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April 10, 2006

Abstract

Morris and Shin (2002) have shown that a central bank may be too transparent if the private sector pays too much attention to its possible imprecise signals simply because they are common knowledge. In their model, the central bank faces a binary choice: to reveal or not to reveal its information. This paper extends their model to the more realistic case where the central bank must anyway convey some information by setting the interest rate. This situation radically changes the conclusions. In many cases, full transparency is socially optimal. In other instances the central bank can distill information to either manipulate private sector expectations in a way that reduces the common knowledge effect or to reduce the unavoidable information content of the interest rate. In no circumstance is the option of only setting the interest rate socially optimal.

1 Introduction

In a series of papers Amato, Morris and Shin (2002, 2003) and Morris and Shin (2002, 2005) have argued that central banks should not reveal all the information at their disposal.[1] This is result stands in contrast with another view, developed by Blinder (1998), that central banks should be as transparent as possible. The case for transparency rests on the need for the central

[1] These papers share a common model and differ only by their emphasis on particular issues. In what follows we refer to all of them when we quote Morris and Shin (2002), denoted as M&S.

bank to affect expectation-driven channels of transmission such as long-term interest rates, asset prices and exchange rates. In this view, monetary policy effectiveness is enhanced when the private sector understands the intentions of the central bank. The same conclusion can be derived when considering expectations-augmented Phillips curves; the inflation-output trade-off is reduced when the central bank is credible and credibility is best served by transparency.

M&S are not the first ones to argue for less than full transparency. The case for "constructive ambiguity" has been initially advanced by Cukierman and Meltzer (1986). It rests on two assumptions: 1) only unanticipated money matters (Kydland and Prescott, 1977) and 2) the central bank preferences are not precisely known by the public (Vickers, 1986). Under these combined assumptions, some degree of opacity enhances monetary policy effectiveness because a fully transparent central bank cannot create surprises.[2] These assumptions have become less appealing. The 'only unanticipated money matter' view has been convincingly criticized by McCallum (1995) and Blinder (1998). The idea that the central bank conceals its preferences to pursue its own agenda may have been realistic, but it does not sit well with the tendency to clearly specify the objectives of monetary policy, as is the case with the increasingly popular inflation targeting strategy. In addition central bank secrecy raises a huge range of issues about accountability when monetary policy is delegated to independent central banks.

The striking feature of the results of M&S is that they do not require any of these assumptions. The argument rests instead on three different assumptions: 1) the information available to both the central bank and the private sector is noisy; 2) the central bank's signals are seen by everyone in the private sector; and 3) private sector agent forecasts must not just be as precise as possible, they must be as close as possible to the consensus forecast. The last assumption, which goes back to Keynes' celebrated beauty contest effect, reflects the basic principle that it is relative prices that matter in competitive markets. An implication of the beauty contest assumption is that everyone knows that everyone else observes the same central bank signals. As a consequence is the common knowledge effect: relative to private information, central bank signals receive undue attention in the sense that their impact will not just reflect their quality. It follows that when the quality of central bank signals is not good enough to overcome the weight that they receive, it may be desirable for the central bank to withhold releasing its information. Svensson (2005) observes that, in practice, the quality of central bank signals is unlikely to be sufficiently poor to justify withholding

[2] For review of this literature, see Geraats (2002).

information.

We accept the three basic assumptions because they are obviously realistic, but we question a fourth assumption made by M&S. In their model the central bank faces the choice of releasing either no information at all or all its information. It is unrealistic that a central bank can withhold *all* of its information. If only by setting the interest rate, the central bank is forced to reveal *some* information. The question then is not black-and-white - reveal or not reveal - and this changes the question to be asked and, it turns out, the answers.

The question now becomes: given that the central bank anyway releases some noisy information, the interest rate, should it also reveal all the detailed information that was used in its decision? The answer is that it is generally desirable for the central bank to release some information in addition to setting the interest rate. We find many cases where, in fact, it is desirable for the central bank to be fully transparent and to release all its information. This result emerges even in the presence of the common knowledge effect implied by the three assumptions previously mentioned.

The next section presents our model, an extension of M&S to the case where there are two price fundamentals. This extension is needed as we assume throughout that the central bank optimally sets and publishes the interest rate. Section 3 considers the case where the variances of the central bank and private sector signals are perfectly known to both the central bank and the private sector. Depending on the strength of the common knowledge effect, it may or may not be desirable for the central bank to be transparent. Section 4 introduces one more degree of possible transparency by allowing for the precision of the private sector signals to be unknown to the central bank. Since the central bank must set the interest rate, and is assumed to do so by minimizing its expectation of the private sector loss function, it must form a forecast of the private sector information precision. In this situation, the central bank may simply publish its chosen interest rate, or release its forecast of the private sector information precision, or be fully transparent and also release its signals. This section presents the outcome in the three cases and evaluates the welfare implications. Most of these results remain valid when private sector information precision is also unknown to the the private sector itself, as is briefly indicated in Section 5. The last section concludes.

2 Central Banking with an Interest Rate Instrument

2.1 Monetary policy

Since we assume that the central bank publishes a signal by setting the interest rate, we need to extend the model of M&S to allow for more than one fundamental. If there were only one fundamental, the interest rate decision would be fully revealing, unless the central bank were intentionally introducing noise in its monetary policy decisions in order to manipulate private sector expectations, for example by creating surprises or by concealing its true preferences. We do not deal with these issues here as we assume that the central bank maximizes the social welfare, i.e. that its preferences are the same as those of the private sector and thus perfectly well known. To keep matters as simple as possible, we assume that there are two fundamentals that both the central bank and the private sector must forecast.[3]

As M&S, we consider the Lucas-Phelps island parable. There is one good produced by an infinite number of private producers i spread over the unit interval. Each private producer lives on one island and has limited information on the situation in the archipelago. Producer i's supply function is:

$$q_S^i = b(p^i - E^i(\overline{p}))$$

where E^i is the expectation conditional on information available in island i and $\overline{p}$ is the aggregate price index $\int_{i=0}^{1} p^i di$. The demand in island i is:

$$q_D^i = E^i(A\theta) - p_i$$

where $\theta = (\theta_1, \theta_2)'$ is a vector of two fundamentals and $A = (A_1, A_2)$ is a vector of structural parameters. Without any loss of generality, we set $(A_1, A_2) = (1, 1)$. This merely implies normalizing the two fundamentals accordingly. The market equilibrium condition yields:

$$p^i = (1 - r)E^i(\theta) + rE^i(\overline{p}) \tag{1}$$

with $r = b/(1 + b) \in [0, 1]$.[4]

[3] The model can be generalized to more than two fundamentals. It does not seem that this generalization would affect our results.

[4] There are other possible interpretations of 1. For example,Woodford (2003) derives a similar price setting for monopolistically competive firms.

M&S interpret their single fundamental as the money supply. Yet, in their framework the money supply is not controlled by the central bank whose sole function if to forecast this variable, possibly with less precision than the private sector. This characterization of monetary policy is unsatisfactory and, as it turns out, deeply affects the issue of transparency. If the central bank were to directly set the money supply, there would be no issue of transparency. Presumably, the central bank sets the interest rate, which in turn affects the money supply but with significant noise. In that case the private sector must observe the interest rate and both the central bank and the private sector infer from this observation the likely level of the money supply. In that case, the choice of the interest rate must be related to the central bank's forecast of the fundamental.

Short of fully specifying how monetary policy operates (channels of monetary policy effects, the instrument and the targets), we still need to recognize that the central bank uses one instrument to affect some intermediate target(s). To that effect, we depart from M&S in two ways. First, we allow for two fundamentals, which are meant to represent exogenous variables or intermediate targets that affect demand. Among the first category, one can think of foreign demand, fiscal policy or exogenous shocks. The intermediate targets can be the money supply, long-term interest rates, asset prices or the exchange rate. Second, we assume that the instrument is the short-term interest rate R, which is set by the central bank and observed by the private sector. In order to optimally set the interest rate, the central bank makes the best possible use of its information about the fundamentals. Denoting $\widetilde{\theta}_1$ and $\widetilde{\theta}_1$ the information that the central bank has regarding the two fundamentals, we assume that it uses a linear rule of the type:

$$R = \mu\widetilde{\theta}_1 + \nu\widetilde{\theta}_2 \tag{2}$$

Without loss of generality, we impose the restriction $\mu + \nu = 1$. This restriction amounts to normalizing the interest rate R. In accordance with private sector preferences, the central bank chooses the short-term interest rate that minimizes the social loss function:

$$L = E^{CB} \int \left(p^i - A\theta\right)^2 \tag{3}$$

based on its own information set. The central bank observes the fundamentals with additive noise

$$\widetilde{\theta}_k = \theta_k + \varepsilon_k \qquad k = 1, 2$$

where the noise terms ε_1 and ε_2 are identically and independently distributed with zero mean and variance V_ϵ. Obviously, in choosing μ and ν the central bank must take into account how the private sector sets its prices.

2.2 The Private sector

Each island producer i sets its price according to (1), using available privately information on the fundamentals. We assume that producer i receives a noisy signal for each in the fundamentals:

$$x_k^i = \theta_k + \eta_k^i \qquad k = 1, 2$$

where the noise terms η_k^i are independently distributed with zero mean and variance $V_{\eta,k}$. While the signals on each of the two fundamentals are identically distributed across all producers, their perceived variance may differ across the fundamentals (we may have $V_{\eta,1} \neq V_{\eta,2}$) and the variance perceived by the producers may differ from the variance perceived by the central bank (i.e. we may have $V_{\eta,k} \neq V_\varepsilon$). The perceived variances reflect the quality of information, which plays a central role in the analysis. Assuming $V(\varepsilon_1) = V(\varepsilon_2)$ and $V(\eta_1) \neq V(\eta_2)$ is the simplest way of introducing two meaningfully distinct fundamentals. Imposing $V(\varepsilon_1) = V(\varepsilon_2)$ does not restrict the generality of the model, it merely implies a normalization of $\tilde{\theta}_1$ and $\tilde{\theta}_2$. If, in addition we were to allow for $V(\eta_1) = V(\eta_2)$, we would have a symmetric case where the two signals are seen by both the central bank and the private sector drawn from the same distribution but with different accuracy. The resulting asymmetry of treatment of the private sector and the central bank is meant to reflect the notion that the former is heterogenous.

The beauty contest parameter r reflects the reactivity producers to each other's expectations. It implies that each producer must guess what all the other producers will guess, while they themselves guess everyone else's guess. Since each producer receives her own idiosyncratic information η_k^i, this is no trivial task. Iterating (1) infinitely, and denoting $\bar{E}^n$ the n^{th} order expectation, the optimal pricing decision is:

$$p^i = (1-r)\sum_{n=0}^{\infty} r^n E^i\left(\bar{E}^n(A\theta)\right) \tag{4}$$

which exists when $0 < r < 1$.

Thus each producer will set her own price in order to stay as close as possible to her estimate of the fundamental combination $A\theta$ that drives the aggregate price index $\bar{p}$. Her loss will be therefore $L_i = (p_i - A\theta)^2$, which justifies the central bank loss function (3) in this one-shot game. Since all producers are identical save for the white noise terms η_k^i that they receive individually, the central bank objective can be treated as $E^{CB}(p^i - A\theta)^2$.

We can now look for the equilibrium outcome, where the central bank optimally chooses μ and ν to set the short-term interest rate R which is then observed by the private sector. The central bank decision involves forecasting p^i, which requires guessing the private sector price setting behavior (4). The question is whether the central bank should, in addition, reveal its own information about the fundamentals $\tilde{\theta}_1$ and $\tilde{\theta}_2$? In order to deal with that question, we must specify carefully what is known by the central bank and by the private sector about the distribution of noises ε_k and η_k^i, for $k = 1, 2$. We assume that everyone knows that the means are zero but what about the variances V_ε and $V_{\eta,k}$? We consider three cases: when the variances are known to all, when private sector variances are only known by the private sector and when private sector variances are unknown to all.

3 The variances are common knowledge

We start with the simplest case, when the variances of the noise terms V_ε and $V_{\eta,k}$ are known by everyone, both the central bank and the private sector. We first consider that the central bank does not reveal its private information $\tilde{\theta}_1$ and $\tilde{\theta}_2$; it only announces the interest rate R, which the sector known is set optimally as a linear function of $\tilde{\theta}_1$ and $\tilde{\theta}_2$. We then ask whether welfare improves when the central bank releases its information $\tilde{\theta}_1$ and $\tilde{\theta}_2$.

It will prove helpful to simplify the notations as follows. Let $\delta = V_{\eta,1}/V_{\eta,2}$ be the relative variance of private sector signals on the two fundamentals and denote $V_\eta = V_{\eta,2}$ so that:

$$V_{\eta,1} = \delta V_{\eta,2} = \delta V_\eta$$

We define the precision of the central bank and of public sector as α and β, respectively:

$$\begin{aligned} \alpha &= \frac{1}{V_\varepsilon} \\ \beta &= \frac{1}{V_\eta} \end{aligned}$$

Finally, we denote $\lambda = \frac{\mu}{\nu}$. This ratio represents the central bank decision when it sets the interest rate according to (2) since we normalized the signals such that $\mu + \nu = 1$, λ . Note that $\delta = 1$, corresponds to the symmetric case where there is no discrepancy between the central bank and the private sector assessment of relative variances.

3.1 The central bank only reveals its interest rate

Assuming that the central bank sets the interest rate R according to (2), producer i forms her expectations of the two fundamentals using Bayes theorem:

$$\text{(5a)} \qquad E^i(\theta_1) = \gamma_1 \left(\frac{R - \nu x_2^i}{\mu} \right) + (1 - \gamma_1) x_1^i$$

$$\text{(5b)} \qquad E^i(\theta_2) = \gamma_2 \left(\frac{R - \mu x_1^i}{\nu} \right) + (1 - \gamma_2) x_2^i$$

with

$$\text{(6a)} \qquad \gamma_1 = \frac{\lambda^2 \delta \alpha}{(1 + \lambda^2 \delta)\alpha + \left(1 + \lambda^2\right) \beta}$$

$$\text{(6b)} \qquad \gamma_2 = \frac{\alpha}{(1 + \lambda^2 \delta)\alpha + \left(1 + \lambda^2\right) \beta}$$

Agent i's expectation of the combined fundamentals, which enters her pricing decision (1), is:

$$E_i(A\theta) = \frac{1+\lambda}{\lambda} \left(\gamma_1 + \lambda\gamma_2\right) R + \left[(1-\gamma_1) - \gamma_2\lambda\right] x_1^i + \left[(1-\gamma_2)\,\lambda - \gamma_1\right] \frac{x_2^i}{\lambda}$$

The Appendix shows how to iterate this relation and find agent's i pricing decision :

$$\text{(7)} \qquad p^i = \frac{1+\lambda}{\lambda} \left(\varphi_1 + \lambda\varphi_2\right) R + \left[(1-\varphi_1) - \varphi_2\lambda\right] x_1^i + \left[(1-\varphi_2)\,\lambda - \varphi_1\right] \frac{x_2^i}{\lambda}$$

where φ_1 and φ_2 are:

$$\text{(8)} \qquad \varphi_k = \frac{\gamma_k}{1 - r[1 - (\gamma_1 + \gamma_2)]}$$

Because the interest rate is common knowledge, it receives more weight than it should if based only on the precision of available information, as given by (6). This can be seen by considering the case where $r = 0$, when agents do not care about other agents' expectations. In that case $\varphi_k = \gamma_k$ and the weights on R and the private signals x_k^i in the pricing equation (7) are exactly those that correspond to Bayesian signal extraction as in (5). When $r > 0$ instead, $\varphi_k > \gamma_k$ and the weight on R in (7) increases with r while the weights on x_1^i and x_2^i decline.

The publication of the interest rate provides the central bank with a tool to mitigate the common knowledge problem. Knowing that the interest rate R excessively influences the private sector, like a benevolent flute-payer sure to be followed by the crowds, the central bank can use its power to steer private expectations in a socially desirable direction. Even though the interest rate does not have any macroeconomic effect, it can play a useful signaling role.[5] Formally, the central bank sets λ to minimize the loss function $E^{CB}(p_i - A\theta)^2$:

$$
\begin{aligned}
E^{CB}(p_i - A\theta)^2 &= L_1(\delta, \lambda) \\
&= \frac{\left(1+\lambda^2\right)(1+\delta\lambda)^2\alpha\beta + \delta[(1-\lambda)\alpha + (1-r)\left(1+\lambda^2\right)\beta]^2}{\beta[\left(1+\delta\lambda^2\right)\alpha + (1-r)\left(1+\lambda^2\right)\beta]^2} \\
&\quad + \frac{[\lambda(\lambda-1)\delta\alpha + (1-r)\left(1+\lambda^2\right)\beta]^2}{\beta[\left(1+\delta\lambda^2\right)\alpha + (1-r)\left(1+\lambda^2\right)\beta]^2}
\end{aligned}
\tag{9}
$$

The resulting loss is denoted $L_1(\delta) = Arg\min L_1(\delta, \lambda)$. The first order condition $\partial L_1(\delta, \lambda)/\partial\lambda = 0$ is a fourth-order equation that cannot be easily solved for an arbitrary value of δ. We use the observation that the problem at hand degenerates into a one-dimension fundamental when $\delta = 1$, in which case a solution is $\lambda = 1$ (we discuss another solution below). Accordingly we focus on a first-order Taylor expansion of the first-order condition around $\delta = 1$ and $\lambda = 1$:

$$
\lambda - 1 = \frac{(1-3r+2r^2) + \dfrac{\alpha}{\beta}}{(1-3r+2r^2) + (2-3r)\dfrac{\alpha}{\beta} + (\dfrac{\alpha}{\beta})^2}(\delta - 1)
\tag{10}
$$

In order to understand this result, consider two limit cases. First let $\dfrac{\alpha}{\beta} \to \infty$, which corresponds to the case where the central bank information is infinitely more precise than the private sector information. The result is

[5]Could the central bank eliminate the externality? This would require that $\varphi_k = \gamma_k$ $\forall r$. This occurs when $\gamma_1 + \gamma_2 = \dfrac{(1+\lambda^2\delta)\alpha}{(1+\lambda^2\delta)\alpha + (1+\lambda^2)\beta} = 1$, which is impossible unless $\beta = 0$, the limit case where the private sector signal precision is nil.

$\lambda \simeq 1$. Since relatively accurate central bank signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are of equal precision ($V(\varepsilon_1) = V(\varepsilon_2)$), it treats them equally. In the second polar case, $\dfrac{\alpha}{\beta} \to 0$, the private sector is infinitely better informed than the central bank. By choosing $\lambda \simeq \delta$, the central bank weighs the signals according to the relative precision of the private sector forecasts. For intermediate values of $\dfrac{\alpha}{\beta}$ the central bank chooses λ to optimally shape private sector forecasts of θ_1 and θ_2 given the known precision of the signals. In this sense the central bank manipulates private sector expectations to mitigate the common knowledge effects.

For $\lambda \simeq 1$ to be optimal (when $\delta \simeq 1$) the second order condition $1 - 2r + \dfrac{\alpha}{\beta} \geq 0$ must be satisfied.[6] This requires that the "beauty contest" weight r be not too large given the ratio of the central bank information precision α relative to the private sector precision β. When this is the case, private agents are paying attention to each other expectations but not excessively so. The maximum value of r declines when $\dfrac{\alpha}{\beta}$ becomes smaller since the combination of a high degree of reactivity and poor central information precision results in a very powerful and welfare-reducing beauty contest effect. When $1-2r+\dfrac{\alpha}{\beta} < 0$, the optimal choice is $\lambda = -1/\delta$ when δ is in the neighborhood of 1**.** This solution implies that μ and ν become infinitely large in absolute value.[7] We further discuss this case below.

3.2 The central bank reveals the interest rate and its signals

Transparency is achieved when the central bank fully reveals its information, i.e. $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$. In that case, the interest rate is not informative anymore. Producer i now receives two signals for each of the two fundamentals: her own signals x_k.and the central bank signals $\widetilde{\theta}_k$ for $k = 1, 2$. Optimal Bayesian signal extraction leads to:

$$
\begin{aligned}
&\text{(11a)} \qquad & E^i(\theta_1) &= \overline{\gamma}_1 \widetilde{\theta}_1 + (1 - \overline{\gamma}_1) x_1^i \\
&\text{(11b)} \qquad & E^i(\theta_2) &= \overline{\gamma}_2 \widetilde{\theta}_2 + (1 - \overline{\gamma}_2) x_2^i
\end{aligned}
$$

[6] When $\delta = 1$ we have: $\dfrac{\partial^2 L_1(1,\lambda)}{\partial \lambda^2} = \frac{1}{2}\alpha^2 \frac{(1-2r)\beta+\alpha}{((1-r)\beta+\alpha)^2}$

[7] $\mu = \dfrac{\lambda}{1+\lambda}$ and $\nu = \dfrac{1}{1+\lambda}$.

with :

(12a) $$\overline{\gamma}_1 = \frac{\delta\alpha}{\delta\alpha+\beta}$$

(12b) $$\overline{\gamma}_2 = \frac{\alpha}{\alpha+\beta}$$

Since individual noises are white noise, we have $\int \bar{E}(x_k^i)di = \theta_k$ and therefore:

$$\bar{E}(A\theta) = \overline{\gamma}_1\widetilde{\theta}_1 + \overline{\gamma}_2\widetilde{\theta}_2 + (1-\overline{\gamma}_1)\theta_1 + (1-\overline{\gamma}_2)\theta_2$$

Iterating this relation to compute $p^i = (1-r)\sum_{n=0}^{\infty} r^n E^i\left(\bar{E}^n(A\theta)\right)$ we have:

(13) $$p_i = \overline{\varphi}_1\tilde{\theta}_1 + \overline{\varphi}_2\tilde{\theta}_2 + (1-\overline{\varphi}_1)\, x_1^i + (1-\overline{\varphi}_2)\, x_2^i$$

where:

$$\overline{\varphi}_1 = (1-r)\,\overline{\gamma}_1 + r\frac{\alpha+(1-r)\beta\overline{\gamma}_1}{1+(1-r)\beta\overline{\gamma}_1} \text{ and } \overline{\varphi}_2 = (1-r)\,\overline{\gamma}_2 + r\frac{\alpha+(1-r)\beta\overline{\gamma}_2}{1+(1-r)\beta\overline{\gamma}_2}$$

This is exactly the same result as in M&S except that we allow for two signals with different variances for the private sector. We find the familiar result that, because $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$ are common knowledge, they receive excessive weight in (13). To see this, note that when $r = 0$, i.e. when individual producers have no incentive to guess what the others expect, $\overline{\varphi}_k = \overline{\gamma}_k$ for $k = 1, 2$ and the weights on $\widetilde{\theta}_k$ and x_k^i in (13) exactly reflect the precision of the respective signals as given in (12). When $r > 0$, $\overline{\varphi}_k > \overline{\gamma}_k$ and the weights on $\widetilde{\theta}_k$ are increased at the expense of the weights on x_k^i.

The individual loss of agent i is:

(14) $$E\left(p^i - A\theta\right)^2 = L_2(\delta) = \frac{\alpha+\beta\,(1-r)^2}{[\alpha+\beta\,(1-r)]^2} + \delta\frac{\delta\alpha+\beta\,(1-r)^2}{[\delta\alpha+\beta\,(1-r)]^2}$$

The central bank does not have any optimization to do since the interest rate does affect the economy other than through its signal content, which is uninformative with full transparency.

3.3 Welfare comparisons

The central bank has the choice between only revealing an optimally chosen interest rate and revealing all its information ($\widetilde{\theta}_1$ and $\widetilde{\theta}_2$). In the latter case, it gives up any possibility of manipulating private expectations to cope with the common knowledge effect. Which strategy is best is therefore more complicated than in the case where the central bank does not have to set the interest rate. In order to determine which is the best policy, we need to compare $L_1(\delta) = Arg \min L_1(\delta, \lambda)$ and $L_2(\delta)$. Given how unwieldy these expressions are, once again we examine the situation in the neighborhood of the symmetric case where $\delta = 1$.

3.3.1 Symmetric case

To start with, we consider the benchmark case $\delta = 1$. Recall that the central bank's optimal interest decision is $\lambda = 1$ when the second-order condition $1 - 2r + \frac{\alpha}{\beta} \geq 0$ is satisfied. Then (9) and (14) show that:

$$L_2(1) = L_1(1,1) = 2\frac{\alpha + \beta(1-r)^2}{[\alpha + (1-r)\beta]^2} \tag{15}$$

When the two fundamentals are drawn from the same distribution, it makes no difference whether the central bank only publishes the interest rate or whether it reveals all its information. In this case, as a linear combination of the two signals, the interest rate is as informative as the signals themselves. This equivalence result holds when the second-order condition is met, i.e. when the relative information of the central bank is good enough given the reactivity of individual producers to each other forecasts ($\frac{\alpha}{\beta} \geq 2r - 1$).

What happens instead when the precision of the central bank is lower than $2r - 1$? In that case, $\lambda = 1$ maximizes $L_1(1, \lambda)$ while $\lambda = -\frac{1}{\delta} = -1$ delivers the minimum. It follows that:

$$L_1(1, -\frac{1}{\delta}) < L_1(1, \lambda) < L_1(1, 1) = L_2(1)$$

Any λ will make releasing only the interest rate a better solution than being fully transparent. The best solution is for the central bank to choose $\lambda = -\frac{1}{\delta} = -1$; given that we are in the neighborhood of $\delta = 1$, this implies that μ and ν become infinitely large in absolute value and so does the variance of R.

M&S observe that when the quality of the central bank information is poor, it should not release any signals. Here, however, the central bank has no choice, it must issue some signal as it cannot avoid setting the interest rate. Aware that the interest rate is common knowledge and that its relative precision is relatively poor, the central bank's best strategy is to make the interest rate as uninformative as it can. Of course, this conclusion only holds because we assume that the interest rate plays no macroeconomic role. If it were, the central bank would face a trade-off between the need for opacity and good monetary policy. This important trade-off is left for future research.

3.3.2 Asymmetric case

Now, consider the asymmetric case where $\delta \neq 1$ so that the two signals received by the private sector are drawn from different distributions and each have a separate information value. We distinguish again between the two solutions for λ. When $1 - 2r + \dfrac{\alpha}{\beta} \geq 0$, in the neighborhood of $\delta = 1$, the optimal choice of the central bank is in the neighborhood of $\lambda = 1$ as in (10). In this case:

$$(16) \qquad L_1(\delta) - L_2(\delta) \simeq -\frac{1}{2}\frac{r^2(\delta-1)^2}{\beta}\frac{(\frac{\alpha}{\beta})^3}{\left[(1-r)+\frac{\alpha}{\beta}\right]^3\left[1-2r+\frac{\alpha}{\beta}\right]} < 0$$

Irrespective of the relative precision of central bank and private sector information, it is preferable for the central banks to only publish the interest rate. This result differs from M&S who find that when $\dfrac{\alpha}{\beta}$ is large the central bank should be fully transparent. The reason is that the central bank can effectively use the interest rate to orient private sector expectations in a way that mitigates the unavoidable common knowledge effect. Transparency, the release of the signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$, creates a common knowledge effect and eliminates the information content of the interest rate and the associated possibility of manipulating private expectations.

When $1-2r+\dfrac{\alpha}{\beta} < 0$, the optimal choice of the central bank is $\lambda = -\dfrac{1}{\delta} \simeq -1$, which implies a highly volatile interest rate. Then, the Appendix shows that:

$$(17) \qquad L_1(\delta) - L_2(\delta) \simeq \frac{2}{\beta}\frac{1-2r+\frac{\alpha}{\beta}}{\left[(1-r)+\frac{\alpha}{\beta}\right]^2}\frac{\alpha}{\beta} < 0$$

Here again, the central bank does not reveal its information. As in Section 3.3.1, it seeks to become as opaque as it can since it is under obligation to publish the interest rate even though it is relatively poorly informed given the reactivity of the previous sector ($\dfrac{\alpha}{\beta} < 2r - 1$).

3.3.3 Assessment

When the precision of central and bank information is known, the case considered by M&S, we find that an interest-setting central bank should never be transparent. We summarize our finding so far as follows:

Proposition 1 *When its information precision is known to be high, the central bank does better by not being transparent. Instead it uses the interest rate publication to manipulate the relatively poorly-informed private sector expectations. When its information precision is low, it still does not release its private information, but it also endeavors to become opaque by making the interest rate highly volatile.*

Note that these results differ from M&S in two respects. First, even when its information is highly precise, an interest rate-setting central bank should not be transparent. Second, when its information is poor, the central bank can still use the interest rate to limit the common knowledge effect.

4 The private sector precision is not known

So far, uncertainty only concerned the first moments of the fundamentals. The second moments, the precision of information received by the central banks and the private sector, were assumed to be common knowledge. It is quite plausible, however that the precision of the signals itself is unknown.

Given the inherent complexity of this situation, we consider a simple example. We now assume that neither the central bank nor the private sector know V_η. On the other hand, both the central bank and the private sector know V_ε, the variance of the central bank signal. This postulated asymmetry is logical as we focus on the role of common knowledge. Since the central bank actions are closely watched by a wide group of observers, it is plausible that much more is known about central bank signal precision than about private signal precision.

We also need to specify who knows what. We assume that both the central bank and the private sector mistakenly believe that their own estimates of private signal variance, $E^{CB}V_\eta$ and $E^{pr}V_\eta$, respectively, are accurate. In line with the postulated asymmetry meant to capture central bank watching, we further assume that the private sector realizes that the central bank is mistaken, i.e. $E^{pr}E^{CB}V_\eta \neq E^{pr}V_\eta$.

As before, we define the two private sector signal as $x_k^i = \theta_k + \eta_k^i$ and we assume that the (unknown) signal quality is the same among all producers:

$$Var\left(\eta_k^i\right) = V_{\eta,k} \quad \forall i \qquad k = 1, 2$$

Note that we still allow for the private sector signal variances of η_1 and η_2 to differ (i.e. $V_{\eta,1} \neq V_{\eta,2}$). To keep matters simple, we assume that the estimates of these variances are exogenous and differ between the central bank and the private sector:

$$\begin{aligned} V_{\eta,k}^{pr} &\equiv E^{pr} V_{\eta,k} \\ V_{\eta,k}^{CB} &\equiv E^{CB} V_{\eta,k} \end{aligned}$$

We start with the simplest case. We assume that the private sector in fact knows its own precision, i.e. $V_{\eta,k}^{pr} = V_{\eta,k}$. Since the private sector knows that $V_{\eta,k}^{CB} \neq V_{\eta,k}$, it must forecast the central bank's estimate. Again, we assume that this forecast is exogenous:

$$\tilde{V}_{\eta,k}^{pr} \equiv E^{pr} E^{CB} V_{\eta,k} = E^{pr} V_{\eta,k}^{CB} \tag{18}$$

A more elaborate model would allow for endogenous expectation formations, but this would require a much heavier setup with little additional insight. Section 4 shows that removing the assumption $V_{\eta,k}^{pr} = V_{\eta,k}$ does not significantly alter the conclusions. As previously, we normalize the private sector variances around $\tilde{V}_{\eta,2}^{pr}$ and adopt the following notation:

$$\tilde{V}_\eta = \tilde{V}_{\eta,2}^{pr} \qquad \tilde{\delta} = \frac{\tilde{V}_{\eta,1}^{pr}}{\tilde{V}_{\eta,2}^{pr}} \qquad \alpha = \frac{1}{V_\varepsilon} \qquad \widetilde{\beta} = \frac{1}{\tilde{V}_\eta}$$

The uncertainty about signal precision now changes the notion of central bank transparency. The private sector is interested in knowing not just the central bank signals $\tilde{\theta}_k$ but also the central bank forecasts $V_{\eta,k}^{CB}$ of the private sector signals. The latter only matters when the central bank does not reveal its signals. Thus there are now three degrees of transparency: 1) the central bank only publishes the interest rate; 2) the central bank reveals its estimates of private sector variances $V_{\eta,k}^{CB}$; and 3) the central bank reveals its signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$.

4.1 The central bank only reveals the interest rate

When she observes the interest rate R and try to figure out what is the central bank reaction function, producer i must distinguish between her own estimates $V_{\eta,k}^{pr} = V_{\eta,k}$ of $V_{\eta,k}$ and those $\tilde{V}_{\eta,k}^{pr}$ that she believes are made by the central bank. To do so, each producer applies the usual Bayesian signal extraction method to interpret the interest rate on the basis of its own signals x_k^i. This allows producer i to form her expectation $E^i(A\theta)$ of the fundamentals. But the producer also knows that the central bank reaction function is set on the basis of its own mistaken expectations $V_{\eta,k}^{CB}$ that the private sector estimates as $\tilde{V}_{\eta,k}^{pr}$. Producer i must therefore also extract information from the observed interest rate to infer the parameters of the central bank reaction function that enter $E^i(A\theta)$. We now examine this procedure.

Private sector inference conditional on the central bank reaction function

As before, producer i observes the interest rate, knowing that the central bank follows a linear reaction function $R = \widetilde{\mu}\tilde{\theta}_1 + \widetilde{\nu}\tilde{\theta}_2$, and receives the signals x_k^i, $k = 1, 2$, corresponding to the two fundamentals. Using Bayes theorem, her estimates of the two fundamentals is as in (5) and (6). The price decision of producer i is given by (7).

Producer i must estimate the central bank choice of λ. She knows that the central bank chooses λ to minimize $E^{CB}(p^i - A\theta)^2$ but that $\tilde{V}_{\eta,k}^{pr} = E^{pr}E^{CB}V_{\eta,k} \neq V_{\eta,k}$. Accordingly, following (9), the private sector expects the central bank to choose $\widetilde{\lambda} = \dfrac{\widetilde{\mu}}{\widetilde{\nu}}$ to minimize:

$$
\begin{aligned}
E^{CB}\left(p^i - A\theta\right)^2 &= L_1\left(\widetilde{\delta}, \widetilde{\lambda}\right) \\
&= \frac{\left(1+\widetilde{\lambda}^2\right)(1+\widetilde{\delta}\widetilde{\lambda})^2\alpha\widetilde{\beta} + \widetilde{\delta}[(1-\widetilde{\lambda})\alpha + (1-r)\left(1+\widetilde{\lambda}^2\right)\widetilde{\beta}]^2}{\widetilde{\beta}[\left(1+\widetilde{\delta}\widetilde{\lambda}^2\right)\alpha + (1-r)\left(1+\widetilde{\lambda}^2\right)\widetilde{\beta}]^2} \\
&\quad + \frac{[\widetilde{\lambda}(\widetilde{\lambda}-1)\widetilde{\delta}\alpha + (1-r)\left(1+\widetilde{\lambda}^2\right)\widetilde{\beta}]^2}{\widetilde{\beta}[\left(1+\widetilde{\delta}\widetilde{\lambda}^2\right)\alpha + (1-r)\left(1+\widetilde{\lambda}^2\right)\widetilde{\beta}]^2}
\end{aligned}
\tag{19}
$$

In the neighborhood of $\widetilde{\delta} = 1$, the solution is in the neighborhood of $\widetilde{\lambda} = 1$ when the second order condition $1 - 2r + \dfrac{\alpha}{\beta} \geq 0$ is satisfied. Section 4.4 deals with the alternative case. Following (10), a first order approximation is:

$$
\widetilde{\lambda} - 1 = \frac{(1-3r+2r^2) + \dfrac{\alpha}{\widetilde{\beta}}}{(1-3r+2r^2) + (2-3r)\dfrac{\alpha}{\widetilde{\beta}} + (\dfrac{\alpha}{\widetilde{\beta}})^2}\left(\widetilde{\delta} - 1\right) \tag{20}
$$

This is similar to (10) and has the same interpretation.

Pricing

Once producer i has determined the value of $\widetilde{\lambda}$ that she expects the central bank to choose in deciding on the interest rate, she estimates $E^i(\theta_k)$ from observing the interest rate and receiving the signals x_k^i on the two fundamentals. This leads her to modify the coefficients γ_k of her Bayesian inference (5):

$$\text{(21a)} \qquad \widetilde{\gamma}_1 = \frac{\delta\widetilde{\lambda}^2\alpha}{\left(1+\widetilde{\lambda}^2\right)\beta+\left(1+\widetilde{\lambda}^2\delta\right)\alpha}$$

$$\text{(21b)} \qquad \widetilde{\gamma}_2 = \frac{\alpha}{\left(1+\widetilde{\lambda}^2\right)\beta+\left(1+\widetilde{\lambda}^2\delta\right)\alpha}$$

Using these parameters in (7) delivers the pricing decision:

$$\begin{aligned} p^i &= (1-r)\sum_{n=0}^{\infty} r^n E^i\left(\bar{E}^n(A\theta)\right) \\ \text{(22)} &= \frac{1+\widetilde{\lambda}}{\widetilde{\lambda}}\left(\tilde{\varphi}_1+\widetilde{\lambda}\tilde{\varphi}_2\right)R+\left[(1-\tilde{\varphi}_1)-\tilde{\varphi}_2\widetilde{\lambda}\right]x_1^i+\left[(1-\tilde{\varphi}_2)\widetilde{\lambda}-\tilde{\varphi}_1\right]\frac{x_2^i}{\widetilde{\lambda}} \end{aligned}$$

where:

$$\tilde{\varphi}_k = \frac{\widetilde{\gamma}_k}{1-[1-r(\widetilde{\gamma}_1+\widetilde{\gamma}_2)]}$$

These are exactly the same as in (7) and (8), the only difference being that $\widetilde{\lambda}$, the producer estimate of the central bank action, is based on $\tilde{V}_{\eta,k}^{pr} \equiv E^{pr}E^{CB}V_{\eta,k}$, her estimate of the central bank forecast instead of $V_{\eta,k}$ itself (even though producer i knows $V_{\eta,k}$).

The true central bank reaction function

So far, we have computed the private sector estimate of central bank expectations (20), but the private sector does not know the later for sure (and knows that it does not know). What will be the central bank choice of λ? The central bank will first extract information from its own signals $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$, using its forecasts $V_{\eta,k}^{CB} \equiv E^{CB}V_{\eta,k}$. As before, we normalize on $V_{\eta,2}^{CB}$ and adopt the following notation:

$$V_\eta^{CB} = V_{\eta,2}^{CB} \qquad \delta' = \frac{V_{\eta,1}^{CB}}{V_{\eta,2}^{CB}} \qquad \beta' = \frac{1}{V_\eta^{CB}}$$

This is exactly the same situation as in Section 3.1. The loss expected by the central bank $E^{CB}(p^i - A\theta)^2$ is as in (9) with β' and δ' instead of β and δ, respectively. When δ' is in the neighborhood of 1, subject to the second-order condition $1 - 2r + \dfrac{\alpha}{\beta} \geq 0$, the optimal policy parameter λ' is close to 1 and given by the suitably adjusted equation (10):

$$\lambda' - 1 \simeq \frac{(1 - 3r + 2r^2) + \dfrac{\alpha}{\beta'}}{(1 - 3r + 2r^2) + (2 - 3r)\dfrac{\alpha}{\beta'} + (\dfrac{\alpha}{\beta'})^2}(\delta' - 1) \tag{23}$$

Here again, we find the same result as (10), with the same interpretation.

Market equilibrium: the systematic bias

We now have a situation where the central bank chooses λ' and sets the interest rate according to $R = \dfrac{\lambda_1'\tilde{\theta} + \tilde{\theta}_2}{1 + \lambda'}$ while the private sector looks at it believing that it is $R = \dfrac{\widetilde{\lambda}\tilde{\theta}_1 + \tilde{\theta}_2}{1 + \widetilde{\lambda}}$. Inserting the true value of the interest rate into (22) gives the actual market price equilibrium. As a deviation from its fundamentals, it is:

$$\begin{aligned} p^i - A\theta &= \frac{\tilde{\varphi}_1 + \tilde{\varphi}_2\widetilde{\lambda}}{\widetilde{\lambda}(1 + \lambda')}(\tilde{\theta}_1 - \tilde{\theta}_2)(\lambda' - \widetilde{\lambda}) \\ &\quad + \frac{\tilde{\varphi}_1 + \tilde{\varphi}_2\widetilde{\lambda}}{\widetilde{\lambda}}\left[\widetilde{\lambda}\left(\tilde{\theta}_1 - \theta_1\right) + \left(\tilde{\theta}_2 - \theta_2\right)\right] \\ &\quad + \left[(1 - \tilde{\varphi}_1) - \tilde{\varphi}_2\widetilde{\lambda}\right]\left(x_1^i - \theta_1\right) + \left[(1 - \tilde{\varphi}_2)\,\widetilde{\lambda} - \tilde{\varphi}_1\right]\frac{x_2^i - \theta_2}{\widetilde{\lambda}} \end{aligned} \tag{24}$$

This expression includes three terms. The second and third ones involve the signal errors of the central bank and of the private sector, respectively. Such mispricing is unavoidable but is zero on average. The first term, on the other hand is not nil on average. The discrepancy between the private sector and the central bank information sets implies that the central bank's choice λ' given by (20) systematically differs from the corresponding private sector forecast $\widetilde{\lambda}$ shown in (23). This discrepancy creates a systematic bias between the price level and its fundamentals.[8]

Proposition 2 *When the central bank and the private sector differ in their forecasts of the precision of private sector information, the interest rate becomes the source of a systematic bias between the price level and its fundamentals. The bias occurs irrespective of the relative precision of central bank and private sector information.*

The size of the systematic bias is proportional to the size of $\lambda' - \widetilde{\lambda}$. Consider again the two polar limit cases in the neighborhood of 1 for $\widetilde{\delta}$, δ', $\widetilde{\lambda}$ and λ'. From (20) and (23), we know that there is no bias ($\lambda' \simeq \widetilde{\lambda} \simeq 1$) when $\dfrac{\alpha}{\beta} \to \infty$, while $\lambda' - \widetilde{\lambda} \simeq \delta' - \widetilde{\delta}$ when $\dfrac{\alpha}{\beta} \to 0$. When the central bank is well informed it relies on its own signals; with $V(\varepsilon_1) = V(\varepsilon_2)$ it weighs both signals equally and the private sector knows that, so the bias is eliminated. On the contrary, when the central bank is poorly informed, it relies on its estimate of private sector signals and sets $\lambda' = \delta'$; the private sector is aware of this strategy but it imagines $\tilde{\lambda} = \tilde{\delta}$. This suggests that the systematic bias is larger the less well-informed the central bank is.

4.2 The central bank is partially transparent

We consider now the case where the central bank reveals the interest rate and its estimates of private sector variances $V^{CB}_{\eta,k}$ - i.e. β' and δ' - but not its signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$. The central bank choice, λ' given by (23), rests on its mistaken estimates β' and δ' but this is now known to the private sector. Accordingly $\tilde{\lambda} = \lambda'$ and the systematic bias in (24) is eliminated:

$$\begin{aligned} p^i - A\theta &= \frac{\tilde{\varphi}_1 + \tilde{\varphi}_2\widetilde{\lambda}}{\widetilde{\lambda}} \left[\widetilde{\lambda}\left(\tilde{\theta}_1 - \theta_1\right) + \left(\tilde{\theta}_2 - \theta_2\right)\right] \\ &\quad + \left[(1-\tilde{\varphi}_1) - \tilde{\varphi}_2\widetilde{\lambda}\right]\left(x_1^i - \theta_1\right) + \left[(1-\tilde{\varphi}_2)\widetilde{\lambda} - \tilde{\varphi}_1\right]\frac{x_2^i - \theta_2}{\widetilde{\lambda}} \end{aligned} \tag{25}$$

where $\tilde{\varphi}_k$ is as in (6) and (8) with λ' instead of λ, and the loss is $L_1(\delta, \lambda')$. We evaluate this case in Section 4.5 below.

[8] The bias disappers when $\widetilde{\theta}_1 = \widetilde{\theta}_2$. This woud imply that there is a single fundamental. In that case, revealing R would fully reveal the central bank choice up to the signal errors that appear in the second and third terms.

4.3 The central bank is fully transparent

The central bank now reveals its estimates β' and δ' as well as its own signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$. This situation has already been treated in Section 3.2. Since $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are known, the interest rate has no information value anymore and the discrepancy of information precision between the central bank and the previous sector does not create any systematic bias anymore. The optimal forecast of producer i is given by (11) and (12), and the individual loss remains as in (14).

When the central bank is fully transparent, the private sector knows everything that the central bank knows, even though the central bank is mistaken about the precision of private forecasts. Since the interest rate is not informative anymore, the private sector ignores the flawed signal that it conveys.

Proposition 3 *When the central bank and the private sectors have different information about the private sector precision, by revealing all its information a poorly-informed central bank allows the private sector not to misinterpret the interest rate signal.*

4.4 Polar beliefs

So far we have considered the case where the second order condition $1 - 2r + \frac{\alpha}{\beta} \geq 0$ is satisfied and the optimal λ is in the neighborhood of 1 when δ is close to unity. When this condition is not satisfied $\lambda \simeq 1$ maximizes the loss function and the optimal choice is $\lambda = -\frac{1}{\delta}$. Section 3.3.2 examines this case for the case where the variances are known. When the central bank estimates of private sector variances are not known and the central bank sets λ' while the private sector believes that the central bank has chosen $\tilde{\lambda}$, three new possibilities may arise: 1) the central bank chooses $\lambda' = -\frac{1}{\delta'}$ and the private sector believes $\tilde{\lambda} = -\frac{1}{\tilde{\delta}}$; 2) the central bank chooses $\lambda' = -\frac{1}{\delta}$ and the private sector believes that $\tilde{\lambda} \simeq 1$; 3) the central bank chooses $\lambda' \simeq -1$ and the private sector correctly believes that $\tilde{\lambda} \simeq -1$. We know from (24) that $\lambda' \neq \tilde{\lambda}$ produces a systematic bias and is therefore the source of a welfare loss. This loss is largest when the central bank and the private sector each aim at polar policy settings $-\frac{1}{\delta'}$ and 1. This is not so.

From (24) we know that the bias it is proportional to $\frac{\lambda' - \tilde{\lambda}}{\tilde{\lambda}(1 + \lambda')}$. With

$\lambda' = -\frac{1}{\delta'}$and $\tilde{\lambda} = -\frac{1}{\tilde{\tilde{\delta}}}$, the systematic bias is proportional to $\frac{\tilde{\delta}-\delta'}{\delta'-1}$, which is indeterminate but likely finite. With $\lambda' = -\frac{1}{\delta'}$ and $\tilde{\lambda} \simeq 1$, the bias is proportional to $-\frac{1+\delta'}{\delta'-1}$, which is close in absolute value to ∞. Finally, $\lambda' \simeq 1$ and $\tilde{\lambda} \simeq -\frac{1}{\tilde{\tilde{\delta}}}$ leads to a bias proportional to $\frac{1+\tilde{\delta}}{2} \simeq 1$. Thus the bias is infinite only when the central bank believes that it is poorly informed in the face of strong private sector reactivity ($\frac{\alpha}{\beta'} < 1-2r$) while the private sector knows that this is not the case ($\frac{\alpha}{\tilde{\tilde{\beta}}} \geq 1-2r$). The private sector attaches value to the interest rate signal while the central bank purposefully attempts to remove any information content from the rate. In the opposite case, when central bank considers itself relatively well informed while the private sector knows that it is not so, the latter simply discards the information that the former attempts to convey *via* the interest rate, and the bias is finite.

4.5 Welfare comparisons

We now compare the welfare properties of the three degrees of transparency previously studied. To summarize:

1) No transparency: the central bank only publishes the interest rate. It chooses λ' given by (23), but the private sector believes that it is $\tilde{\lambda}$ as in (20). The corresponding loss, denoted $\tilde{L}_1\left(\delta, \tilde{\lambda}, \lambda'\right)$, is the unconditional expectation $E(p^i - A\theta)^2$ where p^i is given by (24).

2) Partial transparency: in addition to the interest rate, the central bank reveals its (mistaken) estimates of private sector variances $V_{\eta,k}^{CB}$, hence β' and δ'. It chooses λ' and the private sector is aware of this. The loss, which is computed using the correct (and private estimates of) variances $V_{\eta,k}$, hence using β and δ, is denoted $L_1(\delta, \lambda') = E(p^i - A\theta)^2$ where p^i is set according to (25).

3) Full transparency: the central bank additionally reveals its signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$, which renders the choice of R, and therefore of λ, uninformative. The corresponding loss $L_2(\delta)$ is given by (14).

Note first that the central bank's erroneous forecast of private variances implies $L_1(\delta, \lambda') > Arg\min L_1(\delta, x) = L_1(\delta)$, where $L_1(\delta)$, the optimal loss when the variances are known to both the central bank and the private sector, is computed in Section 3.1. It follows that whenever $L_1(\delta) > L_2(\delta)$ full transparency is the socially optimal strategy. However, at least in the polar cases studied in Section 3.3.2, this condition is never satisfied, so we need to compare directly the corresponding values of the loss function. This is not possible in general. Accordingly we focus on the neighborhood of $\delta = 1$ and make some additional simplifying assumptions.

4.5.1 Partial vs. full transparency

We compare $L_1(\delta, \lambda')$ and $L_2(\delta)$. With λ as the optimal policy choice when the variances are known (see Section 3), we assume that the central bank decision error $\lambda' - \lambda$, which depends on its information errors, is an exogenous random variable with zero mean and variance $E(\lambda' - \lambda)^2 = V$. We consider again two cases.

- When $1 - 2r + \dfrac{\alpha}{\beta} > 0$, the optimal λ is close to 1, and a Taylor expansion around $\lambda = 1$ gives:

$$L_1(\delta, \lambda') = L_1(\delta) - \frac{V}{\beta} \frac{2\left(\lambda^2 - 3\right)\left[\frac{\alpha}{\beta}(1-2r) + (\frac{\alpha}{\beta})^2\right]\lambda}{\left[(1-r) + \frac{\alpha}{\beta}\right]^2 \left(1+\lambda^2\right)^3}$$

Using (16), we have in the neighborhood of $\lambda \simeq \delta \simeq 1$:
(26)

$$L_1(\delta, \lambda') - L_2(\delta) \simeq \frac{V\left[1 - 2r + \frac{\alpha}{\beta}\right]^2 \left[1 - r + \frac{\alpha}{\beta}\right]^2 - r^2(\delta-1)^2(\frac{\alpha}{\beta})^2}{2\beta\left[1 - r + \frac{\alpha}{\beta}\right]^4 \left[1 - 2r + \frac{\alpha}{\beta}\right]} \frac{\alpha}{\beta}$$

Several possibilities arise, as shown in the Appendix.

When $r < \dfrac{1}{2}$ i.e. when the private sector is not too reactive, the beauty contest effect is weak. In that case, it matters whether the variance of the central bank decision error $E(\lambda' - \lambda)^2 = V$ is large or not relatively to the asymmetry of information distance, both of which are taken as exogenous.

If $\sqrt{\frac{V}{(\delta-1)^2}} < 1$, there is some scope for interest manipulation.[9] In this case we have:

[9] When r is very small, full transparency is always optimal because the beauty contest effect is too weak for interest rate manipulation. This is clear by continuity starting from $r = 0$.

$$
\begin{aligned}
\frac{\alpha}{\beta} &< a_1 \qquad L_1(\delta,\lambda') > L_2(\delta) \\
a_1 &< \frac{\alpha}{\beta} < a_2 \qquad L_1(\delta,\lambda') < L_2(\delta) \\
\frac{\alpha}{\beta} &> a_2 \qquad L_1(\delta,\lambda') > L_2(\delta)
\end{aligned}
$$

where a_1 and a_2 depend on r and $\sqrt{\frac{V}{(\delta-1)^2}}$(see the Appendix for details).

Except for a range $a_1 < \dfrac{\alpha}{\beta} < a_2$, it is preferable for the central bank for the be fully transparent. Within the range, however, the central bank should be partially transparent. When $\dfrac{\alpha}{\beta}$ is low, the central bank is too poorly informed to be able to effectively manipulate the interest rate, so it should be fully transparent. When $\dfrac{\alpha}{\beta}$ is large, the high precision of central bank information justifies revealing all its information, as in M&S. In the intermediate range the central bank raises welfare by setting the interest rate iin a way that correctly orient private sector expectations.

If $\sqrt{\frac{V}{(\delta-1)^2}} > 1$, $L_1(\delta,\lambda') > L_2(\delta)$, the central bank decision error is relatively large, which means that its effectiveness at manipulating the interest rate is weak. In that case, it is always preferable that the central bank be fully rather than partially transparent. The common knowledge effect affecting the interest rate is worse, from a welfare viewpoint, than the common knowledge effect from the signals themselves. This result holds independently of the quality of central bank information.

When $r > \dfrac{1}{2}$, i.e. when the beauty contest effect is strong, we have:

$$
\begin{aligned}
\frac{\alpha}{\beta} &< a_3 \qquad L_1(\delta,\lambda') < L_2(\delta) \\
\frac{\alpha}{\beta} &> a_3 \qquad L_1(\delta,\lambda') > L_2(\delta)
\end{aligned}
$$

When its relative precision is low the central bank should be partially transparent, and fully transparent when the relative precision is above a threshold. In comparison with the case above, even when its precision is very low the central bank can use the interest rate to orient private expectations because the private sector is highly reactive.

- When $1 - 2r + \dfrac{\alpha}{\beta} < 0$, the optimal $\lambda = -\dfrac{1}{\delta}$. The Appendix shows that $L_1(\delta,\lambda') - L_2(\delta)$ is the same as the sign of $(4 - V)$. Although we assume that

$V = E(\lambda' - \lambda)^2$ is exogenous, we know that, in this case, the central bank optimally chooses $\lambda' = -\dfrac{1}{\delta}$ and that, indeed, the optimal $\lambda = -\dfrac{1}{\delta}$. Since V is unconditional, it is reasonable to consider that it is small, which implies that $L_1(\delta, \lambda') < L_2(\delta)$. Half transparency now dominates full transparency because it allows a relatively poorly informed central bank to conceal the signal value of the interest rate by increasing its variance. Indeed a small V means that the difference of information sets between the central bank and the private sector does not create any large discrepancy between λ', chosen by the central bank sets the interest rate, and the interpretation λ of the private sector.

4.5.2 Partial vs. no transparency

We now compare $\tilde{L}_1\left(\delta, \tilde{\lambda}, \lambda'\right)$ and $L_1(\delta, \lambda')$. As before we assume that $\lambda' - \lambda$ is white noise u_1 with exogenous variance V. We also need to consider the private sector forecast error $\tilde{\lambda} = E^{pr}(\lambda')$. We assume that this step involves an additive error u_2 which is white noise with unconditional variance V_2 and u_1 and u_2 are taken as orthogonal. Intuitively, we consider that expectations of expectations additively increase the variance of forecast errors.

Using (24) and (25) we obtain in the neighborhood of $\delta = 1$:

$$\begin{aligned}\tilde{L}_1\left(\delta, \tilde{\lambda}, \lambda'\right) - L_1(\delta, \lambda') &= L_1\left(\delta, \tilde{\lambda}\right) - L_1(\delta, \lambda') \\ &+E\left(\frac{\frac{\left(\tilde{\lambda}-\lambda'\right)^2}{\left(\tilde{\lambda}(1+\lambda')\right)^2}\left(\frac{2}{\alpha} + (\theta_1-\theta_2)^2\right)\tilde{\lambda}^2(\delta\tilde{\lambda}+1)^2(\frac{\alpha}{\tilde{\beta}})^2}{\left[(1-r)\left(1+\tilde{\lambda}^2\right) + \left(1+\tilde{\lambda}^2\delta\right)\frac{\alpha}{\tilde{\beta}}\right]^2}\right) \qquad (27)\end{aligned}$$

When $1 - 2r + \dfrac{\alpha}{\beta} > 0$ and the optimal choice is $\tilde{\lambda} \simeq 1$, the Appendix shows that:

$$\begin{aligned}&\tilde{L}_1\left(\delta, \tilde{\lambda}, \lambda'\right) - L_1(\delta, \lambda') \\ (28) \quad \simeq\ &\frac{1}{4}V_2\left[\frac{2}{\alpha} + (\theta_1-\theta_2)^2\right]\frac{(\frac{\alpha}{\tilde{\beta}})^2}{\left[1-r+\frac{\alpha}{\tilde{\beta}}\right]^2} + \frac{1}{2}\frac{V_2}{\beta}\frac{1-2r+\frac{\alpha}{\beta}}{\left[1-r+\frac{\alpha}{\beta}\right]^2} > 0\end{aligned}$$

Even though its information is of poor quality, by releasing its forecasts of private sector precision, the central bank eliminates the systematic bias. In addition, it saves the private sector from having mistaken forecast the policy choice λ', which gives rise to V_2.

When $1 - 2r + \dfrac{\alpha}{\beta} < 0$ and the optimal choice is $\lambda' = -\dfrac{1}{\delta'}$. The same procedure leads to **:**

$$\tilde{L}_1\left(\delta, \tilde{\lambda}, \lambda'\right) - L_1\left(\delta, \lambda'\right) \simeq \frac{1}{4} V_2 \left(\frac{2}{\alpha} + (\theta_1 - \theta_2)^2\right) \frac{(\frac{\alpha}{\bar{\bar{\beta}}})^2}{\left[1 - r + \frac{\alpha}{\bar{\bar{\beta}}}\right]^2} - \alpha V_2 \frac{1 - 2r + \frac{\alpha}{\beta}}{2\left(\alpha + \beta(1-r)\right)^2} > 0$$

This establishes that, in comparison to only setting the interest rate, the central bank raises welfare by revealing its forecasts of private sector precision, even though its precision is relatively poor.

4.5.3 Implication and interpretation

Summing up so far, with three exceptions, we find $\tilde{L}_1\left(\delta, \tilde{\lambda}, \lambda'\right) > L_1\left(\delta, \lambda'\right) > L_2\left(\delta\right)$.

Proposition 4 *The precision of the central bank information is unknown, from a welfare viewpoint, full transparency generally dominates partial transparency, which dominates no transparency. It is better for the central bank to be half transparent - issuing variance forecasts that affect the private sector interpretation of the interest rate - when the central bank information precision is relatively low and the private sector is highly reactive and therefore manipulable.*

These results stand in sharp contrast with those of M&S. Partial transparency improves over no transparency because in the latter case the private sector misinterprets the interest rate, which introduces a systematic bias between the price level and its fundamentals. The bias can be large because of the common knowledge effect. Full transparency improves over partial transparency because we assume here that the central bank forecasts of private sector precision are erroneous. As a consequence, it chooses an interest rate that is both misleading and a source of common knowledge effect. When it directly releases its signals, a transparent central bank does not avoid the common knowledge effect but, at least, the signals are not shrouded in fog.

Two of the three exceptions occur when the central bank is poorly informed and when the private sector is highly reactive. In both cases, the poor signal quality leads to large common pool effects as in M&S while the reactivity of the private sector provides an opportunity to manipulate the interest rate, in one case even to use the interest rate signal to deliberately being opaque and thus avoid any misinterpretation. The third exception concerns a situation of intermediate relative central bank precision and low public sector reactivity where the central bank makes a sufficiently small error in setting the interest rate that its ability to affect private sector forecasts compensate the common knowledge effect attached to the interest rate. [10]

5 The private sector is mistaken about its own variances

In Section 4 we assume the private sector precision is unknown to the central bank but known to the private sector. This section briefly looks at a more symmetric setup where the private sector does not know either its own variance. Its estimates of δ and β are denoted $\hat{\delta}$ and $\hat{\beta}$. Unaware of its erroneous forecasts, the private sector therefore uses $\hat{\delta}$ and $\hat{\beta}$ to estimate λ in the central bank interest rate rule. Note that the losses are still computed using the true values δ and β.

We use the following notation for the welfare losses:

- The central bank is not transparent, i.e. it only publishes the interest rate: $\tilde{L}_1(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta)$

- The central bank is partially transparent, i.e. it additionally reveals its variance estimates: $L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta)$

- The central bank is fully transparent, i.e. it additionally reveals its signals: $L_2(\beta', \hat{\delta}, \delta)$.

The following results are established in the Appendix:

When the second order condition is verified and the optimal λ is close to 1.

$$\frac{\alpha}{\beta} \to \infty$$

$$\tilde{L}_1(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) > L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) > L_2(\hat{\beta}, \hat{\delta}, \delta)$$

$$\frac{\alpha}{\beta} \to 0$$

$$\tilde{L}_1(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) > L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) \gtrless L_2(\hat{\beta}, \hat{\delta}, \delta)$$

[10] The first two cases are $r > \frac{1}{2}$ and $\frac{\alpha}{\beta} < a_3$, and $1 - 2r + \frac{\alpha}{\beta} < 0$. The third case is $r < \frac{1}{2}$, $E\left(\lambda' - \lambda\right)^2 = V$ and $a_1 < \frac{\alpha}{\beta} < a_2$.

Thus we reach the same conclusion as in Section 4: it is generally preferable for the central bank to be transparent whether the relative precision of central bank information is very high or very low. The ambiguity concerning the limit case $\frac{\alpha}{\beta} \to 0$ reflects two sources of welfare losses that can be seen from the Appendix. When the central bank is half transparent, its mistaken choice of λ' plays a major role in determining $L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta)$ while with full transparency $L_2(\hat{\beta}, \hat{\delta}, \delta)$ is mostly influenced by the private sector error $\hat{\beta}$ (the central bank's error on $\hat{\delta}$ affects both losses).

When the second order condition is not verified and the optimal $\lambda = -\frac{1}{\delta}$, we only consider the polar case $\frac{\alpha}{\beta} \to 0$. Here again, the conclusions from Section 4 generally hold.

6 Conclusions

The influential result by M&S, that a central bank should not be transparent, rests on the intuitive flute-player effect. In a world of imperfectly-informed agents who face a strong incentive to converge to the mean (the beauty contest effect) signals that are known to be observed by all (common knowledge) strongly affect expectations. Everyone follows the flute-payer. If these signals are of poor quality, the effect can be welfare-reducing as the flute-player becomes the head-sheep that jumps off the cliff. As noted by Svensson (2005), the conclusion that central bank transparency may be undesirable rests on the dubious assumption that the central bank is relatively less well informed than the previous sector. Yet, the flute-player effect stands.

In this paper, we show that the flute-player effect is not necessarily welfare-reducing. An interest rate-setting central bank can use this effect to mitigate the common knowledge effect. Put differently, an optimizing flute player does not jump of the cliff; he may also decide to charm away his followers in the direction or even to hide in the bushes to distill his music when the fog is thick.

The key intuition is that a central bank must anyway set a publicly visible interest rate, so complete opaqueness of the sort envisioned by M&S is technically impossible (the flute player must play the flute). Thus the central bank must release *some* information, the interest rate, and it faces two decisions: how to set the interest rate and whether to *additionally* reveal some or all of its own information? As it sets the interest rate, and is well aware that this will serve as a signal that can be over-interpreted because it is common knowledge, the central bank may use the interest rate to mitigate the undesirable effect of the signal.

In addition, we allow for some fog when we let the precision of the signals received by the central bank and by the private sector be unknown. As a consequence, the signal extraction efforts of the private sector may not only unduly respond to the central bank, they may also be distorted by a mistaken appraisal of the precision of the signals. This is why, the central bank can dissipate the fog in which it is caught by releasing its own forecast of the private sector precision. Even if this forecast is erroneous, it dissipates the fog.

The conclusions depend very much on what is known about the precision of private and central bank information. If the precision is known, the flute-player effect is upheld but the central bank can optimally set the interest rate to mitigate the common knowledge effect; the flute player here is a shrewd optimizer. In this case, as a first approximation, the desirability of releasing the signals depend as in M&S on the quality of the central bank information and on the reactivity of the private sector to each other's expectations. If, however, the precision is unknown, the interest rate is not as fully informative anymore. It becomes a noisy signal of the central bank information. The release of additional information by the central bank - actually all its information - now generally improves the situation. The reason is that the information eliminates the mistake that the private sector makes when interpreting the observed interest rate. This example vindicates those who, like Blinder (1998), argue that the central bank ought to be fully transparent precisely to prevent mistaken inferences of its actions by the private sector.

The difference between this paper and M&S carries an intriguing interpretation. Their central bank is like the Federal Reserve before 1993, when it was not publishing the interest rate. Our central bank, like any modern central bank, immediately indicates its interest rate decisions. While the old Fed may have found it desirable to be fully opaque, as Morris and Shin suggest that it may be desirable when the central bank information is poor, we find that, in general, the new Fed may raise welfare by being transparent.

The paper suffers from several limitations that should be kept in mind before drawing firm policy conclusions. To start with, the interest rate plays no macroeconomic role in our model. Its only function is to convey some information about the central bank signals. This explains why we find that, in some instances, the central bank should considerably raise the variance of the interest rate, as if the flute player would be hiding to avoid being followed. This is quite unrealistic and requires an extension, but it succeeds in allowing us to isolate the information content of the interest rate and to see how common knowledge of the interest rate affects the results of M&S. In addition, we treat as exogenous the variances of the forecast errors. Still, we believe that our paper shows that the conclusions of M&S are not robust to reasonable extensions of their papers.

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Appendix

Derivation of (7)

Iterating expectations as in (4), we have:

$$p_i = (1-r)\sum r^k E^i\left(\bar{E}^k(A\theta)\right) = AM\frac{1-r}{1-rM}\begin{bmatrix} R \\ x_1^i \\ x_2^i \end{bmatrix}$$

where:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\alpha_1}{\mu} & \beta_1 & -\frac{\nu\alpha_1}{\mu} \\ \frac{\alpha_2}{\nu} & -\frac{\mu\alpha_2}{\nu} & \beta_2 \end{bmatrix}$$

Note that:

$$\bar{E}\begin{bmatrix} R \\ \theta_1 \\ \theta_2 \end{bmatrix} = M\begin{bmatrix} R \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

Using:

$$1-rM = \begin{bmatrix} 1-r & 0 & 0 \\ -r\frac{\alpha_1}{\mu} & 1-r\beta_1 & r\frac{\nu\alpha_1}{\mu} \\ -r\frac{\alpha_2}{\nu} & r\frac{\mu\alpha_2}{\nu} & 1-r\beta_2 \end{bmatrix}$$

we obtain:

$$(1-r)(1-rM)^{-1} = \frac{1}{\Delta}\begin{bmatrix} \Delta & \left(r\frac{\alpha_1}{\mu}\right)(1-r\beta_2) - r^2\frac{\alpha_1\alpha_2}{\mu} & r\frac{\alpha_2}{\nu}(1-r\beta_1) - r^2\frac{\alpha_1\alpha_2}{\nu} \\ 0 & (1-r)(1-r\beta_2) & -(1-r)\,r\frac{\mu\alpha_2}{\nu} \\ 0 & -(1-r)\,r\frac{\nu\alpha_1}{\mu} & (1-r)(1-r\beta_1) \end{bmatrix}^t$$

with:

$$\Delta = \left[(1-r\beta_1)(1-r\beta_2) - r^2\alpha_2\alpha_1\right]$$

As a consequence:

$$
\begin{aligned}
& M\frac{1-r}{1-rM} \\
= \quad & \frac{1}{\Delta}\begin{bmatrix} 1 & 0 & 0 \\ \frac{\alpha_1}{\mu} & \beta_1 & -\frac{\nu\alpha_1}{\mu} \\ \frac{\alpha_2}{\nu} & -\frac{\mu\alpha_2}{\nu} & \beta_2 \end{bmatrix} \times \\
& \times \begin{bmatrix} \Delta & 0 & 0 \\ \left(r\frac{\alpha_1}{\mu}\right)(1-r\beta_2) - r^2\frac{\alpha_1\alpha_2}{\mu} & (1-r)(1-r\beta_2) & -(1-r)\,r\frac{\nu\alpha_1}{\mu} \\ r\frac{\alpha_2}{\nu}(1-r\beta_1) - r^2\frac{\alpha_1\alpha_2}{\nu} & -(1-r)\,r\frac{\mu\alpha_2}{\nu} & (1-r)(1-r\beta_1) \end{bmatrix} \\
= \quad & \begin{bmatrix} 1 & 0 & 0 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{bmatrix}
\end{aligned}
$$

with :

$$X_1 = \frac{\alpha_1}{\mu} + \frac{\beta_1}{\Delta}\left[\left(r\frac{\alpha_1}{\mu}\right)(1-r\beta_2) - r^2\frac{\alpha_1\alpha_2}{\mu}\right] - \frac{\alpha_1}{\Delta\mu}\left[r\alpha_2(1-r\beta_1) - r^2\alpha_1\alpha_2\right]$$

$$X_2 = \frac{\alpha_2}{\nu} - \frac{\alpha_2}{\nu\Delta}\left[(r\alpha_1)(1-r\beta_2) - r^2\alpha_1\alpha_2\right] + \frac{\beta_2}{\Delta}\left[r\frac{\alpha_2}{\nu}(1-r\beta_1) - r^2\frac{\alpha_1\alpha_2}{\nu}\right]$$

$$Y_1 = \frac{\beta_1}{\Delta}(1-r)(1-r\beta_2) + (1-r)\,r\frac{\alpha_1}{\Delta}\alpha_2$$

$$Y_2 = -\frac{\mu\alpha_2}{\nu\Delta}(1-r)(1-r\beta_2) - \frac{\beta_2}{\Delta}(1-r)\,r\frac{\mu\alpha_2}{\nu}$$

$$Z_1 = -\frac{\beta_1}{\Delta}(1-r)\,r\frac{\nu\alpha_1}{\mu} - \frac{\nu\alpha_1}{\mu}(1-r)(1-r\beta_1)$$

$$Z_2 = \frac{\beta_2}{\Delta}(1-r)(1-r\beta_1) + (1-r)\,r\frac{\alpha_1}{\Delta}\alpha_2$$

Multiplying on the right hand-side by $\begin{bmatrix} R \\ x_1^i \\ x_2^i \end{bmatrix}$ and on the left hand-side by A leads directly, after some simplifications, to (7).

Derivation of (17)

Combining (9) and (14), we have:

$$
\begin{aligned}
L_1(\delta) - L_2(\delta) = \frac{1+\delta}{\beta} - \left(\frac{\frac{\alpha}{\beta} + (1-r)^2}{\beta(\frac{\alpha}{\beta} + (1-r))^2} + \delta \frac{\delta\frac{\alpha}{\beta} + (1-r)^2}{\beta(\frac{\alpha}{\beta} + (1-r))^2} \right) \\
= \; -\frac{\alpha}{\beta} \frac{(\delta+1)\,2r - (\delta+1)\frac{\alpha}{\beta} - 2\delta - 1 + \delta^2}{\beta\left(\frac{\alpha}{\beta} + (1-r)\right)^2} \\
= \; \left(2\frac{\alpha}{\beta} \frac{1 - 2r + \frac{\alpha}{\beta}}{\beta\left(\frac{\alpha}{\beta} + (1-r)\right)^2} \right) + O(\delta - 1) < 0
\end{aligned}
$$

Derivation of results of Section 4.5.1

We start with the case $1 - 2r + \dfrac{\alpha}{\beta} > 0$. In order to determine the sign of (26), we first look at two limit cases. When $\dfrac{\alpha}{\beta} \to 0$, with the second order condition now $1 - 2r > 0$, $L_1(\delta, \lambda') - L_2(\delta) \to \dfrac{V}{2\beta} \frac{1-2r}{(1-r)^2} \dfrac{\alpha}{\beta}$ is of the same sign as $1 - 2r$. When $\dfrac{\alpha}{\beta} \to \infty$, $L_1(\delta, \lambda') - L_2(\delta) \to \frac{V}{2\beta} > 0$. Next note that the denominator of (26) is positive when $1 - 2r + \dfrac{\alpha}{\beta} > 0$. Denote N the numerator and let $W = \sqrt{\frac{V}{(\delta-1)^2}}$:

$$
\begin{aligned}
N &= (\delta-1)^2 \left[\frac{V}{(\delta-1)^2} \left(1 - 2r + \frac{\alpha}{\beta}\right)^2 \left(1 - r + \frac{\alpha}{\beta}\right)^2 - r^2 (\frac{\alpha}{\beta})^2 \right] \\
&= (\delta-1)^2 \left[W \left(1 - 2r + \frac{\alpha}{\beta}\right) \left(1 - r + \frac{\alpha}{\beta}\right) + r(\frac{\alpha}{\beta}) \right] \\
&\quad \times \left[W \left(1 - 2r + \frac{\alpha}{\beta}\right) \left(1 - r + \frac{\alpha}{\beta}\right) - r(\frac{\alpha}{\beta}) \right]
\end{aligned}
$$

Since the first factor in N is always positive, the sign of N is given by the sign of:

$$
W \left(1 - 2r + \frac{\alpha}{\beta}\right) \left(1 - r + \frac{\alpha}{\beta}\right) - r(\frac{\alpha}{\beta})
$$

This expression has two roots in $\dfrac{\alpha}{\beta}$. Since $\dfrac{\alpha}{\beta} > 0$ we are interested in positive roots only. The discriminant of this expression is $-4Wr + W^2r^2 + 6Wr^2 + r^2$.

It is positive for $r > 4\frac{W}{W^2+6W+1}$. Since $4\frac{W}{W^2+6W+1} \leq \frac{1}{2}$, a sufficient condition for this to be the case is $1-2r<0$, there is one positive root :

$$a_3 = \frac{1}{2W}\left(-2W + 3Wr + r + \sqrt{(-4Wr + W^2r^2 + 6Wr^2 + r^2)}\right)$$

We know that when $1-2r<0$, $L_1(\delta,\lambda') - L_2(\delta) < 0$ when $\dfrac{\alpha}{\beta} \to 0$ and $L_1(\delta,\lambda') - L_2(\delta) > 0$ when $\dfrac{\alpha}{\beta} \to \infty$. This establishes the result reported in Section 4.5.1.

When $1-2r>0$, we know that $L_1(\delta,\lambda') - L_2(\delta) > 0$ when $\dfrac{\alpha}{\beta}$ goes to zero and infinity.

- If $r > \frac{2W}{3W+1}$, (only possible if $W<1$) there are two positive roots

$$\begin{aligned} a_1 &= \frac{1}{2W}\left(-2W + 3Wr + r - \sqrt{(-4Wr + W^2r^2 + 6Wr^2 + r^2)}\right) \\ a_2 &= \frac{1}{2W}\left(-2W + 3Wr + r + \sqrt{(-4Wr + W^2r^2 + 6Wr^2 + r^2)}\right) \end{aligned}$$

and the limit cases determine the sign of $L_1(\delta,\lambda') - L_2(\delta)$.

- If $r < \frac{2W}{3W+1}$, there is no positive root. (This case is only possible if : $4\frac{W}{W^2+6W+1} < \frac{2W}{3W+1}$ that is $W>1$.) The limit cases imply that $L_1(\delta,\lambda') - L_2(\delta) > 0\ \forall\dfrac{\alpha}{\beta}$.

Consider now the case $1 - 2r + \dfrac{\alpha}{\beta} < 0$.

$$L_1(\delta,\lambda') = L_1(\delta,\lambda) + \frac{\alpha\left(2r-1-\dfrac{\alpha}{\beta}\right)}{2\left(\alpha+\beta(1-r)\right)^2} E\left(\lambda' + \frac{1}{\delta}\right)^2$$

so:

$$\begin{aligned} L_1(\delta,\lambda') - L_2(\delta) &= L_1(\delta,\lambda) - L_2(\delta) + \frac{\alpha\left(2r-1-\dfrac{\alpha}{\beta}\right)}{2\left(\alpha+\beta(1-r)\right)^2} V \\ &\simeq \frac{\alpha}{\beta}\frac{\left(2r-1-\dfrac{\alpha}{\beta}\right)}{2\beta\left(\frac{\alpha}{\beta}+(1-r)\right)^2}(V-4) \end{aligned}$$

Derivation of (28)

Considering the case $\lambda \approx \tilde{\lambda} \approx \lambda' \approx 1$ the Taylor expansion of (27) is:

$$L_1\left(\delta,\tilde{\lambda}\right)-L_1\left(\delta,\lambda'\right)=E\left[\left(\lambda'-\lambda\right)^2-\left(\tilde{\lambda}-\lambda\right)^2\right]\frac{2\left(\lambda^2-3\right)\left(\alpha^2+\alpha\beta\left(1-2r\right)\right)\lambda}{\beta\left(\beta\left(1-r\right)+\alpha\right)^2\left(1+\lambda^2\right)^3}$$

Computing the expectations yields:

$$\begin{aligned}
&\tilde{L}_1\left(\delta,\tilde{\lambda},\lambda'\right)-L_1\left(\delta,\lambda'\right)\\
=\;& E\frac{\left(\tilde{\lambda}-\lambda'\right)^2}{\left(\tilde{\lambda}(1+\lambda')\right)^2}\left(\frac{2}{\alpha}+\left(\theta_1-\theta_2\right)^2\right)\frac{\alpha^2\tilde{\lambda}^2(\delta\tilde{\lambda}+1)^2}{\left((1-r)\left(1+\tilde{\lambda}^2\right)\tilde{\beta}+\left(1+\tilde{\lambda}^2\delta\right)\alpha\right)^2}\\
&+E\left[\left(\lambda'-\lambda\right)^2-\left(\tilde{\lambda}-\lambda\right)^2\right]\frac{2\left(\lambda^2-3\right)\left(\alpha^2+\alpha\beta\left(1-2r\right)\right)\lambda}{\beta\left(\beta\left(1-r\right)+\alpha\right)^2\left(1+\lambda^2\right)^3}
\end{aligned}$$

At the lowest order in $\lambda-\lambda'$ we have:

$$\begin{aligned}
&\tilde{L}_1\left(\delta,\tilde{\lambda},\lambda'\right)-L_1\left(\delta,\lambda'\right)\\
=\;&\frac{1}{4}E\left(\tilde{\lambda}-\lambda'\right)^2\left(\frac{2}{\alpha_\varepsilon}+\left(\theta_1-\theta_2\right)^2\right)\frac{\alpha^2}{\left(\tilde{\beta}\left(1-r\right)+\alpha\right)^2}\\
&-\frac{1}{2}E\left[\left(\lambda'-\lambda\right)^2-\left(\tilde{\lambda}-\lambda\right)^2\right]\frac{\left(\alpha+\beta\left(1-2r\right)\right)}{\left(\beta\left(1-r\right)+\alpha\right)^2}\\
=\;&\frac{1}{4}V_2\left(\frac{2}{\alpha}+\left(\theta_1-\theta_2\right)^2\right)\frac{\alpha^2}{\left(\tilde{\beta}\left(1-r\right)+\alpha\right)^2}+\frac{1}{2}V_2\frac{\left(\alpha+\beta\left(1-2r\right)\right)}{\left(\beta\left(1-r\right)+\alpha\right)^2}
\end{aligned}$$

Derivation of results in Section 5

The second order condition is satisfied When the central bank is partially transparent, the unconditional expectation of the loss is:

$$\begin{aligned}
L_1(\lambda',\hat{\beta},\hat{\delta},\delta)\;=\;& E\frac{\left(1+\lambda'^2\right)(\hat{\delta}\lambda'+1)^2\alpha\hat{\beta}+\delta\frac{\hat{\beta}}{\beta}\left((1-r)\left(1+\lambda'^2\right)\hat{\beta}+(1-\lambda')\alpha\right)^2}{\hat{\beta}\left((1-r)\left(1+\lambda'^2\right)\hat{\beta}+\left(1+\lambda'^2\hat{\delta}\right)\alpha\right)^2}\\
&+\frac{\frac{\hat{\beta}}{\beta}\left((1-r)\left(1+\lambda'^2\right)\hat{\beta}+(\lambda'-1)\lambda'\hat{\delta}\alpha\right)^2}{\hat{\beta}\left((1-r)\left(1+\lambda'^2\right)\hat{\beta}+\left(1+\lambda'^2\hat{\delta}\right)\alpha\right)^2}
\end{aligned}$$

So:

$$L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) = L_1(\lambda', \delta) + B$$

with :

$$B = 2\alpha \frac{(1-r)^2 (2\beta r + \beta + \alpha)}{\beta (\alpha + (1-r)\beta)^4} E\left(\hat{\beta} - \beta\right)^2 - \frac{1}{2}\alpha\beta \frac{(1-r)(3\alpha r + \beta r - \beta - \alpha)}{(\alpha + (1-r)\beta)^4} E\left(\hat{\delta} - \delta\right)^2$$

When the central bank is not transparent, the unconditional expectation of the loss is:

$$\begin{aligned} \tilde{L}_1(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) &= \tilde{L}_1(\tilde{\lambda}, \delta) + A \\ &= L_1(\tilde{\lambda}, \delta) + \text{Bias term} + A \end{aligned}$$

The loss decomposes into three terms where A.comes from the correction due to $\hat{\beta}, \hat{\delta}$. At our order of approximation $A = B$. $L_1(\tilde{\lambda}, \delta)$ and the bias term is similar to the one obtained in Section 4.

When the central bank is fully transparent, the unconditional expectation of the loss is:

$$L_2(\beta', \hat{\delta}, \delta) = E\left(\frac{\alpha + \frac{(\hat{\beta})^2}{\beta}(1-r)^2}{\left(\alpha + (1-r)\hat{\beta}\right)^2} + \frac{\left(\hat{\delta}\right)^2 \alpha + \delta \frac{(\hat{\beta})^2}{\beta}(1-r)^2}{\left(\hat{\delta}\alpha + (1-r)\hat{\beta}\right)^2}\right)$$

that can be written as:

$$L_2(\hat{\beta}, \hat{\delta}, \delta) = L_2(\delta) + C$$

with:

$$C = -\alpha\beta \frac{(1-r)(3\alpha r + \beta r - \alpha - \beta)}{(\alpha + (1-r)\beta)^4} E\left(\hat{\delta} - \delta\right)^2 + 2(1-r)^2 \alpha \frac{\beta + \alpha + 2\beta r}{(\alpha + (1-r)\beta)^4 \beta} E\left(\hat{\beta} - \beta\right)^2$$

Given the results from Section 4, noting that $A = B$, we have:

$$\begin{aligned} \tilde{L}_1(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) - L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) &= \tilde{L}_1(\tilde{\lambda}, \delta) + A - L_1(\lambda', \delta) - B \\ &= L_1(\tilde{\lambda}, \delta) - L_1(\lambda', \delta) + \text{Bias term } > 0 \end{aligned}$$

The other comparison is:

$$\begin{aligned} L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) - L_2(\hat{\beta}, \hat{\delta}, \delta) &= L_1(\lambda', \delta) - L_2(\delta) + B - C \\ & L_1(\lambda', \delta) - L_2(\delta) + \frac{1}{2}\alpha\beta \frac{(1-r)(3\alpha r + \beta r - \alpha - \beta)}{(\alpha + (1-r)\beta)^4} \left(\hat{\delta} - \delta\right)^2 \end{aligned}$$

Using our previous results about the losses $L_1(\lambda', \delta)$ and $L_2(\delta)$, we can study our limit cases:

When $\frac{\alpha}{\beta} \to 0$ $\quad L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) - L_2(\hat{\beta}, \hat{\delta}, \delta) = E(\lambda' - \lambda)^2 \frac{\alpha(1-2r)}{2(\beta(1-r))^2} - \frac{\alpha}{2((1-r)\beta)^2} E\left(\hat{\delta} - \delta\right)^2$

which can be positive or negative, depending on the relative magnitudes of $E(\lambda' - \lambda)^2$ and $E\left(\hat{\delta} - \delta\right)^2$.

$$\text{When } \frac{\alpha}{\beta} \to \infty \qquad L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) - L_2(\hat{\beta}, \hat{\delta}, \delta) \approx E(\lambda' - \lambda)^2 \frac{1}{2\beta} > 0$$

The second order condition is not satisfied When the optimal policy is $\lambda = -\frac{1}{\delta}$:

$$\begin{aligned} L_2(\hat{\beta}, \hat{\delta}, \delta) &= \frac{1+\delta}{\beta} + \left(\delta^2 + 1\right) \frac{(2r-1)\alpha}{\beta^2(-1+r)^2} + \frac{1}{\beta^2} \frac{\alpha}{(1-r)^2} E\left(\hat{\delta} - \delta\right)^2 \\ &\quad + \frac{1}{\beta^4} \alpha \left(\left(\delta^2 + 1\right)(1+2r)\right) \frac{E\left(\hat{\beta} - \beta\right)^2}{(1-r)^2} \\ &\approx L_2(\delta) + \frac{1}{\beta^2} \frac{\alpha}{(1-r)^2} E\left(\hat{\delta} - \delta\right)^2 + \frac{1}{\beta^4} 2\alpha (1+2r) \frac{E\left(\hat{\beta} - \beta\right)^2}{(1-r)^2} > L_2(\delta) \end{aligned}$$

for δ close to 1.

In Section 4 we have seen that $\tilde{L}_1\left(\delta, \tilde{\lambda}, \lambda'\right) - L_1(\delta, \lambda') > 0$. By the same arguments as above, we have:

$$\tilde{L}_1(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) - L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) > 0$$

Now computing $L_1(\lambda', \delta, \beta', \hat{\delta})$ as previously leads to:

$$L_1(\lambda', \hat{\beta}, \hat{\delta}, \delta) = \frac{1+\delta}{\beta} + \frac{1}{\beta^2} \alpha \,(\text{Variance terms})$$

where the variance terms are of order $E\left(\hat{\delta} - \delta\right)^2$, $E\left(\hat{\beta} - \beta\right)^2$, $E(\lambda' - \lambda)^2$. This formula yields the same result as in Section 4.