation this difference reduces to: $-\Omega^2(a-1)(1-\Omega)(1+g)-y^{1+k-s}(b^{1-s}-1)((1-\Omega^2)(1-g\Omega)+\Omega^2(g-\Omega)(a-1))$, which is always negative, as all terms in brackets are positive.

$$\frac{n_{C(OS)}}{n_{C(NOS)}}=\frac{n_{(OS)}}{n_{(NOS)}}\frac{1+\Omega\frac{F_D}{F_X}\frac{(g-\Omega)(1-\Omega)-g\Omega^2(a-1)-\Omega(1-\Omega)y^{1+k-s}(b^{1-s}-1)(1+g)}{1-\Omega-g\Omega+ag\Omega^2}}{1+\Omega\frac{F_D}{F_X}\frac{g-\Omega}{1-g\Omega}}$$

$$\frac{n^*_{C(OS)}}{n^*_{C(NOS)}}=\frac{n^*_{(OS)}}{n^*_{(NOS)}}\frac{1+\Omega\frac{F_D}{F_X}\frac{1-\Omega-g\Omega+ag\Omega^2}{(g-\Omega)(1-\Omega)-g\Omega^2(a-1)-\Omega(1-\Omega)y^{1+k-s}(b^{1-s}-1)(1+g)}}{1+\Omega\frac{F_D}{F_X}\frac{1-g\Omega}{g-\Omega}}$$

It can be shown that $\frac{(g-\Omega)(1-\Omega)-g\Omega^2(a-1)-\Omega(1-\Omega)y^{1+k-s}(b^{1-s}-1)(1+g)}{1-\Omega-g\Omega+ag\Omega^2}<\frac{g-\Omega}{1-g\Omega}$, implying that the relative change of H-consumed varieties is smaller than of H-produced varieties and vice versa for F-country. These results are not surprising. To see this, note again that the mass of firms operating at each productivity level increases in H and decreases in F while the cut-off coefficients $a_D$ and $a^*_X$ decrease in the same proportions. As a consequence, the fraction of F-made varieties exported into H, given by $n^*_X/n=m^*{a^*_X}^k/ma^k_D$, is lower. This effect may be so strong, that the mass of all varieties available to a typical H-consumer falls even if the mass of H-produced varieties increases. On the other hand, although the relative cut-off levels of productivity, $a_X/a^*_D$, remain the same, offshoring raises the relative mass of firms operating in H-country per each level of $a$ increasing thus the ratio of H-firms importing into F. This increase can be so large that the mass of varieties bought by foreign consumers increases despite an unambiguous decrease in the locally produced varieties.

We finally examine the welfare implications of offshoring, which can be summarized by the changes in the indirect utility function $V=E/P$. As previously highlighted, the manufacturing price index decreases at Home and increases in Foreign. This immediately reveals that the overall impact of offshoring on the welfare at Home is unambiguously positive. On contrary, F-country incurs a welfare loss as a result of hosting H-firms' production facilities. Observe that Home nation continues to enjoy a lower price index and thus an improved welfare comparing to Foreign and, following our discussion, the gap between H's and F's price indices and welfare expands. This gap is shown by the ratio of the manufacturing price indices between the two nations:

$$\frac{P^k_{(OS)}}{{P^*}^k_{(OS)}}=\left(\frac{1}{g}\right)^{\frac{k}{1-s}}\frac{1-a\Omega}{1-\Omega}<1$$

### *5.4 Employment in the manufacturing sector*

In this section we focus on the consequences that offshoring has on the labour employment in the manufacturing goods sector, excluding the fixed costs component. The usual accounting is that one job offshored equals one job lost for the economy. In this paper, we argue that these concerns are unfounded. The model shows that employment in the sector may actually increase despite the fact that one of the tasks is performed abroad.

Before illustrating the implications for the total employment in the sector, we analyze the impact on the labour employment within an individual firm. We start by contrasting the amounts of labour used by D-type firms, which remain non-offshoring according to the Case 4 set up. As discussed previously, a typical firm $j$ employs $l_j^{HH} = a_j c_j^{HH}$ units of labour with consumption and price levels described by (3) and (6). This suggests an unambiguous fall in per-firm employment with relative magnitude equal to:

$$\frac{l_{i(OS)}^{D}}{l_{i(NOS)}^{D}} = \frac{P_{(NOS)}^{1-s}}{P_{(OS)}^{1-s}} < 1$$

The results for X-type firms are not as straightforward because offshoring may have both a positive and a negative impact on the amount of labour employed at the firm level. Specifically, the change in per-firm employment in less efficient X-types, which continue to perform both tasks at home, is:

$$\begin{aligned}\frac{l_{i(OS)}^{X-NOS}}{l_{i(NOS)}^{X-NOS}} &= \frac{1+f(1-a\Omega)/(1-\Omega))^{(1-s)/k}}{1+f}\frac{P_{(NOS)}^{1-s}}{P_{(OS)}^{1-s}} \\ &= \frac{1+f(1-a\Omega)/(1-\Omega))^{(1-s)/k}}{1+f}\left(\frac{1-a\Omega^2}{(1-a\Omega)(1+\Omega)}\right)^{(1-s)/k}\end{aligned}$$

The first fraction is larger than unity and the second is less than unity, suggesting an ambiguous labour effect for this category of firms. For more efficient X-type firms that choose to offshore, the respective change in per-firm labour demand is given by:

$$\begin{aligned}\frac{l_{i(OS)}^{X-OS}}{l_{i(NOS)}^{X-NOS}} &= b^{-s}\,\frac{1+f(1-a\Omega)/(1-\Omega))^{(1-s)/k}}{1+f}\frac{P_{(NOS)}^{1-s}}{P_{(OS)}^{1-s}} \\ &= b^{-s}\,\frac{1+f(1-a\Omega)/(1-\Omega))^{(1-s)/k}}{1+f}\left(\frac{1-a\Omega^2}{(1-a\Omega)(1+\Omega)}\right)^{(1-s)/k}\end{aligned}$$

Comparing to the ratio just above, this fraction has an additional term, $b^{-s}$ , which is larger than unity, but it continues to be the case that a firm may experience both an increase and a decrease in its employment levels. If we only focus on calculating the change in employment of the local work force, i.e. excluding labour usage in the second task which is offshored, then the change is given by:

$$\frac{l_{i(OS)}^{X-OS}}{l_{i(NOS)}^{X-OS}} = \frac{b^{-s}}{1+d}\frac{1+f(1-a\Omega)/(1-\Omega))^{(1-s)/k}}{1+f}\frac{P_{(NOS)}^{1-s}}{P_{(OS)}^{1-s}}$$
$$= \frac{b^{-s}}{1+d}\frac{1+f(1-a\Omega)/(1-\Omega))^{(1-s)/k}}{1+f}\left(\frac{1-a\Omega^2}{(1-a\Omega)(1+\Omega)}\right)^{(1-s)/k}$$

Obviously, the change is smaller: the first term may be less or larger than unity, but it is certainly less than $b^{-s}$ . Yet, looking at the overall result, domestic labour demand by an X-type firm that goes offshore may both increase and decrease.

Given the unambiguous changes in firm-level employment of larger firms as well as the overall mass of firms operating at Home, the overall effect of offshoring on the aggregate employment in the sector (excluding fixed costs) is also uncertain. Taking the ratio of the domestic labour used by manufacturing firms in variable costs post- and prior to offshoring, we get:

$$\frac{L_{VC(OS)}}{L_{VC(NOS)}} = \frac{n_{(OS)}}{n_{(NOS)}}\frac{1+\dfrac{\Omega(1-\Omega)}{1-a\Omega}\left(1+\dfrac{(b^{-s}/(1+d)-1)(a-1)}{b^{1-s}-1}\right)}{1+\Omega}$$

As inspection of the terms reveals, the total labour used in the performance of non-offshored tasks may increase or decrease as a result of offshoring. First, if $b^{-s}/(1+d)>1$, then the magnitude of change in the labour ratio is unambiguously larger than the change in the total mass of firms active at Home. Even if $b^{-s}/(1+d)<1$, we may still get the same result given that $(1-\Omega)/(1-a\Omega)>1$ as $a>1$. Otherwise, $L_{VC(OS)}/L_{VC(NOS)} < n_{(OS)}/n_{(NOS)}$ is also possible. Therefore labour usage may increase or decrease in the sector, independently of whether or not the mass of active firms increases or decreases. And this of course follows from the ambiguous effects of offshoring on the per-firm employment levels by X-types.

We now investigate how model parameters affect the employment in the sector. Clearly, the increase in the employment in the sector is directly related to how much labour is offshored, i.e. the parameter $d$ : the lower is the labour requirement in the task that is being offshored relative to the task performed onshore, the smaller is the adverse effect on the employment in the sector from allowing firms to offshore. In addition, numerical simulations show that low fixed offshoring costs, $F_{OS}$, also encourage larger employment in the sector.

Furthermore, notice that a decrease in trade costs (whether it is caused by higher $f$ and/or lower $F_X / F_D$), has a positive impact on the labour employment in the sector in both pre- and post-offshoring equilibria. This is easy to see for the pre-offshoring situation, where derivative with respect to $\Omega$ is positive:

$$\frac{\partial L_{VC(NOS)}}{\partial \Omega} = \frac{mE(s-1)}{s} \frac{1-g}{(1-\Omega)^2} > 0$$

Due to low tractability of the model, however, we can not analytically determine the effect of trade costs on the labour employed under offshoring scheme, but numerical simulations show that freer trade leads to higher employment in the sector in the offshoring equilibrium as well. Simulations also show that, in general, the labour ratio, $L_{VC(OS)} / L_{VC(NOS)}$, increases as trade barriers fall. Yet, when the ratio is less than unity, the relationship may not be monotonic for the lowest values of trade costs supported by the Case 4 requirements. When trade freeness is very low, incremental decline in trade costs (i.e. higher $f$ or lower $F_X / F_D$) increases employment in the pre-offshoring equilibrium faster than in the post-offshoring equilibrium, with the labour ratio decreasing overall. When trade costs fall beyond a certain threshold, the ratio starts to increase and further increases in $f$ and/or decreases in $F_X / F_D$ bring the ratio to a value above unity. Our conclusion then is that freer trade, i.e. large $f$ and low $F_X / F_D$, combined with low fixed offshoring costs and low $d$ are the necessary conditions for an increase in the employment in variable costs of the manufacturing sector.

## 6. Concluding remarks

We introduced GRH offshoring into the Helpman *et al* (2004) model with heterogeneous firms. In this set up, firms of the technologically advanced nation find it advantageous to engage in trade in tasks, because in doing so they are able to combine superior technology with lower factor costs. Firms' heterogeneity, which materializes via differences in productivity levels, coupled with fixed offshoring costs allows for four distinct long-run equilibrium outcomes. In each outcome there is an efficiency (and therefore size) threshold beyond which firms offshore and model parameters categorize exactly of which type the offshoring and non-offshoring firms are.

The central message of our model is that, although some jobs are lost within offshoring firms, the incidence of offshoring on the aggregate labour demand in the manufacturing sector is ambiguous. This is because the mass of firms operating in the home country and employment in larger firms may increase or decrease, moving in the same or opposite directions. In addition, we showed that international fragmentation of production brings welfare benefits to the offshoring nation, depresses prices and increases productivity of its firms; these effects are reversed for the recipient country.

Notably, the model highlights a number of testable hypotheses. One that has not been yet explored by the empirical literature relates to employment dynamics at the individual firm level. Offshoring unambiguously reduces labour demand in the smallest locally selling firms that stay onshore. On the contrary, both job creation and job destruction are both possible in larger firms.

There are several limitations in our model that create avenues for future theoretical research. First, we did not address the issue of how offshoring affects different skill groups, because we only assume one factor of production. Second we did not analyze the incidence of offshoring on nominal wages, because factor returns are pinned down by the outside good. Finally, it is straightforward to extend the model to two-way offshoring, as in Baldwin and Robert-Nicoud (2007b), to determine the consequences of offshoring between developed nations.

## Appendix 1

In Appendix 1 we show one of the ways to solve the model in the pre-offshoring case. Note, that we follow analogous procedure to derive equilibrium solutions in all four cases that are possible under offshoring.

We start by solving the free entry conditions (21) and (22) into which we plug the expressions for price indices from the cut-off conditions (15)-(18). The resulting expressions are:

$$a_D^k = \frac{F_I(1+k-s)a_0^k}{-F_D(1-s)} + \frac{F_X a_X^k}{-F_D} \text{ and } a_D^{*\,k} = \frac{F_I(1+k-s)a_0^k}{-F_D(1-s)} + \frac{F_X a_X^{*\,k}}{-F_D}$$

These expressions together with the ratios of the cut-off conditions are solved for the equilibrium cut-off unit labour requirements. The cut-off ratios that we use in derivations are obtained by dividing the RHS and the LHS of the equations (15) and (18) as well as (16) and (17), to cancel out the price indices, i.e.:

$$\frac{a_D^{1-s}}{a_X^{*\,1-s}} = \frac{fF_D}{F_X} \text{ and } \frac{a_X^{1-s}}{a_D^{*\,1-s}} = \frac{F_X}{fF_D}$$

From this system of four equations we get the solutions for the cut-off levels $a_D$, $a_X$, $a_D^*$ and $a_X^*$ provided in the text.

Further we solve for the equilibrium mass of firms in two countries using the solutions of the cut-off unit labour requirements, the definitions of the price indices and the cut-off conditions (15) and (17). More precisely, we work out the equations (19) and (20) to get:

$$P^{1-s} = \frac{k}{(1-1/s)^{1-s}(1+k-s)}\left\{na_D^{1-s} + fn^* a_D^{*\,1-s}\left(\frac{a_X^*}{a_D^*}\right)^{1+k-s}\right\}$$
$$P^{*1-s} = \frac{k}{(1-1/s)^{1-s}(1+k-s)}\left\{n^* a_D^{*\,1-s} + fna_D^{1-s}\left(\frac{a_X}{a_D}\right)^{1+k-s}\right\}$$

and substitute these price indices expressions into (15) and (17), which together with the equilibrium cut-offs imply the equilibrium expressions for $n$ and $n^*$ given in the text.

Finally, to find the manufacturing price indices we simply plug in the equilibrium values of the cut-off unit labour requirements into the cut-off conditions (15) and (17).

## Appendix 2: Case 1 solution

In this Appendix we lay out the short run and long run equilibrium solutions for the Case 1 set-up, where all H-firms choose to offshore the performance of the second task. We also provide comparative statics results between the pre- and post-offshoring equilibria.

As Figure 1 shows, all H-produced varieties are supplied by offshoring firms. The cut-off conditions for Home firms are then replaced by (32) and (33). Foreign firms continue to operate under identical cut-off conditions in (17) and (18). Under these circumstances, the price indices become:

$$P^{1-s} = \int_0^{a_D^{OS}} \left( \frac{ba}{1-1/s} \right)^{1-s} ng\left[a \middle| a_D^{OS}\right] da + \int_0^{a_X^*} f \left( \frac{a}{1-1/s} \right)^{1-s} n^* g\left[a \middle| a_D^*\right] da$$

$$P^{*1-s} = \int_0^{a_D^*} \left( \frac{a}{1-1/s} \right)^{1-s} n^* g\left[a \middle| a_D^*\right] da + \int_0^{a_X^{OS}} f \left( \frac{ba}{1-1/s} \right)^{1-s} ng\left[a \middle| a_D^{OS}\right] da$$

Although there is no change in the free entry condition for prospective entrants in F-country, for which (22) remains valid, entry and exit of Home offshoring firms is now governed by:

$$\int_0^{a_D^{OS}} \left( \frac{(ba)^{1-s}}{(1-1/s)^{1-s}} \frac{mE}{sP^{1-s}} - (F_D + F_{OS}) \right) dG(a) + \int_0^{a_X^{OS}} \left( \frac{f(ba)^{1-s}}{(1-1/s)^{1-s}} \frac{mgE}{sP^{*1-s}} - F_X \right) dG(a) = F_I$$

where we have taken into account that offshoring firms' profit functions are given by (30) and (31) and the new cut-off labour requirements.

As before, we solve the system of free entry conditions and the cut-off conditions for the four unknown cut-off unit input coefficients:

$$a_D^{OS\,k} = \frac{F_I(1+k-s)a_0^k}{(F_{OS}+F_D)(s-1)b^k} \frac{b^k - \Omega}{1-\Omega\Omega_{OS}}\, ; \; a_X^{OS\,k} = \frac{F_I(1+k-s)a_0^k}{F_X(s-1)b^k} \frac{\Omega(1-b^k\Omega_{OS})}{1-\Omega\Omega_{OS}}$$

$$a_D^{*\,k} = \frac{F_I(1+k-s)a_0^k}{F_D(s-1)} \frac{1-b^k\Omega_{OS}}{1-\Omega\Omega_{OS}}\, ; \; a_X^{*\,k} = \frac{F_I(1+k-s)a_0^k}{F_X(s-1)} \frac{\Omega_{OS}(b^k - \Omega)}{1-\Omega\Omega_{OS}}$$

where $\Omega_{OS} \equiv \left(\frac{1}{f}\right)^{\frac{k}{1-s}} \left(\frac{F_X}{F_{OS}+F_D}\right)^{\frac{1+k-s}{1-s}}$ and $\Omega_{OS} \geq \Omega$, by definition. The long run equilibrium mass of firms operating in each country are then found by substituting the expressions of price indices along with the equilibrium values of the cut-offs into the cut off conditions (32) and (17):

$$n = \frac{mE(1+k-s)}{sk(F_{OS}+F_D)} \frac{1}{(1-\Omega\Omega_{OS})} \left( 1 - g\Omega_{OS} \frac{b^k - \Omega}{1-b^k\Omega_{OS}} \right)$$

$$n^* = \frac{mE(1+k-s)}{skF_D} \frac{1}{(1-\Omega\Omega_{OS})} \left( g - \Omega \frac{1-b^k\Omega_{OS}}{b^k - \Omega} \right)$$

As in the main text, we derive the long run solutions for the mass of varieties available to Home and Foreign consumers:

$$n_C = n\left(1+\Omega_{OS}\,\frac{F_{OS}+F_D}{F_X}\,\frac{gb^k - g\Omega - \Omega + b^k\Omega\Omega_{OS}}{1-b^k\Omega_{OS} - gb^k\Omega_{OS} + g\Omega\Omega_{OS}}\right)$$
$$n_C^* = n^*\left(1+\Omega\,\frac{F_D}{F_X}\,\frac{1-b^k\Omega_{OS} - gb^k\Omega_{OS} + g\Omega\Omega_{OS}}{gb^k - g\Omega - \Omega + b^k\Omega\Omega_{OS}}\right)$$

where $n$ and $n^*$ are the equilibrium mass of firms operating in each country provided just above.

Furthermore, since offshoring firms perform only the first task of the manufacturing production process at Home while all firms offshore in Case1, the total sector's employment, excluding fixed costs is given by:

$$L_{VC} = \int_0^{a_D^{OS}} \frac{nb^{-s}\,a^{1-s}}{(1+d)(1-1/s)^{-s}}\,\frac{mE}{P^{1-s}}\,dG\left[a\middle|a_D^{OS}\right] + \int_0^{a_X^{OS}} \frac{nfb^{-s}\,a^{1-s}}{(1+d)(1-1/s)^{-s}}\,\frac{mgE}{P^{*1-s}}\,dG\left[a\middle|a_D^{OS}\right]$$

In the long run equilibrium, this amounts to:

$$L_{VC} = \frac{nk(s-1)(F_{OS}+F_D)}{(1+d)(1+k-s)b}\,\frac{b^k(1-\Omega\Omega_{OS})}{b^k-\Omega}$$

or

$$L_{VC} = \frac{mE(s-1)b^k}{s(1+d)b}\,\frac{1-g\Omega_{OS}(b^k-\Omega)/(1-b^k\Omega_{OS})}{b^k-\Omega}$$

if we also substitute for the equilibrium mass of firms active at Home.

Finally, a number of parameter restrictions are in order to ensure the validity of Case 1 equilibrium solutions. Given that profit functions increase faster for offshoring firms, the necessary and sufficient condition for the Case 1 to apply boils down to the requirement that offshoring firms break even at lower levels of productivity, i.e. ${a_D^{OS}}^{1-s} \le a_D^{1-s}$ [38]. Taking the ratio of the cut off conditions (32) and (15) and rearranging the powers, this entails the following restriction on the $F_{OS}$ and coefficient $b$:

$$b^k \le \left(\frac{F_{OS}+F_D}{F_D}\right)^{\frac{k}{1-s}}$$

Using the definition of $\Omega_{OS}$, the restriction can be alternatively stated as:

$$b^k \le \frac{\Omega}{\Omega_{OS}}\,\frac{F_D}{F_{OS}+F_D} \qquad \text{(A2-1)}$$

[38] At the point where ${a_D^{1-s}}^{OS} = a_D^{1-s}$ firms are indifferent whether or not to offshore; it is our assumption that they all choose to offshore.

Importantly, (A2-1) suggests that $b^k\Omega_{OS} \le \Omega\dfrac{F_D}{F_{OS}+F_D}$, which clearly implies that $b^k\Omega_{OS} < 1$ as both $\Omega$ and $F_D/(F_D+F_{OS})$ are less than unity. Hence, for the cut-off solutions to be positive, $\Omega\Omega_{OS} < 1$ and $b^k > \Omega$ must be true.

As before, the requirement that not all producing firms engage in exporting amounts to $a_D^{OS} > a_X^{OS}$ and $a_D^* > a_X^*$. The regularity condition for Home and Foreign firms then entails:

$$b^k > \Omega\frac{1+F_X/(F_{OS}+F_D)}{F_X/(F_{OS}+F_D)+\Omega\Omega_{OS}} \tag{A2-2}$$

and

$$b^k < \frac{F_X/F_D+\Omega\Omega_{OS}}{\Omega_{OS}(1+F_X/F_D)} \tag{A2-3}$$

Condition (A2-1) appears to be more binding than (A2-3). To see this, we compare the RHS terms of both inequalities, which after some manipulation requires showing whether $\dfrac{F_{OS}+F_D}{F_D}\left(\dfrac{F_X}{F_D}+\Omega\Omega_{OS}\right)$ is larger or less than $\Omega\left(\dfrac{F_X}{F_D}+1\right)$. Setting $F_{OS}=0$ (which means that $\Omega=\Omega_{OS}$), the comparison is between $F_X/F_D+\Omega^2$ and $\Omega(F_X/F_D+1)$. The inequality (24) then implies that $F_X/F_D+\Omega^2 > \Omega(F_X/F_D+1)$. As fixed offshoring costs increase, the term $\dfrac{F_{OS}+F_D}{F_D}\left(\dfrac{F_X}{F_D}+\Omega\Omega_{OS}\right)$ increases while $\Omega\left(\dfrac{F_X}{F_D}+1\right)$ remains constant, suggesting that the former term becomes even larger than the latter. Thus $\dfrac{F_{OS}+F_D}{F_D}\left(\dfrac{F_X}{F_D}+\Omega\Omega_{OS}\right) > \Omega\left(\dfrac{F_X}{F_D}+1\right)$ for any fixed costs applicable under Case 1.

From (A2-2), it follows that $b^k > \Omega$ necessarily holds, as $\Omega < \Omega\dfrac{1+F_X/(F_{OS}+F_D)}{F_X/(F_{OS}+F_D)+\Omega\Omega_{OS}}$ due to $\Omega\Omega_{OS} < 1$ requirement. In addition, (A2-1) ensures that $\Omega\Omega_{OS} < 1$ is always valid. To see this we re-write (A2-1) as $\Omega_{OS} \le \dfrac{\Omega}{b^k}\dfrac{F_D}{F_{OS}+F_D}$ and $\Omega\Omega_{OS} < 1$ as $\Omega_{OS} < 1/\Omega$. Now note that the RHS in the former inequality is less than unity while $1/\Omega > 1$. All in all, the cut-offs are always positive if the condition for Case 1 and the regularity condition for offshoring H-firms are satisfied.

Two more conditions are in order to ensure that the mass of firms operating in each country is positive. Given that $\Omega\Omega_{OS} < 1$, $b^k\Omega_{OS} < 1$ and $b^k > \Omega$, these requirements entail $1-b^k\Omega_{OS}-gb^k\Omega_{OS}+g\Omega\Omega_{OS} > 0$ and $gb^k-g\Omega-\Omega+b^k\Omega\Omega_{OS} > 0$ or:

$$b^k < \frac{1+g\Omega\Omega_{OS}}{\Omega_{OS}(1+g)}$$

and

$$b^k > \frac{\Omega(1+g)}{g+\Omega\Omega_{OS}} \tag{A2-4}$$

It can be shown that $\frac{1+g\Omega\Omega_{OS}}{\Omega_{OS}(1+g)} > \frac{\Omega}{\Omega_{OS}}\frac{F_D}{F_{OS}+F_D}$, meaning that as long as (A2-1) holds, the mass of firms operating in the Home country is always positive. However, each of the conditions (A2-2) and (A2-4) may be the most binding one, depending on the actual level of model parameters. Therefore, in all comparisons and simulations it is imperative to ensure that both of these conditions hold. To conclude on inequalities, the utmost requirements for the Case 1 to apply are given by (A2-1), (A2-2) and (A2-4).

Turning to the comparisons of the pre- and post-offshoring productivity cut-offs and prices, the ratios appear as follows:

$$\frac{a_{D\ (OS)}^{OS\,k}}{a_{D(NOS)}^{k}} = \frac{F_D}{F_{OS}+F_D}\frac{(1-\Omega/b^k)(1+\Omega)}{1-\Omega\Omega_{OS}};\ \frac{a_{X(OS)}^{*\,k}}{a_{X(NOS)}^{*\,k}} = \frac{\Omega_{OS}}{\Omega}\frac{(b^k-\Omega)(1+\Omega)}{1-\Omega\Omega_{OS}}$$

$$\frac{a_{D(OS)}^{*\,k}}{a_{D(NOS)}^{*\,k}} = \frac{(1-b^k\Omega_{OS})(1+\Omega)}{1-\Omega\Omega_{OS}};\ \frac{a_{X\ (OS)}^{OS\,k}}{a_{X(NOS)}^{k}} = \frac{1}{b^k}\frac{(1-b^k\Omega_{OS})(1+\Omega)}{1-\Omega\Omega_{OS}}$$

$$\frac{P_{(OS)}^{k}}{P_{(NOS)}^{k}} = \frac{\Omega_{OS}}{\Omega}\frac{(b^k-\Omega)(1+\Omega)}{1-\Omega\Omega_{OS}};\ \frac{P_{(OS)}^{*\,k}}{P_{(NOS)}^{*\,k}} = \frac{(1-b^k\Omega_{OS})(1+\Omega)}{1-\Omega\Omega_{OS}}$$

$$\frac{P_{(OS)}^{k}}{P_{(NOS)}^{*\,k}} = \left(\frac{1}{g}\right)^{\frac{k}{1-s}}\frac{\Omega_{OS}(b^k-\Omega)}{\Omega(1-b^k\Omega_{OS})}$$

Showing whether these ratios are larger or less than unity reduces to determining how $b^k$ compares to $\frac{\Omega(1+\Omega_{OS})}{\Omega_{OS}(1+\Omega)}$ and $\frac{1+\Omega}{1+\Omega_{OS}}$. Given that $\Omega_{OS} \geq \Omega$, it can be shown that:

$$b^k \leq \frac{\Omega}{\Omega_{OS}}\frac{F_D}{F_{OS}+F_D} < \frac{\Omega(1+\Omega_{OS})}{\Omega_{OS}(1+\Omega)} < \frac{1+\Omega}{1+\Omega_{OS}}$$

From this it follows, that $a_{D(OS)} < a_{D(NOS)}$, $a_{X(OS)}^{*} < a_{X(NOS)}^{*}$, $a_{D(OS)}^{*} > a_{D(NOS)}^{*}$, $a_{X(OS)} > a_{X(NOS)}$, $P_{(OS)} < P_{(NOS)}$, $P_{(OS)}^{*} > P_{(NOS)}^{*}$ and $P_{(OS)} < P_{(NOS)}^{*}$, as in the Case 4.

As for the mass of firms operating in each country, offshoring has an ambiguous effect in the Home nation and unambiguously negative impact in the Foreign country[39]:

[39] The fact that $n_{C(OS)}^{*} < n_{C(NOS)}^{*}$ can be proved by first showing that the derivative of the ratio with respect to $b$ (or alternatively $F_{OS}$) is positive and then evaluating the limit of the ratio at the end points for $b$ (or $F_{OS}$) determined by the inequalities above.

$$\frac{n_{(OS)}}{n_{(NOS)}}=\frac{F_D}{(F_{OS}+F_D)}\frac{1-g\Omega_{OS}\dfrac{b^k-\Omega}{1-b^k\Omega_{OS}}}{1-g\Omega}\frac{1-\Omega^2}{(1-\Omega\Omega_{OS})}$$

$$\frac{n^*_{(OS)}}{n^*_{(NOS)}}=\frac{g-\Omega\dfrac{1-b^k\Omega_{OS}}{b^k-\Omega}}{g-\Omega}\frac{1-\Omega^2}{1-\Omega\Omega_{OS}}$$

In terms of the mass of consumed varieties, the same results as in Case 4, $n_{C(OS)}/n_{C(NOS)} < n_{(OS)}/n_{(NOS)}$ and $n^*_{C(OS)}/n^*_{C(NOS)} > n^*_{(OS)}/n^*_{(NOS)}$, hold:

$$\frac{n_{C(OS)}}{n_{C(NOS)}}=\frac{n_{(OS)}}{n_{(NOS)}}\frac{1+\Omega_{OS}\dfrac{F_{OS}+F_D}{F_X}\dfrac{gb^k-g\Omega-\Omega+b^k\Omega\Omega_{OS}}{1-b^k\Omega_{OS}-gb^k\Omega_{OS}+g\Omega\Omega_{OS}}}{1+\Omega\dfrac{F_D}{F_X}\dfrac{g-\Omega}{1-g\Omega}}$$

$$\frac{n^*_{C(OS)}}{n^*_{C(NOS)}}=\frac{n^*_{(OS)}}{n^*_{(NOS)}}\frac{1+\Omega\dfrac{F_D}{F_X}\dfrac{1-b^k\Omega_{OS}-gb^k\Omega_{OS}+g\Omega\Omega_{OS}}{gb^k-g\Omega-\Omega+b^k\Omega\Omega_{OS}}}{1+\Omega\dfrac{F_D}{F_X}\dfrac{1-g\Omega}{g-\Omega}}$$

Lastly, our conclusions are also confirmed for the labour employment in the manufacturing sector, which may increase or decrease independently of whether the mass of firms increases or decreases in the Home nation:

$$\frac{L_{VC(OS)}}{L_{VC(NOS)}}=\frac{n_{(OS)}}{n_{(NOS)}}\frac{1}{(1+d)b}\frac{F_{OS}+F_D}{F_D}\frac{1-\Omega\Omega_{OS}}{(1-\Omega/b^k)(1+\Omega)}$$

$$\frac{L_{VC(OS)}}{L_{VC(NOS)}}=\frac{1}{b}\frac{1}{1+d}\frac{1-\Omega}{1-\Omega/b^k}\frac{1-g\Omega_{OS}\dfrac{b^k-\Omega}{1-b^k\Omega_{OS}}}{1-g\Omega}$$

## Appendix 3: Case 2 solution

In this Appendix we derive the long-run equilibrium for the Case 2.

In Case 2, the cut-off conditions for Home firms are given by (15) and (33). These are now complemented by the condition of the threshold productivity level $\tilde{a}_D^{OS}$ at which operating profits for D-type non-offshoring and offshoring firms are equal, i.e. $p_{local}^{OS} = p_{local}$:

$$-(F_D + F_{OS}) + \left(\frac{b\tilde{a}_D^{OS}}{1-1/s}\right)^{1-s} \cdot \left(\frac{mE}{sP^{1-s}}\right) = -F_D + \left(\frac{\tilde{a}_D^{OS}}{1-1/s}\right)^{1-s} \cdot \left(\frac{mE}{sP^{1-s}}\right)$$

which by rearranging the terms simplifies to:

$$F_{OS} = (b^{1-s} - 1) \cdot \left(\frac{\tilde{a}_D^{OS}}{1-1/s}\right)^{1-s} \cdot \left(\frac{mE}{sP^{1-s}}\right) \tag{A3-1}$$

The cut-off conditions (17) and (18) remain identical for the F-firms. The price indices are replaced by:

$$\begin{aligned} P^{1-s} &= \int_0^{\tilde{a}_D^{OS}} \left(\frac{ba}{1-1/s}\right)^{1-s} ng[a|a_D]da + \int_{\tilde{a}_D^{OS}}^{a_D} \left(\frac{a}{1-1/s}\right)^{1-s} ng[a|a_D]da \\ &\quad + \int_0^{a_X^*} f\left(\frac{a}{1-1/s}\right)^{1-s} n^* g[a|a_D^*]da \\ P^{*1-s} &= \int_0^{a_D^*} \left(\frac{a}{1-1/s}\right)^{1-s} n^* g[a|a_D^*]da + \int_0^{a_X^{OS}} f\left(\frac{ba}{1-1/s}\right)^{1-s} ng[a|a_D]da \end{aligned}$$

For F-firms, the free entry condition (22) continues to hold. However, the free entry condition for H-firms has to be modified to account for the whole range of firms' types that the possibility of offshoring brings in Case 2:

$$\begin{aligned} &\int_0^{\tilde{a}_D^{OS}} \left(\frac{(ba)^{1-s}}{(1-1/s)^{1-s}} \frac{mE}{sP^{1-s}} - (F_D + F_{OS})\right) dG(a) + \int_{\tilde{a}_D^{OS}}^{a_D} \left(\frac{a^{1-s}}{(1-1/s)^{1-s}} \frac{mE}{sP^{1-s}} - F_D\right) dG(a) + \\ &+ \int_0^{a_X^{OS}} \left(\frac{f(ba)^{1-s}}{(1-1/s)^{1-s}} \frac{mgE}{sP^{*1-s}} - F_X\right) dG(a) = F_I \end{aligned}$$

Finally, offshoring H-firms are now performing only the first task locally, so the domestic employment in the performance of this task is a fraction $1/(1+d)$ of the total amount of labour used by these firms, excluding the fixed costs. As before, a typical H-firm employs $l_j^{HH} = a_j c_j^{HH}$ and $l_j^{HF} = a_j c_j^{HF} t$ units of labour in the variable costs sector for its locally sold and exported varieties, respectively. Given the demand functions in (3) and the offshored and non-offshored variety *j* price levels in (6) and (25), the total employment of the manufacturing sector at Home, excluding the fixed costs, becomes:

$$L_{VC}=\int_0^{\tilde{a}_D^{OS}}\frac{nb^{-s}a^{1-s}}{(1+d)(1-1/s)^{-s}}\frac{mE}{P^{1-s}}dG\left[a|a_D\right]+\int_{\tilde{a}_D^{OS}}^{a_D}\frac{na^{1-s}}{(1-1/s)^{-s}}\frac{mE}{P^{1-s}}dG\left[a|a_D\right]$$
$$+\int_0^{a_X^{OS}}\frac{nfb^{-s}a^{1-s}}{(1+d)(1-1/s)^{-s}}\frac{mgE}{P^{*1-s}}dG\left[a|a_D\right]$$

The four cut-off conditions, the condition for the threshold level of $\tilde{a}_D^{OS}$, two free entry conditions together with the price indices definitions can be worked out to get explicit closed form solutions for the nine unknowns determining the long run equilibrium, i.e. $a_D$, $\tilde{a}_D^{OS}$, $a_X^{OS}$, $a_D^*$, $a_X^*$, $n$, $n^*$, $P$ and $P^*$. We start by solving the free entry conditions, the cut-off conditions and (A3-1) for the equilibrium levels for the cut-off unit labour requirements:

$$a_D^k=\frac{F_I(1+k-s)a_0^k}{F_D(s-1)}\frac{1-\Omega/b^k}{1+w-\Omega^2/b^k},\quad a_X^{OS\,k}=\frac{F_I(1+k-s)a_0^k}{F_X(s-1)}\frac{1}{b^k}\frac{\Omega(1+w-\Omega)}{1+w-\Omega^2/b^k},$$
$$\tilde{a}_D^{OS\,k}=\frac{F_I(1+k-s)a_0^k}{F_{OS}(s-1)}\frac{w(1-\Omega/b^k)}{1+w-\Omega^2/b^k},$$
$$a_D^{*\,k}=\frac{F_I(1+k-s)a_0^k}{F_D(s-1)}\frac{(1+w-\Omega)}{1+w-\Omega^2/b^k},\quad a_X^{*\,k}=\frac{F_I(1+k-s)a_0^k}{F_X(s-1)}\frac{\Omega(1-\Omega/b^k)}{1+w-\Omega^2/b^k}$$

where $w\equiv\left(\frac{F_{OS}}{F_D}\right)^{\frac{1+k-s}{1-s}}\left(\frac{1}{b^{1-s}-1}\right)^{\frac{k}{1-s}}$, with $w>0$, given that $b^{1-s}>1$ (as $g<1$)[40].

Further we solve for the equilibrium mass of firms in two countries using the solutions of the cut-off unit labour requirements, the definitions of the price indices and the cut-off conditions (15) and (17):

$$n=\frac{mE(1+k-s)}{skF_D}\frac{1}{(1+w-\Omega^2/b^k)}\left(1-g\Omega\frac{1-\Omega/b^k}{1+w-\Omega}\right)$$
$$n^*=\frac{mE(1+k-s)}{skF_D}\frac{1}{(1+w-\Omega^2/b^k)}\left(g(1+w)-\frac{\Omega}{b^k}\frac{1+w-\Omega}{1-\Omega/b^k}\right)$$

To find the manufacturing price indices we simply plug in the equilibrium values of the cut-off unit labour requirements into the cut-off conditions (15) and (17):

$$P^{1-s}=\frac{mEa_0^{1-s}}{(1-1/s)^{1-s}sF_D}\left(\frac{F_I(1+k-s)}{F_D(s-1)}\right)^{\frac{1-s}{k}}\left(\frac{1-\Omega/b^k}{1+w-\Omega^2/b^k}\right)^{\frac{1-s}{k}}$$

[40] Note that as before, the cut off level $a_D$ represents both the marginal cost and the unit labour requirement; $a_D^*$ and $a_X^*$ are the cut-off levels of marginal costs with the respective unit labour coefficients equal to $a_D^*/g$ and $a_X^*/g$ ; $a_X^{OS}$ is the unit labour requirements, with the marginal costs being $ba_X^{OS}$ ; $\tilde{a}_D^{OS}$ is the marginal cost and unit labour requirement for non-offshoring firms and unit labour requirement for offshoring firms (with $b\tilde{a}_D^{OS}$ the marginal cost for offshoring firms).

$$P^{*1-s} = \frac{mgEa_0^{1-s}}{(1-1/s)^{1-s}\, sF_D}\left(\frac{F_I(1+k-s)}{F_D(s-1)}\right)^{\frac{1-s}{k}}\left(\frac{1+w-\Omega}{1+w-\Omega^2/b^k}\right)^{\frac{1-s}{k}}$$

It is now straightforward to solve for the equilibrium mass of varieties consumed in each nation and employment in the differentiated good sector:

$$n_C = n\left(1+\Omega\frac{F_D}{F_X}\frac{g(1-\Omega/b^k)(1+w)-\Omega(1+w-\Omega)/b^k}{1+w-\Omega-g\Omega(1-\Omega/b^k)}\right)$$

$$n_C^* = n^*\left(1+\Omega\frac{F_D}{F_X}\frac{1}{b^k}\frac{1+w-\Omega-g\Omega(1-\Omega/b^k)}{g(1-\Omega/b^k)(1+w)-\Omega(1+w-\Omega)/b^k}\right)$$

$$L_{VC} = \frac{nk(s-1)F_D}{1+k-s}\left\{\frac{w(b^{-s}/(1+d)-1)}{b^{1-s}-1}+\frac{\Omega(1+w-\Omega)}{b(1+d)(b^k-\Omega)}+1\right\}$$

where $n$ and $n^*$ are the equilibrium mass of firms operating in each country provided just above.

A set of restrictions is suggested by the equilibrium values of the cut-off levels of productivity and the number of firms, which have to be positive. First, note that $1+w-\Omega>0$ since $\Omega<1$ and $w>0$. For the cut offs to be positive, $b^k>\Omega$ must be satisfied. This condition is sufficient to ensure that $1+w-\Omega^2/b^k>0$[41], meaning that all cut offs are positive provided $b^k>\Omega$ holds. Second, the mass of firms operating at Home is positive as long as $1+w-\Omega-g\Omega+g\Omega^2/b^k>0$. The inequality always holds if the initial restrictions on $g$ and $\Omega$ hold[42]. The condition that the mass of firms at Foreign is positive requires that $g+gw-g\Omega/b^k-gw\Omega/b^k-\Omega/b^k-w\Omega/b^k+\Omega^2/b^k>0$. This inequality can be re-written as:

$$b^k > \frac{\Omega(1+w-\Omega)}{g(1+w)}+\Omega \qquad \text{(A3-2)}$$

Observe that the first term on the right hand side of (A3-2) is positive meaning that if (A3-2) holds $b^k>\Omega$ is always satisfied. Therefore, the inequality (A3-2) is the ultimate necessary and sufficient condition to ensure that the equilibrium solutions for the cut-offs and the mass of firms are positive.

It is clear from our analysis above, that additional restrictions on the model parameters must be satisfied to ensure that Case 2 applies. Thus along with regularity conditions $a_D^{OS}>a_X^{OS}$ and $a_D^*>a_X^*$, the following set of restrictions should hold:

$a_D^{OS\,1-s} > a_D^{1-s}$ and $a_X^{OS\,1-s} > \tilde{a}_D^{OS\,1-s}$

[41] $1+w-\Omega^2/b^k>0$ is equivalent to $b^k>\Omega^2/(1+w)$. Since $\Omega<1$ and $w>0$, the fact that $b^k>\Omega$ is sufficient to ensure the inequality always holds.

[42] To see that, re-write inequality as $1+w-\Omega>g\Omega(1-\Omega/b^k)$. Note, that $1+w-\Omega>1-\Omega/b^k$, since $w>0$ and $g<1$ meaning that $b^k<1$. Given that $g<1$ and $\Omega<1$, the inequality always holds.

where $a_D^{OS}$ is the level of productivity at which locally selling offshoring firms just break even, determined by equation (32). Using the cut-off conditions (32) and (15), the constraint $a_D^{OS^{1-s}} > a_D^{1-s}$ implies that[43]:

$$b^k > \left(\frac{F_D + F_{OS}}{F_D}\right)^{\frac{k}{1-s}} \tag{A3-3}$$

Substituting the equilibrium values for the cut-off levels, the condition $a_X^{OS^{1-s}} > \tilde{a}_D^{OS^{1-s}}$ reduces to:

$$b^k > \frac{\Omega(1+w-\Omega)}{w}\frac{F_{OS}}{F_X} + \Omega \tag{A3-4}$$

Furthermore, the manipulation of (32), (15) and (A3-1) shows that conditions $\tilde{a}_D^{OS^{1-s}} > a_D^{1-s}$ and $\tilde{a}_D^{OS^{1-s}} > a_D^{OS^{1-s}}$ are equivalent to $a_D^{OS^{1-s}} > a_D^{1-s}$ : all the three reduce to exactly the same restrictions for $F_{OS}$ and $b$ given in (A3-3). The fact that $\tilde{a}_D^{OS^{1-s}} > a_D^{OS^{1-s}}$ along with the requirement that $a_X^{OS^{1-s}} > \tilde{a}_D^{OS^{1-s}}$ means that $a_X^{OS^{1-s}} > a_D^{OS^{1-s}}$, which simply suggests that the regularity condition for the Home nation is always met ( $s > 1$). The regularity condition for the Foreign firms can be re-written as $\frac{F_X}{F_D}\frac{1}{\Omega}\frac{1+w-\Omega}{1-\Omega/b^k} > 1$ by substituting the equilibrium values for the cut-off levels. The condition always holds. This can be seen by noting that $\frac{F_X}{F_D}\frac{1}{\Omega} = \left(\frac{F_D}{F_X/f}\right)^{\frac{k}{1-s}}$ which is larger than unity by our assumption in (24). Additionally, $1+w-\Omega > 1-\Omega/b^k$ as shown above.

All in all, we are left with three constraints (A3-2), (A3-3) and (A3-4) determining the lowest range of values of possible $b^k$. Of these, only the second one is a closed form inequality between the model parameters. The (A3-2) and (A3-4) can not be solved analytically for $b$ as the definition of $w$ includes $b^{1-s}$. As such we resort to numerical simulations for different model parameters that conform to the Case 2 restrictions and it appears that each of the RHS in the three inequalities ((A3-2), (A3-3) and (A3-4)) can be the highest. Therefore, each of the inequalities can be the most binding one, but all three should be met. On the higher end of the $b^k$ range, $b^k < 1$ should be satisfied.

We now turn to contrasting the pre- and post- offshoring equilibria. First we compare the efficiency levels of operating firms, by taking ratios of the equilibrium values for the cut-off levels of unit input coefficients after and prior offshoring:

$$\frac{a^k_{D(OS)}}{a^k_{D(NOS)}} = \frac{a^{*\,k}_{X\ (OS)}}{a^{*\,k}_{X\ (NOS)}} = \frac{(1-\Omega/b^k)(1+\Omega)}{1+w-\Omega^2/b^k}$$

$$\frac{a^{OS\,k}_{X\ (OS)}}{a^k_{X\,(NOS)}} = \frac{1}{b^k}\frac{(1+w-\Omega)(1+\Omega)}{1+w-\Omega^2/b^k}$$

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[43] Notice that $F_{OS}$ has to be different from zero, because otherwise, (A3-3) implies that $b^k > 1$ which contradicts our assumption for $g < 1$.

$$\frac{a_{D}^{*\,k}{}_{(OS)}}{a_{D}^{*\,k}{}_{(NOS)}}=\frac{(1+w-\Omega)(1+\Omega)}{1+w-\Omega^{2}/b^{k}}$$

The RHS of the first ratio can be re-written as $\frac{1-\Omega^{2}/b^{k}-\Omega(1/b^{k}-1)}{1-\Omega^{2}/b^{k}+w}$ which is less than unity given that $b^{k}<1$ and $w>0$. Thus $a_{D(OS)}<a_{D(NOS)}$ and $a^{*}_{X\,(OS)}<a^{*}_{X\,(NOS)}$. On the other hand, simplifying the ratio $\frac{(1+w-\Omega)(1+\Omega)}{1+w-\Omega^{2}/b^{k}}$ to $\frac{1+w-\Omega^{2}+w\Omega}{1+w-\Omega^{2}/b^{k}}$ makes it clear that the fraction is larger than 1 since $b^{k}<1$ and $w>0$. This suggests that $a^{*}_{D(OS)}>a^{*}_{D(NOS)}$ and $a_{X\ (OS)}^{OS}>a_{X\,(NOS)}$ unambiguously.

The manufacturing price index ratios for Home and Foreign countries can be easily obtained from the cut-off conditions prior and after offshoring and are respectively given by:

$$\frac{P_{(OS)}^{k}}{P_{(NOS)}^{k}}=\frac{(1-\Omega/b^{k})(1+\Omega)}{1+w-\Omega^{2}/b^{k}}$$
$$\frac{P^{*k}_{(OS)}}{P^{*k}_{(NOS)}}=\frac{(1+w-\Omega)(1+\Omega)}{1+w-\Omega^{2}/b^{k}}$$

It follows from the above that offshoring decreases the manufacturing price index for the Home country and increases it for the Foreign. Importantly, this also means that consumers' indirect utility as well as real wages increase at Home and decrease in Foreign. Furthermore, Home consumers continue to enjoy a lower price index compared to foreign:

$$\frac{P_{(OS)}^{k}}{P^{*k}_{(OS)}}=\left(\frac{1}{g}\right)^{\frac{k}{1-s}}\frac{1-\Omega/b^{k}}{1+w-\Omega}$$

whereas the ratio is less than unity in light of the fact that $g<1$, $s>1$ as well as $1+w-\Omega>1-\Omega/b^{k}$ as shown above.

Turning now to the mass of varieties produced in each economy, offshoring changes them in the following proportions:

$$\frac{n_{(OS)}}{n_{(NOS)}}=\frac{1-g\Omega(1-\Omega/b^{k})/(1+w-\Omega)}{1+w-\Omega^{2}/b^{k}}\frac{1-\Omega^{2}}{1-g\Omega}$$
$$\frac{n^{*}_{(OS)}}{n^{*}_{(NOS)}}=\frac{g\,(1+w)-\Omega(1+w-\Omega)/(b^{k}-\Omega)}{1+w-\Omega^{2}/b^{k}}\frac{1-\Omega^{2}}{g-\Omega}$$

To see the effect that offshoring has on the mass of firms operating at Home, we first re-arrange the terms in the $n_{(OS)}/n_{(NOS)}$ ratio to get:

$$\frac{n_{(OS)}}{n_{(NOS)}}=(1+\Omega)\frac{1-g\Omega(1-\Omega/b^{k})/(1+w-\Omega)}{1-g\Omega}\frac{1-\Omega}{1+w-\Omega^{2}/b^{k}}$$

The first two terms of the fraction are larger than unity, while the last one is lower[44]. Overall, the mass of firms may increase or decrease when opportunity to offshore arises. This is supported by numerical simulations for the range of acceptable model parameters and conforming to the restrictions set in (A3-2), (A3-3) and (A3-4). Offshoring however unambiguously lowers the mass of Foreign-made varieties as $n^*_{(OS)}/n^*_{(NOS)} < 1$[45].

On the consumption side, it follows that the mass of varieties available to a typical consumer at Home changes in smaller proportions than the mass of varieties produced at Home:

$$\frac{n_{C(OS)}}{n_{C(NOS)}} = \frac{n_{(OS)}}{n_{(NOS)}} \frac{1+\Omega \dfrac{F_D}{F_X} \dfrac{g(1-\Omega/b^k)(1+w)-\Omega(1+w-\Omega)/b^k}{1+w-\Omega-g\Omega(1-\Omega/b^k)}}{1+\Omega \dfrac{F_D}{F_X}\dfrac{g-\Omega}{1-g\Omega}}$$

This is due to the fact that $\dfrac{g(1-\Omega/b^k)(1+w)-\Omega(1+w-\Omega)/b^k}{1+w-\Omega-g\Omega(1-\Omega/b^k)} < \dfrac{g-\Omega}{1-g\Omega}$. The logic behind the result is that offshoring reduces the mass of H-imported varieties relative to the mass of varieties produced by domestic firms. For the F-country, offshoring raises the fraction of H-exported varieties relative to the mass produced locally and this effect can be strong enough to offset the drop in Foreign-made varieties:

$$\frac{n^*_{C(OS)}}{n^*_{C(NOS)}} = \frac{n^*_{(OS)}}{n^*_{(NOS)}} \frac{1+\Omega \dfrac{F_D}{F_X}\dfrac{1}{b^k} \dfrac{1+w-\Omega-g\Omega(1-\Omega/b^k)}{g(1-\Omega/b^k)(1+w)-\Omega(1+w-\Omega)/b^k}}{1+\Omega \dfrac{F_D}{F_X}\dfrac{1-g\Omega}{g-\Omega}}$$

Overall, the mass of varieties available to consumers in each country may increase or decrease with offshoring.

Lastly, we evaluate the changes that offshoring has on the labour usage in the variable costs sector of manufacturing production. Below we provide the ratio of employment in the performance of locally performed tasks prior and after offshoring:

$$\frac{L_{VC(OS)}}{L_{VC(NOS)}} = \frac{n_{(OS)}}{n_{(NOS)}} \frac{1}{1+\Omega} \left\{ \frac{w(b^{-s}/(1+d)-1)}{b^{1-s}-1} + \frac{\Omega(1+w-\Omega)}{b(1+d)(b^k-\Omega)} + 1 \right\}$$

As inspection of the terms reveals, the total local labour used in variable costs may increase or decrease as a result of offshoring. Observe that the term in curly brackets may be larger or less than unity. Therefore, an increase in the mass of active firms does not necessarily lead to increased labour usage in variable costs, nor a decrease in the mass of firms is detrimental for a higher domestic labour

[44] The second term is larger than unity because $(1-\Omega/b^k)/(1+w-\Omega) < 1$ as shown above. The third term $(1-\Omega)/(1+w-\Omega^2/b^k)$ is less than unity because $b^k > \Omega$ and $w > 0$.

[45] This can be seen by evaluating the difference between the numerator and denominator of the $n^*_{(OS)}/n^*_{(NOS)}$ ratio which reduces to $(1-1/b^k)\{1-g\Omega+\Omega^2/b^k(g-\Omega)+w\}+gw\Omega(\Omega/b^k-1)+w\Omega(\Omega-1)/b^k$. This term is always less than zero, because the term in curly bracket is positive due to $g<1$, $\Omega<1$, $w>0$ and $g>\Omega$ while all the remaining terms in brackets are negative as $b^k<1$, $b^k>\Omega$ and $\Omega<1$.

demand. In other words, labour employment in the variable costs may move in the same or opposite direction to the changes in the mass of operating firms, as it happens in all long-run outcomes under offshoring.

# Appendix 4: Case 3 solution

In this Appendix we derive the short run and long run equilibrium for Case 3. As described above, in this model, the H-firms that choose to be D-types produce locally, while all X-types offshore. Thus the cut-off unit labour requirement where H-firms the break-even for local sales is given by (15); the threshold productivity level, $\tilde{a}^{OS^{1-s}}$, beyond which the firms find it more worthwhile to become X-types offshoring is defined by the equality of operating profits of D-type non-offshoring and X-type offshoring firms, given by (13) and (29). After some manipulation, we have:

$$F_X + F_{OS} = (b^{1-s} - 1)\left(\frac{\tilde{a}^{OS}}{1-1/s}\right)^{1-s}\left(\frac{mE}{sP^{1-s}}\right) + f\left(\frac{b\tilde{a}^{OS}}{1-1/s}\right)^{1-s}\left(\frac{mgE}{sP^{*1-s}}\right) \tag{A4-1}$$

The cut off condition for F-firms remain (17) and (18).

This suggests that the CES price indices characterizing the new equilibrium are expressed as:

$$P^{1-s} = \int_0^{\tilde{a}^{OS}}\left(\frac{ba}{1-1/s}\right)^{1-s} ng\left[a|a_D\right]da + \int_{\tilde{a}^{OS}}^{a_D}\left(\frac{a}{1-1/s}\right)^{1-s} ng\left[a|a_D\right]da$$
$$+\int_0^{a_X^*} f\left(\frac{a}{1-1/s}\right)^{1-s} n^* g\left[a|a_D^*\right]da$$
$$P^{*1-s} = \int_0^{a_D^*}\left(\frac{a}{1-1/s}\right)^{1-s} n^* g\left[a|a_D^*\right]da + \int_0^{\tilde{a}^{OS}} f\left(\frac{ba}{1-1/s}\right)^{1-s} ng\left[a|a_D\right]da$$

Free entry of F-firms is determined by (22), as before. At Home, however, the expected operating profits of a potential entrant now include two alternatives: the entrant either becomes a locally producing firm, satisfying only domestic demand, or an offshoring firm selling into both markets. Accordingly, the free entry condition becomes:

$$\int_0^{\tilde{a}^{OS}}\left(\frac{(ba)^{1-s}}{(1-1/s)^{1-s}}\frac{mE}{sP^{1-s}} + \frac{f(ba)^{1-s}}{(1-1/s)^{1-s}}\frac{mgE}{sP^{*1-s}} - (F_D + F_X + F_{OS})\right)dG(a)$$
$$+\int_{\tilde{a}^{OS}}^{a_D}\left(\frac{a^{1-s}}{(1-1/s)^{1-s}}\frac{mE}{sP^{1-s}} - F_D\right)dG(a) = F_I$$

We begin by solving the free entry conditions together with the cut-off conditions and (A4-1) to get the long run equilibrium solutions for the cut-off coefficients. As in the Case 4, the threshold condition (A4-1) involves a summation sign, which means that it is impossible to express all cut-off and threshold coefficients in identical powers. To simplify matters, we introduce a coefficient $x$, such that $\tilde{a}^{OS} = xa_X^{OS}$ where $a_X^{OS}$ is determined by (33) and $x \le 1$ for the Case 3 to apply. Although we can not get closed form solutions, we are able to find implicit solutions, which are:

$$a_D^k = \frac{F_I(1+k-s)a_0^k}{F_D(s-1)}\frac{b^k - x^k\Omega^{OS}}{b^k - x^k\Omega\Omega^{OS}};\ \tilde{a}^{OS\,k} = \frac{F_I(1+k-s)a_0^k}{F_X(s-1)}\frac{x^k\Omega(1-\Omega)}{b^k - x^k\Omega\Omega^{OS}}$$
$$a_D^{*\,k} = \frac{F_I(1+k-s)a_0^k}{F_D(s-1)}\frac{b^k(1-\Omega)}{b^k - x^k\Omega\Omega^{OS}},\ a_X^{*\,k} = \frac{F_I(1+k-s)a_0^k}{F_X(s-1)}\frac{\Omega(b^k - x^k\Omega^{OS})}{b^k - x^k\Omega\Omega^{OS}}$$

where $\Omega^{OS} \equiv \frac{F_{OS}+F_X}{F_D}\left(\frac{F_X}{fF_D}\right)^{\frac{k}{1-s}}$, with $\Omega < \Omega^{OS}$ by construction, and $x$ which must satisfy:

$$\frac{\tilde{a}^{OS^{1-s}}}{a_D^{1-s}} = \left(\frac{F_{OS}+F_X}{F_D} - x^{1-s}\,\frac{F_X}{F_D}\right)\frac{1}{b^{1-s}-1}$$

We can now compute the equilibrium mass of varieties produced at Home and Foreign nations:

$$n = \frac{mE(1+k-s)}{F_D sk}\,\frac{\left(1-\frac{g\Omega(b^k - x^k\Omega^{OS})}{b^k(1-\Omega)}\right)(b^k - x^k\Omega^{OS})}{b^k - x^k\Omega^{OS} + x^k\Omega(1-\Omega)(1+F_{OS}/F_X) - x^{1+k-s}\,\Omega(b^k - x^k\Omega\Omega^{OS})/b^k}$$

$$n^* = \frac{mE(1+k-s)}{F_D sk}\,\frac{g\,(b^k - x^k\Omega^{OS}) + x^k\Omega(1-\Omega)(g\,(1+F_{OS}/F_X) - x^{1-s}\,(1+g))}{b^k - x^k\Omega^{OS} + x^k\Omega(1-\Omega)(1+F_{OS}/F_X) - x^{1+k-s}\,\Omega(b^k - x^k\Omega\Omega^{OS})/b^k}$$

The mass of varieties consumed in each nation then becomes:

$$n_C = n\left(1+\Omega\frac{F_D}{F_X}\,\frac{g\,(b^k - x^k\Omega^{OS}) + x^k\Omega(1-\Omega)(g\,(1+F_{OS}/F_X) - x^{1-s}\,(1+g))}{b^k(1-\Omega) - g\Omega(b^k - x^k\Omega^{OS})}\right)$$

$$n_C^* = n^*\left(1+\Omega\frac{F_D}{F_X}\,\frac{x^k}{b^k}\,\frac{b^k(1-\Omega) - g\Omega(b^k - x^k\Omega^{OS})}{g\,(b^k - x^k\Omega^{OS}) + x^k\Omega(1-\Omega)(g\,(1+F_{OS}/F_X) - x^{1-s}\,(1+g))}\right)$$

where $n$ and $n^*$ are the equilibrium mass of firms operating in each country given just above.

Noting that only X-type firms offshore, the total labour employment in the sector, accounting only for domestic workers, is given by:

$$\begin{aligned}L_{VC} = {} & \int_0^{\tilde{a}^{OS}} \frac{nb^{-s}\,a^{1-s}}{(1+d)(1-1/s)^{-s}}\,\frac{mE}{P^{1-s}}\,dG[a|a_D] + \int_{\tilde{a}^{OS}}^{a_D} \frac{na^{1-s}}{(1-1/s)^{-s}}\,\frac{mE}{P^{1-s}}\,dG[a|a_D] \\ & + \int_0^{\tilde{a}^{OS}} \frac{nfb^{-s}\,a^{1-s}}{(1+d)(1-1/s)^{-s}}\,\frac{mgE}{P^{*1-s}}\,dG[a|a_D]\end{aligned}$$

Substituting the equilibrium cut-off labour requirements, we have:

$$L_{VC} = \frac{nk(s-1)F_D}{1+k-s}\left\{1+\frac{x^{1+k-s}\,\Omega(1-\Omega)}{b(1+d)(b^k - x^k\Omega^{OS})} + \frac{b^{-s}\;/(1+d)-1}{b^{1-s}-1}\,\frac{x^k\Omega(1-\Omega)}{b^k - x^k\Omega^{OS}}\left(1+\frac{F_{OS}}{F_X} - x^{1-s}\right)\right\}$$

As usual, the solutions of the Case 3 are complemented by a set of restrictions. For the Case 3 to apply we require that $a_X^{OS^{1-s}} \le \tilde{a}^{OS^{1-s}} \le a_X^{1-s}$, excluding a situation when $a_X^{OS^{1-s}} = \tilde{a}^{OS^{1-s}} = a_X^{1-s}$, because $b \neq 1$. Using the definition of the cut-off coefficients, this implies that:

$b^k \le x^k \le 1$, excluding $b = x = 1$ (A4-2)

We continue to assume that only a fraction of firms export, i.e. $a_D^{OS} > a_X^{OS}$ and $a_D^* > a_X^*$ [46]. The former requires[47]:

$$b^k > \frac{x^k \Omega^{OS}}{1 - \left(\frac{F_D}{F_D + F_{OS}}\right)^{\frac{k}{1-s}} \frac{\Omega F_D}{F_X}(1-\Omega)} \tag{A4-3}$$

while the latter requires: $\frac{F_X}{\Omega F_D}\frac{b^k(1-\Omega)}{b^k - x^k \Omega^{OS}} > 1$, which is always the case given (24), $b^k \le x^k \le 1$ and $\Omega < \Omega^{OS}$.

Note, that since $\Omega < 1$, for the cut-off solutions to be positive, we require that $b^k - x^k \Omega^{OS} > 0$. However, it may be easily shown that when (A4-3) holds, this requirement also holds.

Finally, the mass of firms operating in each country is positive if:

$$b^k - x^k \Omega^{OS} + x^k \Omega(1-\Omega)(1 + F_{OS}/F_X) - x^{1+k-s}\Omega(b^k - x^k \Omega\Omega^{OS})/b^k > 0 \tag{A4-4}$$

$$g(b^k - x^k \Omega^{OS}) + x^k \Omega(1-\Omega)(g(1 + F_{OS}/F_X) - x^{1-s}(1+g)) > 0 \tag{A4-5}$$

Inequalities (A4-2) through (A4-5) are the most binding constraints and must be satisfied for Case 3 to hold.

Given that $b^k \le x^k \le 1$, excluding $b = x = 1$ and $\Omega < \Omega^{OS}$, it can be shown that offshoring changes the cut-off productivity and price levels in the same directions as in all other equilibrium outcomes:

$$\frac{a^k_{D(OS)}}{a^k_{D(NOS)}} = \frac{a^{*k}_{X\ (OS)}}{a^{*k}_{X\ (NOS)}} = \frac{P^k_{(OS)}}{P^k_{(NOS)}} = \frac{(b^k - x^k \Omega^{OS})(1+\Omega)}{b^k - x^k \Omega\Omega^{OS}} < 1$$

$$\frac{a^{*k}_{D\ (OS)}}{a^{*k}_{D\ (NOS)}} = \frac{P^{*k}_{(OS)}}{P^{*k}_{(NOS)}} = \frac{b^k(1-\Omega^2)}{b^k - x^k \Omega\Omega^{OS}} > 1$$

$$\frac{\tilde{a}^{OS\,k}_{\ (OS)}}{a^k_{X\,(NOS)}} = \frac{x^k(1-\Omega^2)}{b^k - x^k \Omega\Omega^{OS}} > 1$$

$$\frac{P^k_{(OS)}}{P^{*k}_{(OS)}} = \left(\frac{1}{g}\right)^{\frac{k}{1-s}} \frac{b^k - x^k \Omega^{OS}}{b^k(1-\Omega)} < 1$$

The ratios of the mass of varieties produced in each economy are as follows:

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[46] $a_D^{OS}$ is determined by the equation (32).

[47] Note, that the denominator of the (A4-3) is always positive because otherwise $b^k$ would have to be negative, which is a contradiction with our definition of $b$.

$$\frac{n_{(OS)}}{n_{(NOS)}}=\frac{1-\Omega^2}{1-g\Omega}\frac{\left(1-\frac{g\Omega(b^k-x^k\Omega^{OS})}{b^k(1-\Omega)}\right)(b^k-x^k\Omega^{OS})}{b^k-x^k\Omega^{OS}+x^k\Omega(1-\Omega)(1+F_{OS}/F_X)-x^{1+k-s}\Omega(b^k-x^k\Omega\Omega^{OS})/b^k}$$

$$\frac{n^*_{(OS)}}{n^*_{(NOS)}}=\frac{1-\Omega^2}{g-\Omega}\frac{g(b^k-x^k\Omega^{OS})+x^k\Omega(1-\Omega)(g(1+F_{OS}/F_X)-x^{1-s}(1+g))}{b^k-x^k\Omega^{OS}+x^k\Omega(1-\Omega)(1+F_{OS}/F_X)-x^{1+k-s}\Omega(b^k-x^k\Omega\Omega^{OS})/b^k}$$

As previously, the mass of active firms at Home nation may increase or decrease, while in Foreign nation it unambiguously decreases. In terms of the varieties consumed we have:

$$\frac{n_{C(OS)}}{n_{C(NOS)}}=\frac{n_{(OS)}}{n_{(NOS)}}\frac{1+\Omega\frac{F_D}{F_X}\frac{g(b^k-x^k\Omega^{OS})+x^k\Omega(1-\Omega)(g(1+F_{OS}/F_X)-x^{1-s}(1+g))}{b^k(1-\Omega)-g\Omega(b^k-x^k\Omega^{OS})}}{1+\Omega\frac{F_D}{F_X}\frac{g-\Omega}{1-g\Omega}}$$

$$\frac{n^*_{C(OS)}}{n^*_{C(NOS)}}=\frac{n^*_{(OS)}}{n^*_{(NOS)}}\frac{1+\Omega\frac{F_D}{F_X}\frac{x^k}{b^k}\frac{b^k(1-\Omega)-g\Omega(b^k-x^k\Omega^{OS})}{g(b^k-x^k\Omega^{OS})+x^k\Omega(1-\Omega)(g(1+F_{OS}/F_X)-x^{1-s}(1+g))}}{1+\Omega\frac{F_D}{F_X}\frac{1-g\Omega}{g-\Omega}}$$

It can be shown that $n_{C(OS)}/n_{C(NOS)}<n_{(OS)}/n_{(NOS)}$ and $n^*_{C(OS)}/n^*_{C(NOS)}>n^*_{(OS)}/n^*_{(NOS)}$ suggesting qualitatively identical conclusions to all the previous cases. Similarly, domestic labour demand in the differentiated good sector at Home may increase or decrease independently of the effect of offshoring on the mass of firms operating in the economy. The ratio is given by:

$$\frac{L_{VC(OS)}}{L_{VC(NOS)}}=\frac{n_{(OS)}}{n_{(NOS)}}\frac{1}{1+\Omega}\left\{1+\frac{x^{1+k-s}\Omega(1-\Omega)}{b(1+d)(b^k-x^k\Omega^{OS})}+\frac{b^{-s}/(1+d)-1}{b^{1-s}-1}\frac{x^k\Omega(1-\Omega)}{b^k-x^k\Omega^{OS}}\left(1+\frac{F_{OS}}{F_X}-x^{1-s}\right)\right\}$$

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**Figure 1: Long-run equilibrium – Case 1**

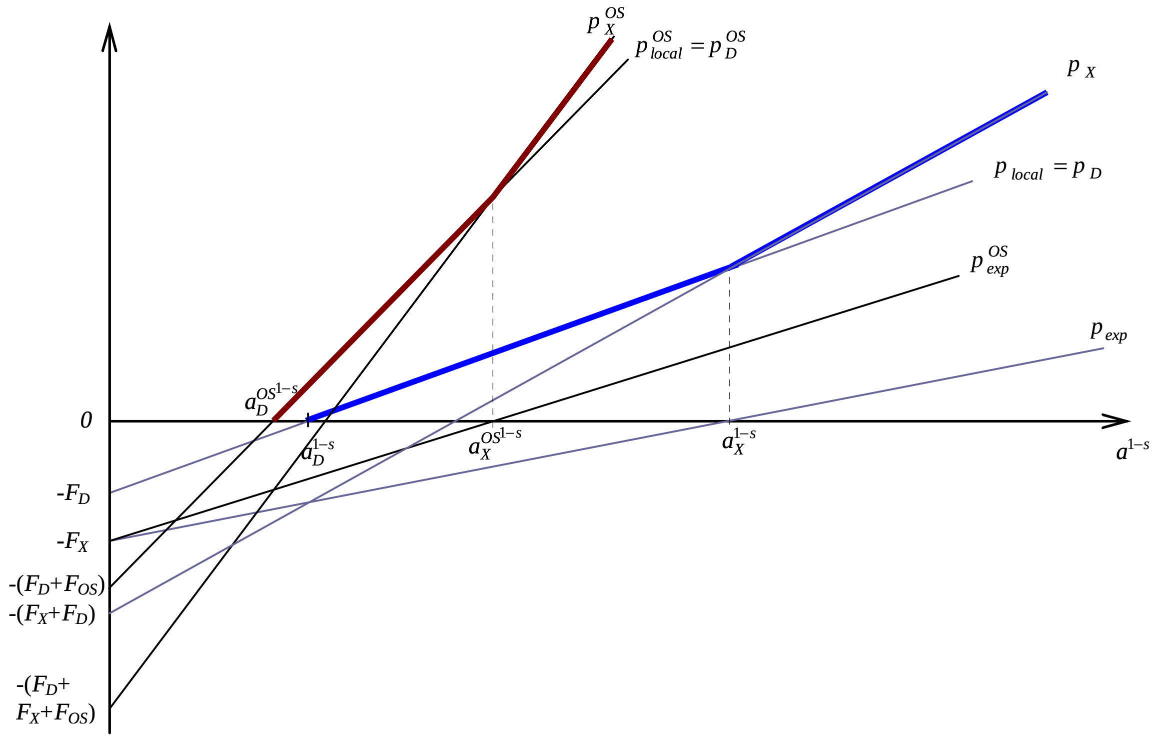

**Figure 2: Long-run equilibrium – Case 2**

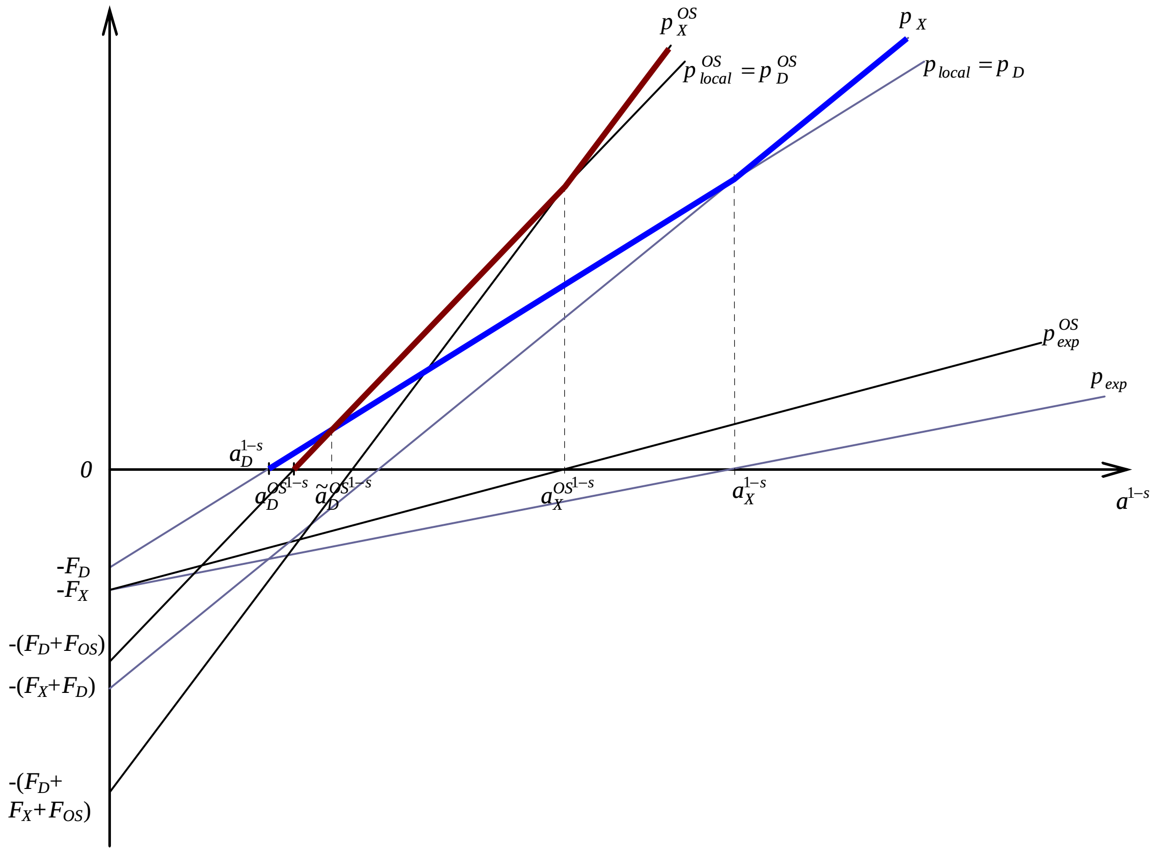

**Figure 3: Long-run equilibrium – Case 3**

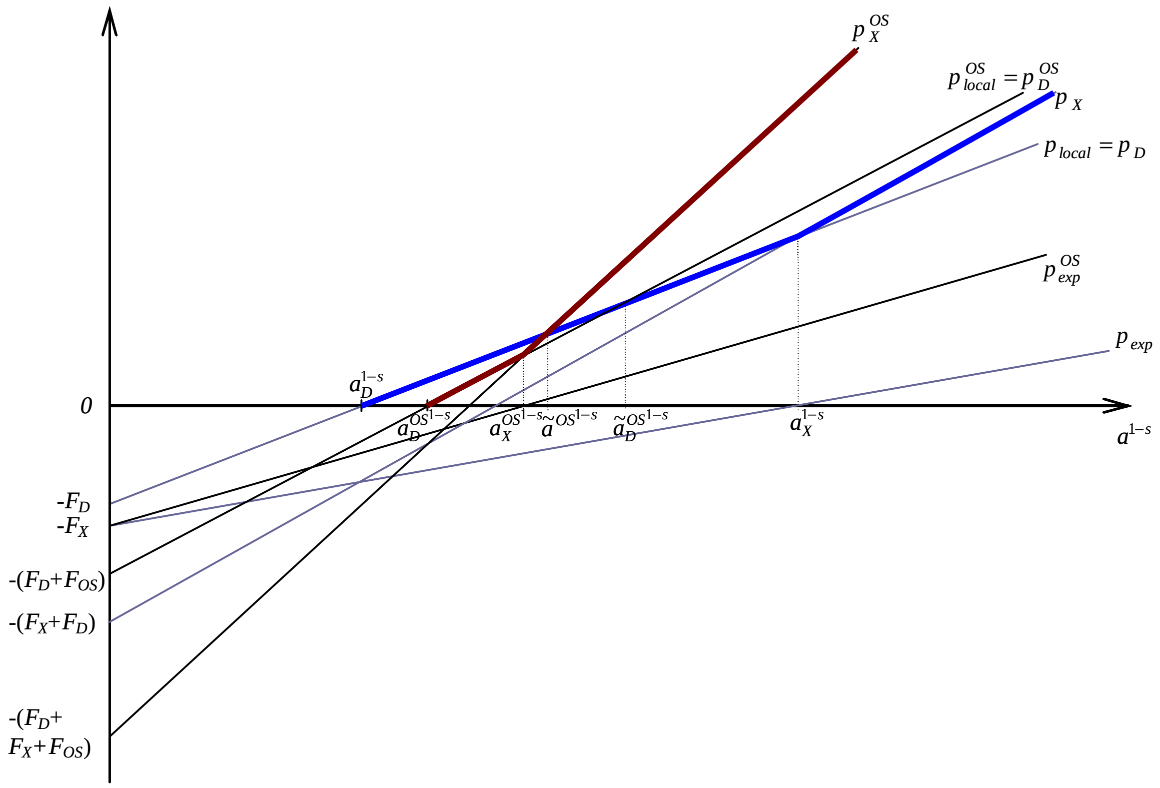

**Figure 4: Long-run equilibrium – Case 4**