The Spatial Pattern of FDI: Some Testable Hypotheses

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Abstract

This paper is a simple extension of the standard FDI model of Markusen and Horstmann (1992). This latter predicts firms would supply nearby markets with exports but far away markets with FDI. Nevertheless, this does not match the spatial pattern in the data for many home nations and industries. We propose a model with heterogeneous firms where the spatial pattern of FDI depends upon distance-linked communications costs as well as trade costs; the resulting model lines up both with the aggregate knowledge-capital model evidence and the firm-level evidence of Helpman-Melitz-Yeaple, while still allowing individual firms to engage in FDI in nearby markets while supplying distant markets via exports.

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1 Introduction

Markusen and Horstmann (1992) developed a model in which market structure is determined endogenously as the outcome of plant location decisions by firms. They incorporated multinational firms (MNFs) into a general equilibrium trade model, where firms benefit from internalization due to increasing returns at the firm level. Brainard (1993) followed a similar line of research in focusing on the location decisions, and she proposed what has become the standard approach for explaining horizontal multinational firms, the so called proximity versus concentration hypothesis\(^1\), or scale versus proximity. This hypothesis put in evidence the trade-off between the advantages from locating near customers and from concentrating production in only one location (which gives rise to scale economies at the plant level). In these models is more likely to be engaged in FDI activities when trade costs are particularly high. Thus, foreign subsidiaries’ sales will be increasing with distance. For the same reason, horizontal FDI are not encouraged by reduction in transport costs. On the contrary, when trade costs fall, scale economy advantages can outweigh the benefit from locating near customers. In this case, export activities could become more profitable.

Comparing this theory with the empirical evidence on FDI, we discover some discrepancies. In fact, despite the reduction in transport costs across different countries, there has been a consistent growth of multinational sales, in particular of FDI inflows. The data tells that multinational enterprises account for a very significant fraction of world trade flows; and trade in intermediate inputs between divisions of the same firm constitutes an important portion of these flows (Hanson, Mataloni and Slaughter, 2001). These data show a broad range of strategies that multinational enterprises can undertake, highlighting the fact that the classical distinction of FDI as either horizontal or vertical is partially misleading (Hanson, Mataloni and Slaughter, 2001). Trade and taxes are important policies which can affect the mix of affiliate strategies, other than the aggregate affiliate activity. But also distance, meant not only as geographical distance but also cultural differences, play an important role in the determination of the strategy chosen by the multinational enterprise.

These empirical findings complicate the usual prediction based on proximity versus concentration hypothesis. In fact, following this theory, the fall in trade costs should reduce FDI and encourage exports. However, what appears from the data is something different: the reduction in trade costs coincided with FDI growth\(^2\) (Buch, Kleinert, Lipponer and Toubal, 2005; Carr, 2019).

\(^1\)The proximity concentration trade-off predicts that “firms are more likely to expand their production horizontally across borders the higher are the transport costs and trade barriers and the lower are investment barriers and the size of scale economies at the level at the plant level relative to the firm level” (Brainard, 1997).

\(^2\)This seems to be confirmed in EU, where under the single market situation a reduction in the trade costs have been achieved.
Markusen, and Maskusen, 2001). Other works stress the theoretical reasons why foreign direct investment (FDI) into a host country may depend on the FDI in proximate countries (Blonigen, Davies, Waddell and Naughton, 2006; Baltagi, Egger, Pfaffermayr, 2006; Mayer and Head, 2004; Mello-Sampayo, 2004).

In what follow we try to reconcile the MNFs theory with empirical findings by combining firm types and destinations, so that to explain the ongoing pattern of supply mode decisions. In order to do so, we introduce the variation by firms by markets as a new element with respect to the existing literature where the variation is by market (homogeneous firm) or by firm in a single market (Helpman, Melitz and Yeaple, 2004). In this model we will have firms switching type from FDI to export in relation to distance: there would not be a one to one correspondence between firm type and its characteristics. As far as we know there are no other papers that look to the spatial implication of the location of FDI.

The paper "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity" by Melitz (2002), added two crucial elements to the new trade theory. The first is the fixed market entry costs that a potential entrant has to pay. The second is heterogeneity in firms' productivity. By introducing firm heterogeneity in the 1980’s Krugman model, he observed how an increase in the exposure to trade leads to reallocation towards the more efficient firms, without necessarily inducing an increase in the productive efficiency of individual firms. His findings are supported by several micro-econometric studies.

After Melitz’s paper, the study of the implications of firm level productivity differences has become an important field of interest in international economics. In fact, the shift from the representative firm framework to the heterogeneous firm framework has allowed to model some aspects of international commerce that until now have only been studied empirically.

In line with this new research is the paper by Helpman, Melitz and Yeaple (2004), "Exports versus FDI". Here, the authors built a multi-country, multi-sectoral general equilibrium model with the intent to analyze the decision of heterogeneous firms to serve foreign markets either through exports or local sales (FDI). Similar to Melitz (2002), they work with heterogeneous firms, identical nations, a single factor, but with more sectors. They find that firm-level heterogeneity plays a relevant role in explaining the choice between export and FDI flows. Their results rely on the assumption of perfectly symmetric countries and on the absence of asymmetries in transport costs or in fixed costs. As a consequence of this, a firm that does export to one single country will also export to every other country. This could limit our comprehension about reality, where usually a firm chooses a mixture of mode of supply.

3This result is partially contradicted by Baldwin et all (2004), where they pointed out that "although freer trade improves industry productivity in a level sense, it harms it in a growth sense".

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Building on this literature, our purpose is to develop a theoretical model where we study the effects of within-sectoral heterogeneity on the decision of firms to supply foreign markets, in a framework where distance plays a role. In order to do this we introduce some vertical linkages within the FDI strategy. In particular, we let the production of the final good variety require a combination of two intermediate goods. We claim that due to technological appropriability issues one of the two intermediate goods can only be made at home. This makes market access strategy through FDI to incur in trade costs: when a subsidiary is built abroad it has to import the intermediate good from the mother located in the home nation. This linkage makes a portion of the total marginal costs of selling via FDI rising with distance. Moreover, we assume there is a distance-related travelling cost of workers from home to host country to coordinate the establishment of the foreign affiliate; we call it communication cost. The communication costs as well as trade costs are the key element of this paper. The interaction between these two can make the supply mode via FDI not be convenient far away. Finally, the paper contributes in explaining the role of distance in affecting industry volume of sales.

In the present analysis the works by Helpman, Melitz and Yeaple (2004) and Antras-Helpman (2004) will be used; nevertheless, some interesting distinctions are introduced. Firstly, we give a role to distance by introducing trade and communication costs in the MNF’s activity. Secondly, we consider N symmetric countries located evenly around a circular trade route. The existence of a circular route generates asymmetries, which are expressed in terms of different country location and therefore different trade costs (this would imply that productivity will be not only firm-specific, but also country-specific). This element introduces a new level of heterogeneity, not only among firms, but also among countries. This higher level of heterogeneity keeps the analysis closer to reality. In fact, converse to the symmetric assumption in Helpman, Melitz and Yeaple (HMY), which yields an equilibrium where if a firm can engage in foreign market activity it will be active in every foreign market independently of the distance, the introduction of spatial distribution of firms gives a role to distance in determining the mode of supply decisions. This is in line with the recent empirical findings which confirm that the number of multinational firms is decreasing with distance. Through this work we will show that spatial distribution of affiliates is much richer than what the scale-versus-proximity model predict.

This paper is organized in the following way. Section 2 elaborates the model. Section 3 characterizes its equilibrium and investigates the role of distance and the effects of trade liberalization. Section 4 considers the effects of distance on sales. The last section concludes.

Footnote: In relation to its own productivity, it will be active as an exporter or as a subsidiary.
2 Theoretical Framework

We study the mode of supply decision between FDI and export in a multi-country framework. For this purpose, we merge the model by Helpman, Melitz and Yeaple (2004) with Antras-Helpman (2004). However, some interesting distinctions are introduced. Firstly, we give a role to distance by introducing trade and communication costs in the MNF’s activity. Secondly, we consider N symmetric countries located evenly around a circular trade route.

2.1 Preferences

Consumers in each country share the same preferences over two final goods: a homogeneous good, $z$, and a differentiated good, $x$. We assume a two-tier preference with Cobb-Douglas in upper tier and CES in the lower tier. A fraction of income, $\beta$, is spent on the differentiated good, $x$, and a fraction $(1 - \beta)$ is spent on the homogeneous good, $z$. The utility function is:

$$U = z^{(1-\beta)} \left[ \int_{v \in V} x(v)^{(\sigma - 1)/\sigma} \right]^{\sigma / (\sigma - 1)} dv$$

where $\sigma > 1$ represents the elasticity of substitution between any two products within the group and $V$ is the set of available varieties.

2.2 Supply

There are $N$ identical countries located evenly around a circular trade route. We assume $N$ to be odd, so that starting from an origin country, there exist for every destination country, another destination country located at the same distance (each destination has a clone). In this framework we have two final goods, two intermediate goods and one factor. Each country is endowed with labor, $L$, which is supplied inelastically.

There are two sectors, one homogeneous and one differentiated. The homogeneous sector, $z$, produces a homogeneous good with constant returns to scale and perfect competition. In this sector the technology is simple. We choose units of $z$ such that one unit of labor is required per unit of output. Thus, the unit cost function is $w$, where $w$ is the wage rate for labor. This unit cost function represents marginal and average costs. In the homogeneous sector, competition determines price equal marginal costs, $p_z = w$. It is convenient to choose good $z$ as the numeraire, so that $p_z = 1$; hence, the pricing condition will become: $1 = w$. As long as the homogeneous good $z$ is produced in every country and it is freely traded on international markets, the cost of producing this homogenous good is equal in every country, and along with this, the wages.

The differentiated sector, $x$, produces a continuum of horizontally differentiated varieties from
two intermediate goods (or tasks), $y_1$ and $y_2$. Both $y_1$ and $y_2$ are produced with one unit of labor, but $y_1$ can only be made at home, due to technological appropriability issues. Each variety is supplied by a monopolistically competitive firm which produces under increasing returns to scale which arise from a fixed cost. As preferences are Dixit-Stiglitz, a single producer competes equally with every other producer. As there are no costs of product differentiation, each firm will produce a different type of variety.

We consider three modes of supply in the $x$-sector; firms which sell only domestically (D-mode); firms who export (X-mode), and firms who supply the foreign market via FDI (M-mode). Hence, when a firm decides to serve the foreign market, chooses to export domestically produced goods, or produce in foreign via affiliate production.

As in Helpman, Melitz and Yeaple (2004), this choice is affected by the classical scale versus proximity trade-off. Nevertheless, in our model, geographical distance between countries matters for separate reasons, namely trade and communication costs. The fact that $y_1$ can only be made at home plays an important role in an open economy situation. In this context, if a firm chooses to supply the foreign market via local sales of foreign affiliates the intermediate good realized using foreign technology, $y_1^*$, cannot be considered a substitute for the intermediate good realized using the home technology, $y_1$. Every foreign affiliate must import the intermediate good $y_1$ from the home nation. This implies that the M-mode does not entirely avoid trade costs. We also assume that workers from home must periodically travel to host country to coordinate final production. This implies that there is a second distance-related cost that we call communication cost.

Following Helpman, Melitz and Yeaple (2004), entering the $x$ sector involves a fixed variety-development cost $f_I$. Subsequently, each entrant draws a labor per unit output coefficient (called $a$) from a cumulative density function $G(a)$, that is common to every country. The support of the continuous random variable $a$ is $0 \leq a \leq a_0$. Upon drawing its own parameter $a$, each firm decides to exit and not to produce (this happens if it has a low productivity draw), or to produce. In this case, the firm must face additional fixed costs linked to the mode of supply chosen. If it chooses to produce for its own domestic market, it pays the additional fixed market entry cost, $f_D$. If the firm chooses to export, it bears the additional costs $f_X$ of meeting different market specific standards (for example, the cost of creating a distribution network in a new country). Finally, if the firm chooses to serve foreign markets through FDI, there would be two types of fixed costs. First of all, there is a fixed cost of creating a distribution network as well as building up new capacities in the foreign country, $f_M$. In addition to this fixed cost, we assume there is a distance-related travelling cost of workers from home to host country to coordinate the establishment of the foreign

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5 Where $I$ stands for innovation.

6 Which in our model means only local production of the $y_2$ intermediate good.
affiliate. We call this $f(d)$ and note that it rises with distance, $d$.

As mentioned, the homogeneous sector is not subject to trade costs, but the $x$-sector incurs in iceberg costs\(^7\) proportional to round-the-circle distance. More precisely, in case of supply through X-mode, the entire final good is subject to iceberg costs, while in case of M-mode only the intermediate good $y_1$, which has to be imported from home nation, is subject to iceberg costs. Selling one unit in the export market $j$, would require shipment from country $i$ of $\tau \geq 1$ units for the exporting sector and $\tau^n$ for the FDI sector. Since FDI is affected by trade costs, its marginal cost is increasing with distance, as will the distance-related communication cost.

In this framework, all firms face a constant probability of death. This event is described by a Poisson distribution with an hazard rate $\delta$: in every period the firm can be hit by this bad event and forced to exit. For simplicity, we assume that there is no time discounting.

### 2.3 Intermediate Results

#### 2.3.1 Demand

Given preferences across varieties have the standard CES form, the demand function is:

$$
x_i(v) = \frac{\beta E^i}{P^{1-\sigma}p_i(v)^{-\sigma}} = A^i p_i(v)^{-\sigma} \tag{1}
$$

where $P^{1-\sigma}$ is the CES price index, $A^i$ and $x_i(v)$ represent the demand shifter and consumption of typical variety $v$ in country $i$. $A^i$ is exogenous from the perspective of the firm and composed by the aggregate level of spending on the differentiated good in country $i$, $\beta E^i$; $n^i$ represents the measure of varieties available in the country and $p_i(v)$ is the consumer price index of variety $v$. The inverse demand function is given by

$$
p_i(v) = A^i x_i(v)^{-\frac{1}{\sigma}} \tag{2}
$$

#### 2.3.2 Organization and Product Variety

Given that $f_I$ have been paid, output of every variety is described by a Cobb-Douglas function of the intermediate goods:

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\(^7\)Trade costs are broadly defined, so as to include different kind of impediments: trade barriers, cultural differences etc.

\(^8\)Where $A^i = \frac{\beta E^i}{P^{1-\sigma}}$. 

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\[ x_i(v) = \frac{1}{a(v)} \left( \frac{y_1}{\eta} \right)^\eta \left( \frac{y_2}{1 - \eta} \right)^{1-\eta}, \quad 0 < \eta < 1 \]  

(3)

where \( a(v) \) is the firm specific marginal cost\(^9\), and \( \eta \) is the Cobb-Douglas cost share of \( y_1 \), common across all nations. When trade is possible, firms that produce decide whether to supply a particular market and how, i.e. via export or FDI strategies. This will depend upon their own productivity and trade costs. As mentioned before, the marginal costs in the exporting sector will be higher than the one in the FDI sector. Hence, despite the existence of \( N \) symmetric countries, the fact that they are located evenly around a circular trade route\(^{10}\) makes distance playing a role in the consumer price. The trade cost is constructed in the following way: between the origin country and the nearest destination country the trade cost is \( \tau \); between the origin country and the second nearest destination country, the trade cost is defined as \( 2\tau \); between the origin country and the third nearest destination country \( 3\tau \) and so on until the most distant country \( (N - 1)/2 \) is reached, here the trade costs is \( [(N - 1)/2] \tau \). The condition in which we are interested in goes from the destination country 1 step away from the origin to the destination country \( (N - 1)/2 \) steps away from the origin. We exploit the mirror image nature of the circle: countries \( (N - 1)/2 \) to \( N - 1 \) steps away are just a mirror image.

Since \( y_1 \) and \( y_2 \) are produced with \( L \) whose wage is unity, the marginal costs, \( mc_{Di} \), for local production in home is\(^{11}\):

\[ mc_D = a(v) \]

where country symmetry allows us to drop the country subscript. The marginal cost for exporting to a market that is \( d \)-steps away is:

\[ mc_{X,d} = a(v) d\tau \]

where \( d \) and \( \tau \) represent distance and trade cost respectively. Finally, the marginal cost for supplying the \( d \) market via local sales of foreign affiliates is:

\[ mc_{M,d} = a(v) (d\tau)^\eta \]

Note that distance matters but only in relation to cost share, \( \eta \), of the intermediate good \( y_1 \) used in the production of the final good. Using the mark up, \( \sigma/(\sigma - 1) \), we can easily derive the price for each particular mode of supply decisions.

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\(^9\) \( 1/a(v) \) represents the firm specific productivity parameter.

\(^{10}\) Since countries are symmetric and located around a circular trade route, destination countries are labelled: 1, 2, 3..., \( N - 1 \).

\(^{11}\) See appendix A1 for details of the cost minimization problem.
2.3.3 Mode of Supply Decisions

The mode of supply decision choice will involve the comparison of profit levels taking account of the various fixed and variable trade costs. A firm can decide to: (i) not supply a market, (ii) supply it via exports, or (iii) supply it via local sales of foreign affiliates. Of course, the local market is supplied by local sales, if the firm is active (iv).

For what concerns the foreign markets, the two types of distance related costs, \( d\tau \) and \( f(d) \), imply that almost anything can happen. To focus on the central case, we assume parameters such that we get the ranking as in HMY when there are only two nations. Namely, only firms with sufficiently high productivity supply the foreign market at all, with the most productive, supplying it via FDI rather than exports. In this way our model is in line with the HMY empirical findings. The regularity condition for this is:

\[
f_D < d\tau^{(\sigma-1)} f_X < d\tau^{(\eta(\sigma-1))} [f_M + f(d)]
\]

The optimal mode of supply depends, as in HMY, on a firm’s productivity. As described above, four cases are relevant.

Case (i). The firm decides not supply a market and exits.

Case (ii). The firm decides to supply a market via exports, so the profit from exporting to a market that is d-steps away is:

\[
\pi_{X,d} = [p_X(v) - a(v) d\tau] x(v) - f_X
\]

Case (iii). The firm decides to supply a market via FDI, so the profit realized by a subsidiary located in the d-steps away market is:

\[
\pi_{M,d} = [p_M(v) - a(v) (d\tau)^\eta] x(v) - f(d) - f_M
\]

where \( f(d) \) represents the fixed communication costs and \( (d\tau)^\eta \) is the trade costs associated with the intermediate good, \( y_1 \), imported from the home country. The subsidiary located in the host country has to face both the communication costs, which rise with distance, and the trade costs that hit the imported intermediate.

Case (iv). The market under consideration is the firm’s home market, so the profits for undertaking D-mode supply are:

\[
\pi_D(v) = [p_D(v) - a(v)] x(v) - f_D
\]

Using the intermediate results for optimal price we calculate the operating profit for the three
modes of supplying a market. The profit from serving the domestic market is:

$$\pi^{*}_{D}(a, A, \eta) = A a^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{(1-\sigma)} - f_D$$

where $A_i$ and $\eta$ are industry (and so country) specific. Using $B = \frac{A}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{(1-\sigma)}$ we obtain:

$$\pi^{*}_{D}(a, A, \eta) = B a^{1-\sigma} - f_D \quad (7)$$

If a firm chooses the X-mode for a given foreign market, then its equilibrium net operating profit on sales in that market is:

$$\pi^{*}_{X,d}(a, A, \eta) = B (d^\tau a)^{1-\sigma} - f_X \quad (8)$$

If a firm chooses the M-mode for a given foreign market, then the equilibrium net operating profit it would earn is:

$$\pi^{*}_{M,d}(a, A, \eta) = B a^{1-\sigma} [(d^\tau)^\eta]^{1-\sigma} - f(d) - f_M \quad (9)$$

These profit functions are depicted in Figure 1, which helps clarify the analysis.
In this figure, we represent how distance affects the modes of supply. On the horizontal axis we have $a^{1-\sigma}$; since $\sigma > 1$, this variable can be used as a firm-level productivity index. All the profits described in (7), (8) and (9) are increasing functions of $a^{1-\sigma}$. The diagram plots $\pi_D$, $\pi_{X_1}$ and $\pi_{M_1}$ which are the operating profits for a firm supplying a market locally ($\pi_D$), or supply a market 1-step away ($\pi_{X_1}$) or supplying the same market via M-mode ($\pi_{M_1}$). Independently of the type of activity, the more productive is a firm, the more profits it will make. The profit function $\pi_{M_1}$ is slightly flatter than $\pi_D$, due to trade costs, and its vertical intercept is lower due to higher fixed costs. The figure also plots the profits for a market 2-steps away as $\pi_{X_2}$ and $\pi_{M_2}$. Consider $\pi_{M_2}$. In this case, the slope of $\pi_{M_2}$ is flatter than $\pi_{M_1}$ due to increase in trade costs and its vertical intercept is lower due to the higher distant dependent communication cost, i.e. $f(2) > f(1)$. The profit function $\pi_{X_1}$ is flatter than $\pi_D$ due to trade costs. In the supply mode via FDI only a part of the intermediate goods incurs in trade costs, this makes $\pi_{M_1}$ steeper than $\pi_{X_1}$; this condition is preserved for any further increase in distance: $\pi_{M_2}$ is also steeper than $\pi_{X_2}$.

From Figure 1 we see that there exist different productivity levels at which a firm is indifferent between supply modes; these change with distance. The cutoff productivity level at which operating profits from domestic sales equal zero is $a_D^{1-\sigma}$. The productivity levels at which exporters and
FDI just break even are generically \( a_{X,d}^{1-\sigma} \) and \( a_{M,d}^{1-\sigma} \). Greater distance will modify these cutoffs. For any given market, say one that is 1-step away, \( a_{X1}^{1-\sigma} < a_{M1}^{1-\sigma} \). For market 2-steps away \( a_{X2}^{1-\sigma} < a_{M2}^{1-\sigma} \). These conditions are ensured by the regularity condition. If the \( a_{X,d}^{1-\sigma} \) rises with the distance of the market "d", we cannot say the same for \( a_{M,d}^{1-\sigma} \). In fact, \( a_{M,d}^{1-\sigma} \) has an ambiguous behavior with respect to distance, which depends on the freeness of trade. We cannot a priori rank the thresholds for X versus M, nor for M at different distances. More precisely, Figure 1 holds for sufficiently high freeness of trade and distance.

2.4 Equilibrium Conditions

We now turn to formal statements of the thresholds illustrate in Figure 1.

2.4.1 The Cutoff Conditions

Firms will choose the optimal supply mode for each market. To relate the choice to firms’ marginal cost we define a threshold marginal costs, \( a(v) \), for each destination and for each mode of supply. Using the equilibrium operating profit of serving the domestic market from (7), we can derive the domestic cutoff condition:

\[
a_D = \left( \frac{f_D}{B} \right)^{\frac{1}{1-\sigma}}
\]

That is firms with \( a(v) \) below \( a_D \) will find it optimal to supply their local market; firms with \( a(v) > a_D \) expect negative profits and exit the industry.

The choice in foreign markets is more complex, so it could be helpful to structure the discussion with the help of Figure 1. As we see from the figure the net operating profits of supplying the foreign market d-steps away, rises under both modes of supply. Firms with \( a_{X,d} < a(v) < a_D \) have positive operating profits from sales in the domestic market, but they lose money if they choose to serve foreign markets. We use the net operating profit from exporting in (8) to derive the X-mode cutoff is:

\[
a_{X,d} = \left( \frac{f_X}{B(\tau d)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}
\]

Thus, only firms with \( a(v) \leq a_{X,d} \) will consider export to the d market.

Notice, from Figure 1, that at \( a(v) = a_{X,d} \), exporting yields a higher net operating profit then FDI. This ordering switches, however, for firms with \( a(v) \leq a_{M,d} \), where this is defined as the \( a(v) \) where:

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\[ a_{M,d} = \left( \frac{f(d) + f_M - f_X}{B \left[ (d\tau)^{\eta(1-\sigma)} - (d\tau)^{1-\sigma} \right]} \right)^{1/\sigma} \]  

(12)

This M-mode cutoff is obtained by equating the operating profits from doing FDI, (9) with the operating profit from doing export, (8). This because by construction, a firm will choose to supply the d-steps away country via FDI if and only if the FDI strategy is more profitable than the export strategy, i.e. if this holds:

\[ \pi_{M,d} - \pi_{X,d} \geq f(d) + f_M - f_X \]

which can be rewritten as,

\[ B a_{M,d}^{1-\sigma} \left[ (d\tau)^{\eta(1-\sigma)} - (d\tau)^{1-\sigma} \right] = f(d) + f_M - f_X \]

Notice that if \[ a(v) \leq a_{M,d} \], M-mode supply yields a higher net operating profit.

From the diagram it is clear that \[ a_{X,d} > a_{M,d} \] for every level of distance. Therefore, considering Figure 1, when \( d = 2 \), both \( \pi_{M,d} \) and \( \pi_{X,d} \) become flatter, but \( \pi_{X,d} \) always by more. Hence, the new crossing point defining the new equilibrium M-mode cutoff, \( a_{M,2} \), will be at the right of \( a_{X,2} \).\(^{12}\)

Nevertheless, some firms that were supplying market 1-step away via M-mode, switch to X-mode as a consequence of higher trade costs and communication costs.

### 2.4.2 The Role of Distance

A key goal of our study is to characterize the spatial pattern of modes of supply. This is implicit in the cutoff conditions, but here we highlight the role of distance in explaining the variation by firms by market. The pattern of organizational forms could be characterized in two main steps. Markusen and Horstmann (1992) proposed a general-equilibrium model where MNFs arise due to a market-access motive to substitute for export flows, or what is termed “horizontal” FDI. Under the assumption of high trade costs, they found export nearby and FDI far away; this result is also called scale versus proximity result. More recently, HMY introduced firms heterogeneity in the pattern of organizational forms; here the nature of serving the market depends on the nature of the firm. Both papers are developed in a two-country framework. In the present work, the scale versus proximity is used in a model with heterogeneous firms and many locations, implying that there would not be a one to one correspondence between firm type and firm characteristic. In this context, a firm could decide to supply via M-mode until a particular destination country, and then when distance becomes too high, to switch to X-mode supply. In Figure 2 is shown the firm level

\(^{12}\)This derives from the regularity condition assumed.
characteristics with multiple destinations.

Figure 2: Modes of Supply and Destinations

In Figure 2 we represent the variation by firms by market with respect to the productivity level, $a^{1-\sigma}$, assuming a certain level of distance and trade openness. In this figure, we refer to the nearest destination country using the label 1. The origin country can reach the other N-1 countries with two different market access strategies: export or FDI. Points $a_{D1}^{1-\sigma}$, $a_{X1}^{1-\sigma}$ and $a_{M1}^{1-\sigma}$ represent the classical HMY model, where location does not play any role: if a firm is doing FDI toward the nearest destination country, country 1, is also doing FDI toward the other $N-1$ countries. For this reason they consider types of firm: X-type, M-type. In this paper a given firm may find it optimal to supply via exports to one market but via FDI to another, so we consider mode of supply X-mode, M-mode by nation.

The existence of distance permits to highlight a new pattern. On the right hand side of $a_{M1}^{1-\sigma}$ there is a new region which represents the spatial variation in M versus X mode supply, for each firm. In this analysis $a_{D}^{1-\sigma}$ represents the minimum productivity level in order to supply the local market; $a_{X1}^{1-\sigma}$ is the minimum productivity level in order to become an exporter to country 1; $a_{M1}^{1-\sigma}$ represents the productivity threshold in order to do FDI in country 1, and so on. These threshold levels change with distance. In relation to its own productivity, a single firm can undertake M-mode supply toward country 1 and then X-mode supply toward country 2 through N-1. This switching is determined by the increasing variable and fixed costs. Only the more productive firm will supply via local sales of foreign affiliates in every destination country.
Remark 1 Under certain conditions, namely high distance and freeness of trade, the area on the right hand side of $a_{M1}^{1-\sigma}$ is shrinking the more important is distance. This implies that few firms will supply the far away market via FDI market access strategy. There exists a critical value of $d$, specific for each firm, at which FDI strategy is not as profitable as export strategy. When this critical level is reached this specific firm will undertake export activities, abandoning FDI.

Remark 2 If the above initial conditions are not respected, we will observe a switch from the X-mode to the M-mode.

Remark 3 If we aggregate all foreign markets into one, we would observe the HMY association between firm level efficiency and supply mode. However, considering a single firm with efficiency $(a')^{1-\sigma}$, then we see that this firm supplies nearby markets by M-mode, but further away markets by X-mode.

It is also possible to consider Figure 2 in terms of export platform. More precisely, the area after $a_{M1}^{1-\sigma}$ could be interpreted in terms of export platform. As we said, when distance increases, some firms stop building foreign affiliates abroad and start to undertake export as a foreign market access strategy. This export activity would be much cheaper if it take place between the last foreign country where it has been built the foreign affiliate and the new destination country. This latter case would imply an export platform strategy, where the foreign affiliate firm located in country $j$ sells in that foreign domestic market, and also in third markets $(j+1, j+2...j+(N-1)/2)$ through export.

2.4.3 Free Entry

It is possible to show the equilibrium which characterizes this economy. In order to do so, we need to specify some other equilibrium equations, namely the free entry condition and the price index.

Free entry ensures equality between the expected operating profits of a potential entrant and the entry cost, $E(\pi) - f_I$. This condition holds for all type of firms. The cumulative density function is $G(a)$, with support: $[0, ... , a_0]$, where for simplicity we can set $a_0 = 1$. The free entry condition can be defined as:

$$\int_{0}^{a_{I}} \pi_D dG(a) + 2 \sum_{d=1}^{N-1} \left\{ \int_{a_{X,d}}^{a_{X,d}} \pi_{X,d} dG(a) + \int_{0}^{a_{M,d}} \pi_{M,d} dG(a) \right\} = f_I$$

Using the profit conditions (7)-(9), we obtain:
\[
\int_{0}^{a_{P}} \left[ \frac{1}{(\sigma - 1)} \beta E a^{1-\sigma} \right] dG(a) + 2 \sum_{d=1}^{N-1} \int_{a_{M,d}}^{a_{X,d}} \left[ \frac{1}{(\sigma - 1)} \phi \gamma a^{1-\sigma} \beta E \right] dG(a) + f_d \]

where \( \phi = \tau^{1-\sigma} \) is freeness of trade, \( \gamma = d^{1-\sigma} \) and \( d \) is the parameter that takes into consideration the different country locations; finally \( P^{1-\sigma} \) is a weighted average of the marginal costs of all firms active in the market. Let’s spend some more words on this term, \( P^{1-\sigma} \).

In every country this weighted average, \( P^{1-\sigma} \), is characterized by all the brands offered in that particular country. Brands offered by domestic firms, for which the consumer price is \( a \sigma / (\sigma - 1) \); brands offered by foreign exporters, for which the consumer price is \( a \sigma d \tau / (\sigma - 1) \); and finally, brands supplied by foreign subsidiaries, with consumer price \( a \sigma (\tau d) \gamma / (\sigma - 1) \). Therefore:

\[
P^{1-\sigma} = \left( \frac{\sigma}{\sigma - 1} \right)^{(1-\sigma)} \int_{0}^{a_{P}} a^{1-\sigma} dG(a/a_D) + \\
\left( \frac{\sigma}{\sigma - 1} \right)^{(1-\sigma)} 2n \sum_{d=1}^{N-1} \left[ \int_{a_{M,d}}^{a_{X,d}} [\phi \gamma]^{\gamma} a^{1-\sigma} dG(a/a_D) + \int_{a_{M,d}}^{a_{X,d}} \phi \gamma a^{1-\sigma} dG(a/a_D) \right] \]

Notice from expressions (13) and (14) that the common term \( \left( \frac{\sigma}{\sigma - 1} \right)^{(1-\sigma)} \) will disappear.

### 2.4.4 Parametrization: Pareto Distribution

The fact that the free entry condition and the price index depend on the probability distribution implies that in order to have explicit solutions for this model, we need to assume a particular functional form for \( G(a) \). Following the empirical literature on firms size distribution (see Axtell 2001; Helpman et al. 2004), we use as an approximation, the Pareto distribution. The cumulative distribution function of a Pareto random variable \( a \) is:

\[
G(a) = \left( \frac{a}{a_0} \right)^k
\]

where \( k \) and \( a_0 \) are the shape and scale parameter, respectively. Note that \( k=1 \) implies a uniform distribution on \([0, a_0]\). The shape parameter \( k \) represents the dispersion of cost draws. An increase
in k would imply a reduction in the dispersion of firm productivity-draws. Hence, the higher is k the smaller is the amount of heterogeneity.

The support of the distribution \([0, ..., a_0]\), is identical for every country, where \(a_0\) represents the upper bound of this distribution. The productivity distribution of surviving firms will also be Pareto with shape \(k\). More precisely, since a firm will start producing only if it has at least a productivity of \(1/a_D\), the probability distribution of supplying as an exporter, or as a foreign affiliate, is conditioned on the probability of successful entry in each market. Hence the truncated cost distribution is given by:

\[
G(a/a_D) = \left( \frac{a}{a_D} \right)^k
\]

where it is exploited the fractal nature of the Pareto. Here the support is \([0, ..., a_D]\). Given the assumed parametrization, we can explicitly solve \(P^{1-\sigma}\) and the free entry.

**Price Index using the Pareto distribution**

As we said, firms will offer a price only if they have at least a productivity of \(1/a_D\). Hence, the cumulative distribution is defined on a support \([0, ..., a_D]\). Solving for the price index we will obtain:

\[
P^{1-\sigma} = \frac{n}{1 - b} a_D^{1-\sigma} \left[ 1 + 2T^{1-b} \sum_{d=1}^{N-1} (\phi\gamma_d)^b + 2 \sum_{d=1}^{N-1} V_d^{1-b} [(\phi\gamma_d)^\eta - \phi\gamma_d]^b \right] (16)
\]

where \(b = \frac{k - 1}{\sigma - 1}\); \(\phi = \tau^{1-\sigma}\); \(\gamma = (d)^{1-\sigma}\); \(T = f_X/f_D\) and \(V_d = (f(d) + f_M - f_X)/f_D\). In order for the integral to converge we assume that \(b > 1\).

**Free entry condition using the Pareto distribution:**

Rewriting the free entry using the Pareto distribution we obtain:

\[
\frac{E}{\sigma P^{1-\sigma}} \left[ \int_0^{a_D} (a^{1-\sigma} - f_D)dG(a) + 2 \sum_{d=1}^{N-1} \int_0^{a_M,a} (a^{1-\sigma}(\phi\gamma)^\eta - (f(d) + f_M)dG(a) + 2 \sum_{d=1}^{N-1} \int_{a_M,d}^{a_X,d} (a^{1-\sigma}(\phi\gamma) - f_X)dG(a) \right] = f_I
\]

**3 General Equilibrium with \(N\) countries**

In order to analyze the main implications of our model, we exploit the fact that all fixed coefficients are the same in every country and that the distribution function is the same. However, the existence of \(N\) countries located evenly around a circular trade route introduces a role for distance
in generating heterogeneity in the supply mode decisions within the same firm. Using the expression for the weighted average of the marginal costs in (16) inside the domestic cutoff condition (10), we find the equilibrium number of varieties (and so the number of surviving firms) consumed in a typical nation:

\[
 n^* = \frac{(b - 1)\beta E}{\sigma bf_D[1 + 2T^{1-b}\sum_{d=1}^{N-1} (\phi\gamma)^b + 2\sum_{d=1}^{N-1} V_d^{1-b}[(\phi\gamma)^n - \phi\gamma_d]^b]}
\]

We define \( \Omega = 2T^{1-b}\sum_{d=1}^{N-1} (\phi\gamma)^b \), and, on the other hand, \( \Psi = 2\sum_{d=1}^{N-1} V^{1-b}[(\phi\gamma)^n - \phi\gamma]^b \). Where \( \Psi \) and \( \Omega \) could be consider as parameters that summarize the impact of the two types of trade barriers on exports and FDI activities. In particular, \( \Omega \) represents the combined effect of higher fixed costs and variable distance costs on export strategy. While \( \Psi \) measures the role of the difference in these costs between a FDI strategy and an export strategy. Using \( \Psi \) and \( \Omega \), the expression for \( n^* \) could be then simplified to:

\[
 n^* = \frac{(b - 1)\beta E}{\sigma bf_D[1 + \Omega + \Psi]}
\]

The equilibrium number of firms described by (18) represents the actual number of survivors in each country, which decreases with \( \Psi \) and \( \Omega \), hence it decreases with higher fixed and variable distance costs. Using the free entry condition in (17), and the cutoff conditions in (10)-(12), we get explicit closed form solutions for \( a_D, a_{X,d} \), and \( a_{M,d} \). In particular,

\[
a_D^* = a_0 \left[ \frac{(b - 1)f_I}{(f_D(1 + \Psi + \Omega))} \right]^{\frac{1}{\xi}}
\]

Using (21) inside the ratio between (11) and (10) we find:

\[
a_{X,d}^* = a_0 \left[ \frac{(b - 1)f_I}{f_X(1 + \Psi + \Omega)} (\phi\gamma)^b T^{1-b} \right]^{\frac{1}{\xi}}
\]

Finally, using (21) inside the ratio between (12) and (10) we obtain the equilibrium cutoff for the M-mode is:

\[
a_{M,d}^* = a_0 \left[ \frac{(b - 1)f_I}{(1 + \Psi + \Omega)} [(\phi\gamma)^n - (\phi\gamma)]^b \frac{V_d^{1-b}}{f(d) + f_M - f_X} \right]^{\frac{1}{\xi}}
\]

The index \( d \) inside these expressions is related to the geographical distance between the origin and a specific destination country.

Differently to Helpman et al. (2004), these cutoffs change in relation to the geographical
location of the destination country. In fact, equations (19)-(21) change in relation to how many
countries belong to this trade bloc and more importantly, (20) and (21) change with respect to the
destination country we consider to reach. Since countries are evenly spaced along the circular trade
route, the above equations are the same for whatever country we pick to be the origin country.

**Remark 4** The existence of different country’s locations generates distance-dependent cutoffs.
Hence, the range of firms choosing the M-mode is shrinking the more distant is the destina-
tion country to reach. Therefore, some firms that are supplying the foreign country, \( j \), via local
sales of foreign affiliates (M-mode), could be forced to supply country \( j + 1 \) via X-mode, where
\( d(j + 1) > d(j) \).

### 3.1 The Impact of Trade

In the present framework, with \( N \) symmetric countries, we observe the effect of opening to trade.
Since \( f_I \) does not change in the transition from autarky to trade, the free entry conditions are
left unaffected by trade: regardless of profit differences across firms (relative to X-mode or M-
mode), the expected value of future profits, in equilibrium, must equal the fixed investment cost
\( f_I \). Hence, as in Melitz (2002), the transition from autarky to open economy, will move up the zero
profit condition curve: the exposure to trade induces an increase in the cutoff productivity level
\( (1/a_D)^T > (1/a_D)^A \). This will modify the productivity level of the least productive firms. In
an open economy situation, a firm with a productivity level between \( 1/a_X \) and \( 1/a_M \) cannot
earn positive profits and so will exit from the market. Moreover, as pointed out by Melitz, another
selection process acts: firms with productivity level above \( 1/a_X \) or above \( 1/a_M \) enter respectively
as exporters or as subsidiary. These three effects are called *domestic market selection effect, export
market selection effect* and *FDI market selection effect*. These effects reallocate market shares
towards more efficient firms, and generate an increase in the overall productivity.

The transition toward the open economy situation generates a reduction in the number of
surviving firms in every country\(^\text{14}\). The total number of firms selling in every country includes:
total number of domestics firms, foreign exporters and multinationals. The number of surviving
firms decreases as a consequence of the domestic market selection effect \( (a_D) \downarrow \). However, as the
entrance of new foreign firms more than compensate this reduction, consumers typically enjoy a
larger amount of varieties.

\(^\text{13}\)Recall that in Melitz (2002) the ZPC are downward sloping and the FE conditions are upward sloping. However,
since here we consider the marginal costs, the slope of these curves will be the opposite.

\(^\text{14}\)As in Melitz, \( n < n_A \), where \( n_A \) represents the number of firms in autarky.
3.2 The Effects of Increase in Distance and Trade Liberalization

We now consider comparative statics with respect to $d$, $N$ and $\phi$: we study how these elements affect the equilibrium marginal costs. We have analytical solutions, but the analysis is facilitated by graphing the changes with respect to $d$, $N$ and $\phi$.

**Increase in N and in Distance** Since the domestic cutoff does not depend on distance, we examine the effect of a change in the dimension of the trade area, $N$. Then we consider the effect of distance on the export cutoffs. These effects are unambiguous.

An increase in the overall dimension of the circle, $N$, implies a decrease in the domestic cutoff. In fact, the increase in the dimension of the economic area generates an increase in competition, and so in the expected profits. Moreover, the increase in distance between the origin and the destination country makes more expensive to reach the destination country through export. Therefore, the cutoff of the exporting firm is decreasing with distance. Hence, only the more productive firms can do export.

The effect of distance on MNF cutoff is more complex; as mentioned in remark 2, it depends
on the degree of openness:

If $\phi$ is sufficiently low, the MNF cutoff, $a_{M,d}$, is a monotonic decreasing function of distance, $d$. This implies that the increase in distance is making more difficult to choose M-mode in order to reach the foreign country. Differently, when $\phi$ is high, the MNF cutoff function has not a monotonous behavior: it increases for low level of distance and then it starts to decrease when distance becomes important. A possible interpretation of this result is that large value of $\phi$ lower the effects of distance on $a_{M,d}$.

**Increase in Trade Openness** We now consider how a progressive exposure to trade affects the supply mode decisions of firms, via the effect on the equilibrium cutoffs conditions. An increase in the exposure to trade, ($\phi \uparrow$ or $\tau \downarrow$), have similar effect to Melitz (2002), for what concern the domestic and export firms,

The domestic cutoff is decreasing as a consequence of the market selection effect, and the export cutoff is increasing as a consequence of the reduction in $\tau$. More complex results are obtain for
what concern the MNF cost cutoff:

For low level of distance, the MNF cost cutoff is not a monotonic function of $\phi$. It increases for low level of $\phi$, and then it decreases. Hence, when distance is not too high, an initial increase in trade openness makes easier to become a MNF. However, a further increase in the trade openness, makes the foreign market strategy too costly: the productivity required to supply via M-mode is now increasing with $\phi$. On the contrary, when distance is sufficiently high, the MNF cost cutoff is a monotonic increasing function of phi. This last result confirms the classical MNF theory.

**Number of Firms** Also the equilibrium number of firms does not depend on distance; therefore we examine the effect of a change in the dimension of the trade area, $N$ and in trade openness, $\phi$, on the number of active firms:

As a consequence of the increase in competition ($N \uparrow$), the number of active firms is decreasing with $N$. In the same way, a further exposure to trade reduces the number of active firms, as a consequence of the increase in domestic productivity cutoff (market selection effect).
3.3 Welfare Effects of Trade Liberalization

From the indirect utility function we can examine the welfare of consumers. Since the indirect utility function\(^{15}\) is given by \(V = \beta E/P\), where \(P\) is the standard CES price index, we can examine the welfare effects simply by examining how \(P\) is changing. A greater openness will increase the welfare by lowering the price index\(^{16}\), as well as a decrease in the domestic cutoff. Instead, a higher distance increases the price index, lowering welfare.

4 Distance and Aggregate Sales

In the following part we will use the analysis presented in Kleinert and Toubal (2006) in order to consider the role of distance on volume of sales. Following the standard scale versus proximity hypothesis, horizontal FDI is chosen when firms prefer to produce abroad through foreign affiliates so to save on trade costs. Hence, in the classical FDI framework we observe foreign affiliates’ sales increase with trade costs. However, empirical findings based on aggregate data (Buch et al. 2005; Carr et al. 2001) find a negative relationship between affiliate sales and distance\(^{17}\). In order to fill this gap, we model a FDI activity which involves a local production of one intermediate good, \(y_2\), and the imports of another intermediate good, \(y_1\); so that it is possible to analyze how affiliate sales could be affected by distance. First of all, let’s define the aggregate affiliate sales in the case of two countries:

\[
S_A = \int_0^{a_{M,d}} A a^{1-\sigma} \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (\phi \gamma)^g (a) \, da \\
= \left( \frac{a_{M,d}}{a_D} \right)^k a_{M,d}^{1-\sigma} \frac{k}{k - \sigma + 1} A \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (\phi \gamma)^g 
\]

The first term, \((a_{M,d}/a_D)^k\), represents the cumulative probability of firms in the origin country to own an affiliate in the destination country. As we briefly mentioned in the previous section, if we multiply this term with the total mass of active firms from the origin country, \(n^*G(a_{M,d}/a_D)\), we obtain the number of affiliates in the destination country. The remaining part of that expression represents average sales. Since we are dealing with \(N\) symmetric countries, and so with \(N-1\) possible

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\(^{15}\)Without loss of generality, in this welfare analysis we are only concerned about the differentiated good.

\(^{16}\)It can nevertheless happen that when trade costs are high and the number of foreign activities is strictly greater than the number of domestic firms, the effect of product varieties on welfare is negative (Melitz, 2002).

\(^{17}\)In these empirical works, distance is used as a proxy for transportation costs.
partners, the overall aggregate affiliate sales are:

\[ S_A = 2 \sum_{d=1}^{N-1} \left( \frac{a_{M,d}}{a_D} \right)^k \left( \frac{a_{1-\sigma}^{M,d}}{k-\sigma + 1} \right) \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (\phi \gamma)^\eta \]  

(23)

Without loss of generality, in the exercise of comparative static, we consider only two countries; the analysis could be extended to N-1 countries, but the conclusions will not change. In observing the change in affiliate sales as a consequence of a change in \( d \), we should keep in mind that the expression above can be rewritten:

\[ S_A = \left( \frac{a_{M,d}}{a_D} \right)^k a_{1-\sigma}^{M,d} \frac{k}{k-\sigma + 1} A \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (\phi d^{1-\sigma})^\eta \]  

(24)

Deriving the expression above with respect to \( d \), we realize that the way distance affects affiliate sales is double:

\[ \frac{\partial S_A}{\partial d} = \frac{\partial S_A}{\partial a_{M,d}} + \frac{\partial S_A}{\partial a_M} \frac{\partial a_{M,d}}{\partial d} \]  

(25)

In the above expression we can see that distance affects \( S_A \) through a direct and an indirect effect. For what concern the direct effect, we see from (24) that there is a negative relationship between \( S_A \) and \( d \). On the contrary, we have more ambiguous results for what concern the indirect effect. In particular the ambiguity is linked to the sign of the last partial derivative, \( \partial a_{M,d}/\partial d \). Equation (24) shows us that \( a_{M,d} \) is positively related to the number of affiliates producing in foreign countries, \( n^x \left( \frac{a_{M,d}}{a_D} \right)^k \), while it is negatively related with the average size of foreign affiliates. Differentiating (24) with respect to \( a_M \) we get:

\[ \frac{\partial S_A}{\partial a_{M,d}} = \left( \frac{a_{M,d}}{a_D} \right)^k a_{1-\sigma}^{M,d} k A \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (\phi \gamma)^\eta > 0 \]

The threshold marginal cost of being a MNF, \( a_{M,d} \), is positively related to aggregate affiliate sales. Let’s now analyze the other partial derivative in the second order effect of (25), i.e. the effect of distance on productivity. What is the sign of \( \partial a_{M,d}/\partial d \) ? Firstly, we use the net operating profits condition of being a MNF with respect to an exporter. Hence,

\[ \pi^*_M(a, A, \eta) = A a_{1-\sigma}^{M,d} A \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} [((\phi \gamma)^\eta - (\phi \gamma)) - f(d) - f_M + f_N] \]
where any particular functional form for \( f(d) \) is assumed. Solving this expression by the threshold marginal cost, \( a_{M,d} \), we get:

\[
a_{M,d} = \left( \frac{f(d) + f_M - f_X}{B \left[ (\phi d^{1-\sigma})^\eta - (\phi d^{1-\sigma}) \right]} \right)^{\frac{1}{1-\sigma}}
\]

where \( B \) was defined before. It is now possible to derive this expression with respect to \( d \). We find the following result:

\[
\frac{\partial a_{M,d}}{\partial d} = \left[ \frac{1}{1 - \sigma} \left( \frac{f(d) + f_M - f_X}{B \left[ (\phi d^{1-\sigma})^\eta - (\phi d^{1-\sigma}) \right]} \right)^{\frac{1}{1-\sigma}-1} \right] \times
\]

\[
\times \left[ \frac{f'(d)}{B \left[ (\phi d^{1-\sigma})^\eta - (\phi d^{1-\sigma}) \right]} - \frac{(f(d) + f_M - f_X) (1 - \sigma) \left[ \eta \frac{(\phi d^{1-\sigma})^\eta}{d} - \phi d^{-\sigma} \right]}{B \left[ (\phi d^{1-\sigma})^\eta - (\phi d^{1-\sigma}) \right]^2} \right] \leq 0 \quad (26)
\]

The sign of the first term is straightforward: since \( \sigma > 1 \), the sign is negative; instead, the second term is more difficult to interpret. In fact, the sign of this second term depends on the behavior of the following term:

\[
(1 - \sigma) \left[ \eta \frac{(\phi d^{1-\sigma})^\eta}{d} - \phi d^{-\sigma} \right] \quad (27)
\]

More precisely, the sign of this term is related to the degree of trade openness\(^{18}\). For low level of trade openness, (27) is always negative. If it is so, the sign in the second bracket in (26) is positive\(^{19}\); thus the overall sign of (25) is negative: the affiliate sales are unambiguously decreasing with distance for low level of trade openness. When \( \phi \) starts to increases the sign of (25) is not so straightforward anymore, since the partial derivative \( \partial a_{M,d}/\partial d \) could be positive. For sufficiently high trade openness and sufficiently small distance, the sign of (27) will be positive; hence the overall sign of (25) depends on the magnitude of its first and second term. However, when distance is not so small, the sign of (25) turns again to negative, since (26) is now negative.

It seems interesting to notice some peculiarities linked to higher level of trade openness. The change in the sign of (26) depends on the degree of trade openness. For example, when \( \phi \) is very high, the change in the sign of (26) occurs at a higher level of distance than for a lower \( \phi \). Therefore, since overall aggregate sales are positively related to the threshold marginal cost, we conclude that

\(^{18}\)See Appendix 2 for a graphical representation.

\(^{19}\)With respect to the second bracket, since the communication costs are increasing with distance and the denominator is smaller in the second ratio, the second term is bigger than the first.
when trade barriers are sufficiently high, aggregate sales are decreasing in distance: the overall effect of distance on $S_A$ is negative. However, the magnitude of this reduction is strictly linked to the level of trade openness. Hence, when distance is sufficiently high, we expect more firms choosing to supply via X-mode. The size of the reduction in affiliates’ sales and so of the increase in export strategies, becomes bigger the more distant is the affiliate’s locations. This results is in line with recent empirical findings.

**Remark 5** The reduction in affiliate sales due to increase distance is more relevant the less open is trade.

It could be interesting to turn our attention to the role of distance on export activity, so as to compare the effect of distance on M-mode supply versus X-mode supply. Since export sales are affected by a combination of $a_{M,d}$ and $a_{X,d}$, we expect a complex relationship between $S_X$ and distance. In order to put in evidence the effect of distance, we first define the aggregate export sales in the case of two countries:

$$S_X = \int_{a_{M,d}}^{a_{X,d}} A(\phi \gamma) a^{1-\sigma} \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma}$$  

$$= A(\phi \gamma^{1-\sigma}) \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} k \frac{k^{k-\sigma+1} - a_{M,d}^{k-\sigma+1}}{a_{D}^{k}}$$  

The effect of distance on the aggregate exports is a combination of different effects,

$$\frac{\partial S_X}{\partial d} = \frac{\partial S_X}{\partial a_{M,d}} \frac{\partial a_{M,d}}{\partial d} + \frac{\partial S_X}{\partial a_{X,d}} \frac{\partial a_{X,d}}{\partial d}$$  

In what follows we confirm the sign of the above partial derivatives. Deriving $S_X$ with respect to $a_{X,d}$ and $a_{M,d}$ we find that:

$$\frac{\partial S_X}{\partial a_{M,d}} = - \frac{(\phi \gamma)}{a_{D}^{k-\sigma+1}} \frac{1}{a_{M,d}} A \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} < 0$$  

and

$$\frac{\partial S_X}{\partial a_{X,d}} = \frac{(\phi \gamma)}{a_{D}^{k-\sigma+1}} \frac{1}{a_{X,d}} A \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} > 0$$  

As expected, export sales are increasing in the threshold marginal cost of being an exporter, $a_{X,d}$, and decreasing in $a_{M,d}$. In order to analyze the relationship between aggregate export sales and
distance, we will use the net operating profits, so to derive the effect of distance on both the threshold \(a_{M,d}\) and \(a_{X,d}\). We already know from the analysis above that the sign of \(\partial a_{M,d}/\partial d\) could be ambiguous: it depends on the degree of trade openness. Thus, as long as distance is not too small the cutoff marginal cost \(a_{M,d}\) is negatively affected by distance, hence \(\partial a_{M,d}/\partial d < 0\). On the other side for what concern \(a_{X,d}\)

\[
a_{X,d} = \left( \frac{f_X}{B(\phi d^{1-\sigma})} \right)^{\frac{1}{1-\sigma}}
\]

hence

\[
\frac{\partial a_{X,d}}{\partial d} = -\left( \frac{f_X}{B(\phi d^{1-\sigma})} \right)^{\frac{1}{1-\sigma}} \frac{1}{d} < 0
\]

the effect of distance, \(d\), on the cutoff marginal cost \(a_{X,d}\) is unambiguously negative. What could be concluded? Similarly to what we found for affiliate sales, the amount of aggregate exports does not linearly depend on distance. The effect of distance on \(S_X\) depends on the magnitude of the partial derivatives in (29). Nevertheless, when \(\partial a_{M,d}/\partial d > 0\), since the sign of (31) will be positive, the overall sign of (29) will be negative. The economic intuition behind this result is that, for high trade openness and low distance, the result of a slight increase in distance between the destination and the origin countries, it causes a decrease in the amount of export sales. Hence, when distance does not play an important role and trade is sufficiently open, the result obtained could be considered as a confirmation of the scale versus proximity hypothesis. On the contrary, when distance becomes important we cannot have a precise conclusion, because the second term in (29) will be positive, since \(\partial a_{M,d}/\partial d < 0\). For high level of distance, if the first and the third terms in (31) are smaller than the second term, the overall sign of that expression will be positive; meaning that when distance plays an important role the export sales are increasing with distance.

To conclude:

**Remark 6** *Under certain circumstances, namely sufficient trade openness, high distance and important reaction of \(S_X\) to changes in \(a_{M,d}\), export sales are increasing with distance.*

### 5 Conclusion

The paper analyses the choice between different supply modes in a framework where the existence of intermediate goods makes MNF activities affected by trade and communication costs. Some asymmetries between countries, in terms of different country locations, are assumed. The production of the final good variety is assumed to depend on a particular intermediate good combination.

\[\text{\textsuperscript{20}}\text{When trade is sufficiently open and distance sufficiently low, the sign of }\partial a_{M}/\partial d\text{ will be positive}\]
This assumption makes the total marginal costs of selling via FDI rising with distance. Moreover, we considered FDI activities as characterized by fixed costs which raises with distance. The existence of intermediate goods and communication costs leads to a result in which the choice of engaging in FDI activities is strictly affected by distance. In particular, in line with recent empirical works, we are able to show a richer pattern of modes of supply the foreign markets.

We found that distance modify the equilibrium cutoffs. In particular, we highlight a process through which for increasing level of distance the same firm can supply via M-mode some markets and via X-mode others. The magnitude of this process increases with distance. This work put in evidence that under certain conditions MNF activities will be nearby concentrated; whereas export activities become more convenient far away. This result relies on the role of the communication costs and the intermediate goods. We also found the conditions which make the aggregate affiliate sales decreasing with distance, and the amount of exports increasing with distance. This result is also consistent with the recent empirical findings.

The existence of "adequate skills" in the host country plus the availability of sufficiently developed communication technologies, would have changed the situation. In that case, MNF firms could find convenient outsourcing parts of the production process to foreign countries. The presence of "adequate skills" would allow a more efficient (time-saving) transmission of knowledge across countries, permitting to the MNFs to avoid communication and trade costs. Nevertheless, at least in this paper, our interest is limited to the case in which a special case of horizontal FDI is undertaken. A possible extension of this work could be to introduce some elements of contract theory in the contest of the offshoring relationship. So that it could be possible to analyze this framework from a different perspective. For example, we could ask what is the profit maximizing way for firms to organize their activities? Do firms prefer internally produce the intermediate goods or to outsource to some local supplier? This idea will be developed in another paper.
References


6 Appendix

A1 Cost Minimization Problem  In order to find the equilibrium operating profits, we solve the minimization problem of the firm. For example, the cost minimization problem for foreign affiliates:

\[
\min_{y_1, y_2, \lambda} L = y_1 d\tau + y_2 + \lambda \left[ x(v) - \frac{1}{a} \left( \frac{y_1}{\eta} \right)^\eta \left( \frac{y_2}{1 - \eta} \right)^{1-\eta} \right]
\]

where the Lagrangian multiplier \( \lambda \) represents the marginal cost of production. The Hicksian factor demands are:

\[
y_1^* = x(v) a \eta \left( \frac{1}{d\tau} \right) \]  \( \eta \)
\[
y_2^* = x(v) a (1 - \eta) \left( d\tau \right)^\eta
\]

Using the Hicksian demands, we can write the total cost of a subsidiary as a function of the final output, \( x(v) \):

\[
TC_{M,j} = y_1^* d\tau + y_2^* + f(d) + f_M
\]

Using (33) inside (6) it is possible to derive an expression for the multinational equilibrium profits, which depends only on the final output \( x(v) \):

\[
\pi_{M,d}(a, A, \eta) = A^i \frac{1}{\sigma} x(v)^{\frac{\sigma - 1}{\sigma}} - x(v) a \left( d\tau \right)^\eta - f(d) - f_M
\]

Equations (34) and (35) refer to this specific multinational framework; the problem above can be solved for each different type of firm. More generically, the final good producer will choose the supply mode that maximizes \( \pi_k^*(a, A, \eta) \) where \( k = M, X \) or \( D \). For this reason, final good producers organize the production so as to minimize both variable and fixed costs.
Appendix 2  Distance and Aggregate sales. The sign of $f = (1 - \sigma) \left[ \eta \left( \frac{\phi d^{1-\sigma}}{d} \right)^{\eta} - \phi d^{-\sigma} \right]$ changes in relation to the degree of trade openness. In particular, when $\phi = 0.9$

![Graph 1](image1.png)

while when $\phi = 0.2$, its behavior is the following:

![Graph 2](image2.png)