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Multinationals, intra-firm trade and FDI: a simple model

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Abstract

This paper models trade and FDI in a world consisting of two symmetric countries. Using a monopolistic competition model of international trade which includes positive trade costs and endogenous multinational firms, we introduce an intermediate good and allow firms to fragment production internationally. The result is that under certain conditions, identical countries engage in both intra-industry FDI and intra-industry, intra-firm trade. This result provides a theoretical explanation for a well-observed but little explained phenomenon in the overlap between the theory of international trade and the theory of multinational enterprises. Examination of welfare demonstrates that firms make location choices that happen to maximise consumer welfare.

1 Introduction

The motivation for this paper is a series of stylized facts about multinationals, trade and FDI that are interesting and often inadequately explained by most trade theory. First, multinationals are of considerable importance in international trade, with some two-thirds of trade being conducted by multinationals. Yet multinationals are largely absent from all but a small branch of trade theory - key papers that deal with multinationals are mentioned below. Secondly, there is considerable similarity between trade flows and patterns of FDI: most trade is between the advanced countries, and so is most FDI. And there appears to be both two-way trade and two-way FDI between pairs of countries at industry level. Yet the papers that do treat multinationals generally posit FDI (or rather, the ensuing sales from foreign subsidiaries) as an explicit alternative to trade. Finally, a styalised fact that has recently caught the attention of others, around a third of world trade is intra-firm trade. A refinement to this last statistic that particularly supports the model developed in this paper is that over 20% of US exports comprises of intra-firm sales of inputs from the American parent company to a foreign subsidiary; and almost 20% of US imports are sales from foreign parent companies to US-based subsidiaries.

Before expounding our model, we examine briefly previous papers that have sought to integrate multinationals into models of international trade, and which attempt to endogenise the location pattern of firms and their decisions to integrate across national borders, either horizontally or vertically.

Prior to the 1980s, the general equilibrium theory of international trade was not well equipped to deal with the rising importance of multinationals in general and to international trade in particular. Batra and Ramachandran (1980) is an isolated, early attempt to integrate multinationals into a perfectly-competitive Heckscher-Ohlin framework. As Markusen (1984) points out, "formal trade theory has largely failed to provide a rationale as to why these corporations exist at all".

There are a very small number of papers from the 1980s that take some steps forward in explaining the existence of multinationals. Helpman (1984) and Markusen (1984) are two papers that integrate multinational enterprises into a general equilibrium mode of international trade. Helpman (1984) develops a simple model to identify circumstances in which corporations find it profitable to establish subsidiaries abroad. Differences in relative factor endowments give rise to differences in factor prices which firms can exploit by shifting activities abroad. This is vertical FDI motivated by differences in factor prices. More innovatively, the contemporaneous paper by Markusen (1984) introduces firm-level economies of scale in the form of an intangible input that can be used simultaneously in more than one production facility. Under certain conditions, a multi-plant equilibrium with horizontal FDI emerges.

The next paper to make an innovative and incisive contribution was Brainard (1993). Using a partial equilibrium model with identical countries, she frames the firm's location decision choice in terms of scale versus proximity, to examine the circumstances under which multinational firms arise endogenously. She finds that, for plausible combinations of parameters, multiple production facilities are a firm's optimal choice.

Markusen and Venables (1998, 2000) contributed two important papers that allow multinationals to arise endogenously. Markusen and Venables (1998) uses a Cournot oligopoly model of the firm with homogeneous goods and positive trade costs, Markusen and Venables (2000) introduces trade costs to the Dixit-Stiglitz monopolistic competition model. In a two country, general equilibrium model, horizontal multinationals arise for selected combinations of endowments as an alternative means of supplying the export market. The result − that multinationals arise as an alternative method of supplying the export market - fits with the concerns of the business research community (see, for example, Chan Kim and Hwang (1992)) which habitually studies mode of supply to export markets from the firm's perspective. The theory seemed to imply that FDI (and the resulting sales from foreign affiliates) is a substitute for international trade.

The past few years have witnessed an upsurge in interest in multinational firms and their behaviour, producing a plethora of interesting papers.[1] This is partly the realisation of the importance of multinationals; and partly the fact that there are now some well-developed tools available to treat multinationals. Most of the papers, for example, Helpman et.al. (2003), continue to explain FDI as an alternative exporting.

Yet empirical investigation, such as Clausing (2000) and Blonigen (2001), has largely failed to find substitution between FDI and trade. Instead, a rather puzzling pattern of complementarity appears to be present in the data.[2]

There are two channels through which the FDI activities of a multinational may stimulate trade. The first is that production abroad may require imported inputs. These inputs can come either from within the firm that is responsible for the FDI; or from other firms from the source country (or from third countries) that are established suppliers to the main firm.

The second channel is that the benefits to the firm of proximity to market may extend beyond the savings in transport or trade costs on the good actually produced in the local market. The local manufacturing presence may lead to spillovers to other products made by the same firm, that in turn increase demand for the other (imported) products. Examples of spillovers that may stimulate

[1] See, for example, Grossman and Helpman (2002); Helpman, et.al (2003); Yeaple (2003); Hanson, et.al. (2004); and Antras and Helpman (2004).

[2] Brainard (1997) identifies two-way FDI between pairs of countries that parallels trade between the two countries.

trade in other products include shorter delivery times due to better organised local warehousing; the ease of adapting a product to the local market, for example the translation of instructions into other languages; and simply an increase in brand acceptability that may come from having a local manufacturing presence.

My own intuition is that firms engaging in FDI tend to rely on inputs sourced from their own country. This could be ongoing, long-term relationships with suppliers, or the import of components from within the firm. Either way, it is plausible that a new, green-field venture abroad stimulates additional trade in components. Whether or not this is a net increase in trade depends on the volume of components required by the new plant, and on the volume of imports that may no longer be required as they have been displaced by the locally-produced articles. This in turn will depend partly on whether a firm is able to produce locally at an overall cost that is lower than that of the imported article, and on the responsiveness of demand to any price change. In a global situation characterised by growing aggregate demand and falling costs of operating abroad, it is plausible to observe simultaneously an increase in FDI and an increase in trade.

The aim of this paper is to create a model in which FDI can occur in combination with international trade. Motivated by the three stylised facts specified above that are largely absent from trade models, this paper develops a model with identical countries that allows international trade and FDI to occur as complements. The specific contribution of this paper is to refine the proximity verses scale trade-off that was first noted by Brainard (1993).

The remainder of this paper is organised as follows. Section 2 presents the model. Section 3 analyses the conditions for the various types of instantaneous equilibria that can occur. Section 4 considers how the equilibria will be influenced by changes in key parameters. Section 5 examines the volumes of trade and FDI, and welfare. Section 6 concludes.

2 The Model

The model has two symmetric countries, Home (h) and Foreign (f). Each country is endowed with the same amount of the single factor, Labour (L). There are two final goods sectors, X and Z, and an intermediate good, C. Z is a homogeneous good, and X is a differentiated good. The intermediate good C is used in the production of the final good X.

2.1 Consumer behaviour

Preferences of the representative consumer in each country are modelled as a Cobb-Douglas nest of Z consumption and a CES composite of X-varieties. Specifically:

$$U = X^{\beta} Z^{(1-\beta)} \tag{1}$$

where:

$$X = \left[\sum_{i=1}^{n} \left(x_i^{1-\frac{1}{\sigma}} \right) \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \sigma > 1 \tag{2}$$

with the subscript i denoting the varieties of X.

Consumers maximise utility U subject to the budget constraint

$$E = P_z Z + \sum_{i=1}^{n} P_{xi} x_i$$

Utility maximisation implies that a fixed proportion β of consumption expenditure E is spent on goods from the X sector, and the rest is spent on Z. The demand function[3] for a typical variety of X is:

$$x_j = \frac{p_j^{-\sigma}}{\sum_{i=1}^{n} p_i^{1-\sigma}} \beta E$$

where the subscript j denotes a single variety of X . Due to the functional form of the utility function, the demand for a particular variety of X depends on the price of that variety of X and also on the price of all the other varieties of X available to the consumer in that market.

[3] See Appendix 1 for the derivation of the demand function.

2.2 Producer behaviour

Production of Z is characterised by constant returns to scale and perfect competition. It takes a one unit of L paid wages w, with a unit input coefficient of $a_z = 1$ We take $w = 1$ as the numeraire. Z is a homogeneous good and can be traded costlessly.

Production of good X comprises two stages: the first stage consists of the production of an intermediate good C; and the second stage, which we denote as A, involves further processing of the intermediate good. Think of C as a Component, such as the chassis or engine, used as an input in the production of the final good, X, such as a car. Think of A as the assembly of car parts to make the car.

The two stages of production are spatially separable, and each stage requires its own plant. We refer to these as the components factory and the assembly plant respectively. A firm establishing production in the X sector incurs three types of fixed costs: there is a firm-specific general cost, G; a fixed cost F_c associated with each components factory; and a fixed cost F_a associated with each assembly plant.

The component C is manufactured using a_c units of labour. It takes one unit of C, plus further processing which requires a_a units of labour, to produce one unit of X. Each firm in the X sector produces its own intermediate good C.[4] The marginal cost of producing a unit of X is therefore split between the marginal cost of producing the component and the marginal cost of assembly, and we define $a_x = a_c + a_a$.

The option to fragment the production of X gives rise to three types of firm endogenously determined within the model. First, there can be national firms, which take advantage of scale economies, producing all their output in a single location, and exporting the finished good X to the other country; we denote these firms as $n - types$. Secondly, there may be firms that concentrate the production of components in a single factory, thus take advantage of scale economies in the production of components; but establish a second assembly assembly plant in the foreign market and thus avoid trade costs associated with the export of the final product. Such firms are benefiting from the separability of the two stages of production, and we denote these firms as $v - types$. Finally there can be traditional horizontal multinational firms, which avoid trade costs completely by producing the whole product from start to finish in each market; we denote these firms as $m - type$.

The table below summarises the characteristics and costs of each type of firm.[5]

[4] Demand for each variety of C is thus determined by demand for the final good.

[5] This table was created in Word and saved as file "table of type of firm.doc" on the m-drive.

Characteristics of firm	**Type of firm**	**Short type**	**Trade**	**FDI**
Manufacturers all components in a single factory in Home; finishes them in an adjacent assembly plant; services the foreign market by shipping finished goods.	traditional **n**ational firm	*n-type*	Exports final good	None
Manufacturers all components in a single factory in Home; finishes those destined for the home market in an adjacent assembly plant; services the foreign market by shipping components and finishing these at a second assembly plant in Foreign.	***vertically fragmented multinational***	***v-type***	***Exports the intermediate good***	***Assembly Plant***
Manufacturers components in both countries; finishes them in separate assmbly plants, one in each country.	traditional **h**orizontal multinational	*m-type*	None	Assembly Plant and Components Factory

Figure 1:

The final good X can be traded; trade costs are of the iceberg type where t represents the additional fraction of X that must be shipped per unit so that one unit of X arrives. Thus for sales abroad, $1+t$ units of X must be produced and shipped for every unit of X that is sold in the foreign market. The good C can be moved across borders, although C is not traded between firms as each firm produces its own intermediate good.[6] The trade cost associated with shipping the component alone is modelled as αt, where $0 < \alpha < 1$.

It is the existence of $v-types$ of firm that is most interesting in terms of the area we are investigating, in that $v-type$ firms are engaged in both FDI and trade. The question we pose is, under what conditions do $v-type$ firms exist? Assuming profit-maximising firms, the answer to this question is that firms will choose to be $v-type$ when this is the choice that results in maximum profits.

We now consider the production decision of a typical firm. The production decision can be thought of as having two stages: in the first stage, firms decide in which market to locate its plant physically; in the second stage firms optimise

[6] We justify the strong assumption that the intermediate good is not traded betwen firms on two counts. First, the incidence of intra-firm trade is very high; and second, to introduce inter-firm trade into the model would cause unnecessary complications.

the price charged to consumers in each market. We solve this by working backwards, by working out first the price each type of firm would charge in each market; we then use these expressions for price to create expressions for operating profits by type of firm; finally we using the expressions for operating profits to inform the location decision.

2.2.1 The pricing decision

The first step is to calculate the prices of a typical variety sold in the home market and abroad for each type of firm. We solve producer j's profit maximisation problem: producer j's profit maximisation problem is:

$$Max \prod_{Pj_{h},Pj_{f}} = x_j^h P_j^h + x_j^f P_j^f - x_j^h c_j^h - x_j^f c_j^f - fixedcosts$$

where x_j^h and x_j^f are the quantities of variety j sold on the home and foreign markets; P_j^h and P_j^f are the price of variety j in the home and foreign markets respectively; c_j^h is the variable cost associated with producing goods for home consumption and is equal to $a_x w$; and c_j^f is the variable production costs for goods to be exported. Note that c_j^f will differ in accordance with the location of production, as will fixed costs. The expressions for c_j^f are $a_x w(1+t)$ for $n-type$ firms; $a_x w(1+\alpha t)$ for $v-type$ firms; and $a_x w$ for $m-type$ firms.[7]

Dropping the subscript j, the first-order conditions are:

$$\frac{\delta \Pi}{\delta P^h} = 0 \Longrightarrow x_p^h P^h + x^h = c^h x_p^h$$

and:

$$\frac{\delta \Pi}{\delta P^f} = 0 \Longrightarrow x_p^f P^f + x^f = c^f x_p$$

where x_p is the partial derivative of x_j with respect to p_j (i.e., $x_p = \frac{\delta x_j}{\delta p_j}$).

Divide through by x_p and replace the subscript j to obtain:

$$p_j + \frac{x_j}{\frac{\delta x_j}{\delta p_j}} = c_j$$

Now multiply top and bottom of the second term by p_j:

[7] The variable cost of production for $v-type$ firms is $a_c w + a_a w^*(1+\alpha t)$; if we assume $w = w^*$, this expression reduces to $a_x w(1+\alpha t)$. Similarly, the variable cost of production for $m-type$ firms is $a_x w^*$ which equals $a_x w$ when $w = w^*$.

$$p_j + p_j \frac{\frac{x_j}{P_j}}{\frac{\delta x_j}{\delta p_j}} = c_j$$

Elasticity of demand is defined $\varepsilon = \frac{\frac{\delta x_j}{x_j}}{\frac{\delta p_j}{P_j}}$ hence $\frac{\delta P_j . x_j}{\delta x_j . p_j} = \frac{-1}{\varepsilon}$. When $\varepsilon = \sigma$, factorising for P_j we can write:

$$p_j = \frac{c_j}{1 - \frac{1}{\sigma}}$$

where c represents marginal cost, which differs according to the country of production and country of sale.

The price of a single variety will be different in the different markets; the price in the export market differs with the location of production. Price can be expressed with the following formula:

$$p_{ij}^{y-type} = \left(\frac{1}{1 - \frac{1}{\sigma}} \right) c_{ij}^{y-type} \tag{3}$$

where i is the country of origin of the firm, j is the market in which the output is sold, and y is the type of firm. The various combinations of suffixes and subscripts provide a specific formula for the price by country of production, country of sale, and type of firm. Recalling our choice of numeraire ($w = 1$); noting that for symmetric countries $w = w^*$; and defining units such that $a_x = \frac{\sigma-1}{\sigma}$, we obtain:

$$P_{hh}^{all-types} = 1 = P_{hf}^{m-type}; \qquad P_{hf}^{n-type} = (1+t); \qquad P_{hf}^{v-type} = (1+\alpha t); \tag{4}$$

The price charged to foreign consumers varies in accordance with the location of production; when production is in H and finished goods are exported to F, varieties of X imported to F are more expensive than locally produced ones. Conversely, when goods from a foreign supplier are manufactured locally, they are available at the same price as local varieties. When components are imported for local assembly, the price is between the two.

2.2.2 The location decision

Given that the price that is charged to the consumer is different depending on where production is located, and that the fixed costs associated with production differs with location, we assume that firms pick the location configuration that maximises their profit. Thus the next step is to calculate the operating profit by type of sale. Operating profit is given by the expression:

$$op\Pi_{ij}^{y-type} = (p_{ij}^{y-type} - c_{ij}^{y-type}) * x_{ij} \tag{5}$$

Substituting the expressions for variable cost and demand from above , we obtain:

$$op\Pi_{ij}^{y-type} = \frac{p_{ij}^{1-\sigma}}{\sum_{i=1}^{N} P_i^{1-\sigma}} * \frac{\beta E}{\sigma} \tag{6}$$

Defining[8]

$$\Phi^n = (1+t)^{1-\sigma} = \Phi; \qquad \Phi^v = (1+\alpha t)^{1-\sigma} = \Phi^1; \tag{7}$$

we obtain expressions for operating profit by sales type and by type of firm.

The operating profit on sales produced and sold at home will be, for all types of firm:

$$op\Pi_{hh}^{all} = \frac{1}{\sum_{i=1}^{N} P_i^{1-\sigma}} * \frac{\beta E}{\sigma} \tag{8}$$

The operating profits on sales to the foreign market vary with the location of production, and can be expressed:

$$op\Pi_{hf}^{n} = \frac{\Phi}{\sum_{i=1}^{N} P_i^{1-\sigma}} * \frac{\beta E}{\sigma}; \quad op\Pi_{hf}^{m} = \frac{1}{\sum_{i=1}^{N} P_i^{1-\sigma}} * \frac{\beta E}{\sigma}; \quad op\Pi_{hf}^{v} = \frac{\Phi 1}{\sum_{i=1}^{N} P_i^{1-\sigma}} * \frac{\beta E}{\sigma} \tag{9}$$

Defining $\Delta = \sum_{i=1}^{N} P_i^{1-\sigma}$,we obtain the following expressions for total operating profits for each type of firm.[9]

$$op\Pi^n = \frac{\beta E}{\sigma} * \frac{(1+\Phi)}{\Delta}; \quad op\Pi^v = \frac{\beta E}{\sigma} * \frac{(1+\Phi 1)}{\Delta}; \qquad op\Pi^m = \frac{\beta E}{\sigma} * \frac{2}{\Delta}; \tag{10}$$

[8] I introduce the symbols Φ and $\Phi 1$ to facilitate comparison with other models that use the same symbol. Φ can be understood as the relative freeness (aka "phi-ness") of trade. It also simplifies the look of several lines of algebra, although the results and propositions in this paper related directly to t.

[9] Consider the sum in the denominator of the expressions above, ie. $\sum_{i=1}^{N} P_i^{1-\sigma}$. Note

that the number of firms will be different in each type of equliibrium, because N depends on fixed cost, and the fixed cost varies depending on the method of supplying the foreign market. Consider for a moment an expression for N. We obtain $N = \frac{\beta L}{\sigma F}$. Therefore note that $N_n \neq N_v \neq N_m$, because $F_n \neq F_v \neq F_m$. For possible future use, we define the number of firms producing in the H market and F maket respectively:

$N = n + m + v; \qquad N^* = n^* + m^* + v^*$

To recap, the table below provides a summary of operating profit and fixed costs by type of firm, where $\Delta = \sum_{i=1}^{N} P_i^{1-\sigma}$

Type of firm	Total operating profit (from sales in two markets)	Total fixed costs (define $F_c = F_a + D$; and $F = F_a$)
$n - type$	$\frac{(1+\Phi)}{\Delta} * \frac{\beta E}{\sigma}$	$2F + D + G$
$v - type$	$\frac{(1+\Phi 1)}{\Delta} * \frac{\beta E}{\sigma}$	$3F + D + G$
$m - type$	$\frac{2}{\Delta} * \frac{\beta E}{\sigma}$	$4F + 2D + G$

I use the above expressions for operating profits and fixed costs in section 4 to

characterise the conditions for each type of equilibrium. Before conducting a formal analysis of the various equilibria, I provide some intuition as to how the model works.

2.2.3 Intuition

Before conducting a formal analysis of the model to establish the various equilibria, I provide some intuition as to how the model works by means of a brief thought-experiment. Consider initially a situation where firms face a simple choice of either supplying the foreign market by shipping the finished product; or producing the finished good locally. There is no option to fragment production, and firms are either $n - type$ or $m - type$.

Begin by imagine trade costs are zero. For an individual firm, this means that there is no cost difference between supplying merchandise to the home market or to the foreign market. Think of this as two small adjacent countries: it may be that a Belgian firm can just as easily deliver to an address in Luxembourg as one in Belgium. In such case, there will never be an incentive for a firm to set up a second plant. In equilibrium, all firms are $n - types$. The export market will always be serviced by production from the home country, but as trade costs are zero,the price of the goods on the foreign market will be the same as that charged at home.

Now imagine that trade costs increase slightly from zero. Let us assume that a very small increase in trade costs will not be sufficient to induce firms to set up a second plant abroad: when trade costs are very low, this will not affect a firm's location decision, and all firms remain $n - types$. But as trade costs grow, the price charged to foreign consumers for imported good increases; foreign demand falls and operating profits are lower.

At some point, trade costs become high enough that it becomes optimal for a firm to incur extra expense of a second production facility abroad and become an $m - type$ firm. This switch occurs because the extra fixed costs due to the second plant are offset by the additional profits that come from the sales abroad, now that the price charged to foreign consumers is lower. This switch occurs at a certain level of trade costs, which I call t^{crit0}. The level of t^{crit0} depends primarily on the fixed costs.

To summarise, so long as trade costs are low enough, the $n-type$ equilibrium prevails. But if trade costs rise sufficiently, the $m - type$ equilibrium prevails. Essentially there is a trade-off between scale and proximity. The advantage of scale is that there is only one plant to sustain, whereas the benefit of proximity is that trade costs are avoided.

Next consider the location choice faced by firms from a starting position of very high trade costs. When trade costs are high enough to be be prohibitive, the only option to serve an export market will be to have a second manufacturing base abroad. The world will be populated by $m - type$ firms. Now imagine trade costs falling gradually. A small change will not alter location choices; but eventually trade costs will be sufficiently low that it will be advantageous to a firm to consolidate production in a single location and supply the foreign market by means of shipping. Again the switch occurs at a certain level of trade costs. It turns out that in the model developed in this paper, there is a single switching point between $n - type$ and $m - type$ firms, which I call t^{crit}.

We now turn to the more complex model, where fragmentation of production is possible. The X good is produced in two stages. Stage 1 is production of the component, which requires a components factory. Stage 2 is assembly, which requires an assembly plant. The components factory and assembly pant each entail a separate fixed cost. When trade costs are zero, an $n-type$ equilibrium will prevail as explained above, as there is no incentive for a firm to established second production units.

Now imagine rising trade costs. Assume for a moment (i) the fixed cost of the components factory is large relative to that of the assembly plant; and (ii) the cost of transporting the component alone is small relative to transporting the finished good. When trade costs reach a certain level, it will become optimal for a firm to set up a second assembly plant abroad. The firm becomes a $v - type$. As a $v - type$, the firm pays trade costs for shipping the component, plus the fixed cost of a second assembly plant. We call this level of trade costs t^{crit1}.

Continue to imagine a steady increase in trade costs. Assume the firm remains a $v-type$ at least for small increases from t^{crit1}. But when trade costs are very high, it becomes optimal for the firm to invest in a second components factory, thus avoiding trade costs altogether. The firm becomes an $m-type$ at a certain level of trade costs, which we call t^{crit2}.

We can think of t^{crit1} and t^{crit2} as "switching points" from $n-type$ to $v-type$ and from $v - type$ to $m - type$ respectively. It turns out that these "switching points" between are each uniquely defined, whether we imagine starting from one type or the other. The levels $t^{crit1}$and t^{crit2} depend on various key parameters in the model. Most importantly, they depend on the relative importance of the fixed costs associated with setting up the second plants; and the proportion of trade cost associated with shipping the component alone.

Finally, relax the assumptions that the components factory is relatively expensive and that trade costs for the component alone are relatively low. Start again by imagining zero trade costs that increase. For low trade costs, firms are $n - types$. As trade costs rise, at some point it becomes optimal for firms to deviate from the $n - type$. But whether it is optimal for the firm to becomes a $v - type$ or an $m - type$ depends critically on the interplay between the parameters. If the fixed cost of the components factors is not sufficiently high relative to the cost of the assembly plant, or if the cost of transporting the component alone is not small enough relative to the cost of transporting the finished good, it is possible to envisage that the firm will switch directly from $n - type$ to $m - type$ as trade costs rise, and that there is no parameter space in which $v - type$ firms are the optimal choice. In a later section we explore the conditions under which there are sets of parameters which can support $v - type$ firms as we imagine trade costs rising. The existence of $v - type$ firms depends on the relationship between the other parameters.

To complete the thought-experiment , consider briefly the binary choice from the extreme perspective of the other key parameter, fixed costs. Imagine for a moment that the fixed costs of establishing a second plant are zero. Zero fixed costs for a plant are not unreasonable if we consider that all costs of production are related to volume, and hence are included in marginal costs. [10] When fixed costs are zero, a firm will always opt to establish a second plant in the export market, and the world is populated by $m - type$ firms. Imagine now fixed cost increasing a little. A small increase in fixed costs would not, we assume, change the equilibrium. But if we imagine fixed costs increasing steadily, then once fixed costs reach at a certain level, a firm would find it beneficial to take advantage of the economies of scale that accrue from producing in a single location.

2.2.4 Summary of firm types

1. Produce the final product in its own country, and sell at home and export abroad. This is an $n - type$ firm, with production only in its home country. In this equilibrium, there is international trade but no multinational firms.

2. Produce the intermediate good at home, assemble in 2 places; this is a vertically integrated firm, or $v - type$, in that it is different stages of production

[10] There is still an overhead, or fixed cost of being a firm, denoted as G in this model. Overheads must remain positive for the model to be defined.

that take place in different countries. When this equilibrium occurs, there will be both trade (in the intermediate input) and multinational firms.

3. Produce the intermediate good in 2 places, and assemble in 2 places. This is a horizontally integrated firm, or traditional multinational, an $m - type$. In this equilibrium, we have multinational firms but no international trade.

4. The fourth option, which I include here for completeness, but will not examine further for the time being, is to produce all abroad, re-import to home. Note that this will occur only if there are differences in factor prices or endowments; for the moment we consider only symmetrical countries, and hence ignore this possibility.

3 Instantaneous Equilibria

This section is set out as follows. First, I begin by assuming all firms are $n - types$, and characterise the $n - type$ equilibrium; then I ask, under what conditions would a firm want to deviate from that equilibrium to become a $v - type$ firm? I next ask the reverse question: if all firms are $v - types$, under what conditions would a firm want deviate to become a $n - type$? Combining the answers to these two questions allows me to derive an expression that defines t^{crit1}. Secondly I assume all firms are $m - types$, and characterise the $m - type$ equilibrium. Then I ask, under what conditions would a firm want to deviate from that equilibrium to become a $v - type$ firm. Then I ask the reverse question: if all firms are if all firms are $v - types$, under what conditions would a firm want deviate to become an $m - type$? These questions allow me to derive an expression for t^{crit2}.

Thirdly, we show that for focal-case parameter values, $t^{crit1}$is less than t^{crit2} and establish a condition on α that defines precisely when $t^{crit1} < t^{crit2}$. In the final part of this section, I establish the conditions under which a $v - type$ equilibrium prevails; specifically we show that a $v - type$ equilibrium can occur for intermediate values of t i.e., when $t^{crit1} < t < t^{crit2}$, provided $t^{crit1} < t^{crit2}$

$n - type$ **equilibrium** I now characterise the conditions for the various types of equilibria that may occur. To begin, imagine an equilibrium in which all firms are $n - types$. An $n - type$ firm concentrates production of final good X in its home country. In this equilibrium, there is international trade but no multinational firms. Assuming free entry, the number of firms will adjust endogenously to yield zero profits for each firm.[11] Hence the free entry condition for an equilibrium in which all firms are $n - types$ is:

$$\left(\frac{1}{\Delta} + \frac{\Phi}{\Delta} \right) \frac{\beta E}{\sigma} = 2F + D + G \tag{11}$$

this means that the operating profit made by an individual firm exactly covers its fixed costs.

Now we ask whether an $n - type$ equilibrium is sustainable, in the sense that if all firms are $n - types$, no single firm would want to become a $v - type$ when all others remained $n - types$. A firm will set up a second assembly plant abroad only if the increase in its fixed cost is more than compensated by the increase

[11] The free entry condition assumes that the number of firms N adjusts until each firm earns zero profits; we can therefore use the condition that, in equilibrium, operating profits equals fixed costs.

in variable profits. The increase in variable profits to the firm comes from the increase in sales to foreign consumers, which in turn results from a reduction in price charged to foreign consumers. Foreign consumers are charged a lower price by $v-type$ firms because assembly in the foreign market saves $(1-\alpha)$ of the trade costs, compared with shipping the finished product. The operating profits[12] of the deviant firm will be

$$\left(\frac{1}{\Delta}+\frac{\Phi 1}{\Delta}\right)\frac{\beta E}{\sigma} \tag{12}$$

whilst the fixed costs will be

$$3F+D+G \tag{13}$$

because there are now two assembly plants. Hence a firm would have an incentive to deviate from $n-type$ to $v-type$ if there are pure profits to be made from operating as a $v-type$ when everyone else is an $n-type$. Pure profits occur if:

$$\left(\frac{1}{\Delta}+\frac{\Phi 1}{\Delta}\right)\frac{\beta E}{\sigma}-3F+D+G\geq\frac{1}{\sigma}\left(\frac{1}{\Delta}+\frac{\Phi}{\Delta}\right)\frac{\beta E}{\sigma}-2F+D+G=0 \tag{14}$$

When the left-hand side of the above inequality is positive, this means there are pure profits to be made from switching from $n-type$ to $v-type$ when all other firms remain $n-type$. Rearranging the above, the test for deviation from $n-type$ to $v-type$ is that the following quantity is positive.[13]

$$\frac{1+\Phi 1}{1+\Phi}-\frac{3F+D+G}{2F+D+G} \tag{15}$$

Divide through by F, define $\delta=\frac{D}{F}$ and $\gamma=\frac{G}{F}$ and write out the full expressions for Φ and $\Phi 1$ to obtain the implicit equation

$$\frac{1+(1+\alpha t^{crit1})^{(1-\sigma)}}{1+(1+\tau^{crit1})^{(1-\sigma)}}-\frac{3+\delta+\gamma}{2+\delta+\gamma}=0 \tag{16}$$

which defines a critical value of t, such that when t is lower than t^{crit1}, all firms will be $n-types$. When trade costs are sufficiently low, it will not be

[12] When a firm changes from being an $n-type$ to a $v-type$, the price charged by that firm in the foreign market will also change. This will affect the denominator in the expressions above. The Δ's are not identical in the various equilibria. But if we consider a case in which only one firm deviates, the impact on the Δ's will be insignificant providing the number of varieties is sufficiently large, hence the expressions can be simplified as in {equation].

[13] To see this, first set the inequality to an equality, so both RHS and LHS of the inequality are zero. At this point a firm will be just indifferent between the two y types of firm. Hence $\left(\frac{1}{\Delta}+\frac{\Phi 1}{\Delta}\right)\frac{\beta E}{\sigma}=3F+D+G$ and $\left(\frac{1}{\Delta}+\frac{\Phi}{\Delta}\right)\frac{\beta E}{\sigma}=2F+D+G$. Now divide one equation by the other, the $\frac{\beta E}{\sigma}$ and Δ cancel (see footnote above), and we have $\frac{1+\Phi 1}{1+\Phi}=\frac{3F+D+G}{2F+D+G}$.

in a firm's interest to switch to being $v-type$. Equation 16 cannot be solved analytically due to the non-integer power, which occurs twice, but it can be solved graphically for for sample parameter values of α, δ, $\gamma$and σ, as in Figure 1 below.[14]

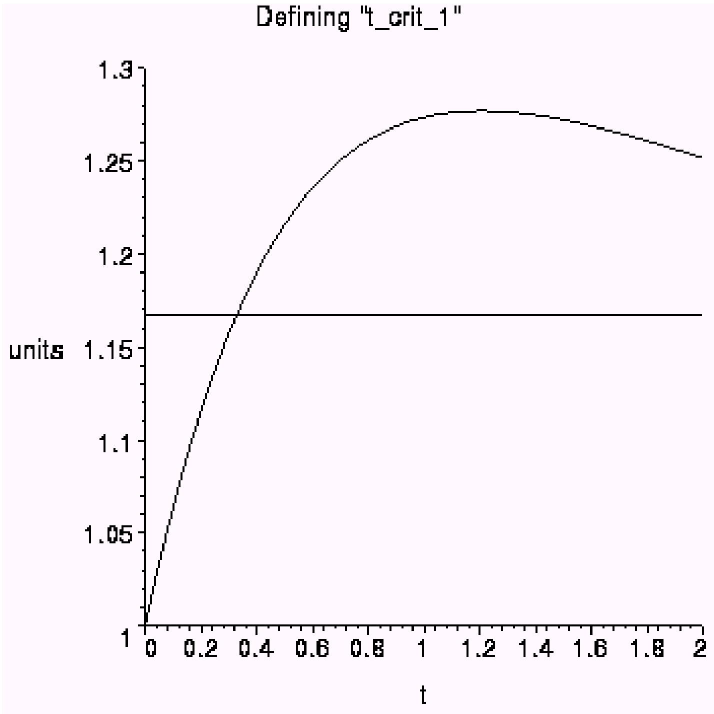

Figure 1

The crossing point of the curve that plots the first term and the line that plots the second term from the equation above illustrates t^{crit1} for the sample parameter values.[15]

[14] Figure 1 uses the following parameter values: $\alpha = 0.3, : \delta = 1$, $\gamma = 3$ and $\sigma = 3$.

[15] The fact that the curve is not monotonically increasing in t misleadingly suggests a second critical value of t. In fact the second crossing point is not relevant to this model. The intuition behind this second crossing point is that when there is a binary choice between $n-type$ and $v-type$ firms, at some rather high level of t the trade costs associated with shipping the component are so high that they seriously diminish the market share of the X_{hf}^{v-type} goods, that it is no longer worth having an assembly plant abroad. This crossing point is irrelevant in our model, as there is a three-way choice of firm type, and a anther level of trade costs, higher than t^{crit1} defined above, but lower than the second crossing point in Figure 1, firms reconfigure to become $m-types$, avoiding trade costs totally.. Note that when α is big, the turning point of the curve occurs at lower values of t. The implicit equation defining the turning point of t is :

$$\frac{1+(1+\alpha t^t)^{1-\sigma}}{1+(1+t^t)^{1-\sigma}} - \frac{\alpha(1+t^t)^\sigma}{(1+\alpha t^t)^\sigma}; \frac{dt^t}{d\alpha} < 0$$

To check that this is a unique value of tcrit1 separating the choice between $n - type$ and $v - type$ firms, consider now an equilibrium in which all firms are $v - types$. Under what condition would a single firm want to deviate from being a $v - type$ firm to an $n - type$, given that all other firms remain $v - types$. The intuition here is that when trade costs are low enough, the best option for the firm is to concentrate production in a single location and export to the foreign market. A firm would only wish to deviate from the $v - type$ equilibrium if there is an increase in profits that comes from the saving in fixed cost of the second assembly plant that more than offsets the reduction in revenue that results from having to charge a higher price to foreign consumers due to increased trade costs on the export of the finished product. This increase in profit occurs when the following quantity is positive:

$$\frac{1+\Phi}{1+\Phi 1} - \frac{2+\delta+\gamma}{3+\delta+\gamma}$$

Writing out the full expressions for Φ and $\Phi 1$ and rearranging, we obtain an implicit equation:

$$\frac{1+(1+\tau^{crit1})^{(1-\sigma)}}{1+(1+\alpha\tau^{crit1})^{(1-\sigma)}} - \frac{2+\delta+\gamma}{3+\delta+\gamma} = 0$$

which provides an identical definition of t^{crit1} as we obtained in 16 above.

To summarise, we can write:

Result 1: when $t < t^{crit1}$, all firms are $n - types$.

$m - type$ **equilibrium** Now imagine an equilibrium in which all firms are m-types which means all firms operate as horizontal multinationals, with both a components factory and an assembly plant abroad, supplying the foreign market with goods produced abroad. The free entry condition for an $m - type$ equilibrium is:

$$\frac{2}{\Delta} * \frac{\beta E}{\sigma} = 2(F+D)+G \tag{17}$$

We now consider, under what conditions would a single firm want to deviate

from being an $m - type$ firm to become a $v - type$ firm when all other firms remain $m - type$. Intuitively, for a firm to want to become $v - type$, the savings in fixed costs due to closing the components factory abroad must more than offset the reduction in variable profits that occurs: as the firm now pays trade

For high values of sigma, the implicit equation also yields a number of negative and imaginary roots, which are irrelevant to our analysis here.

costs on the components, and passes these costs onto the consumers, the higher consumer price for that variety of X is accompanied by a reduction in demand, and hence a fall in variable profits for the firm. The expression for a firm to want to deviate from $m - type$ to $v - type$ is:

$$\left(\frac{1}{\Delta} + \frac{\Phi 1}{\Delta}\right)\frac{\beta E}{\sigma} - (3F + D + G) \geq \frac{1}{\sigma}\left(\frac{2}{\Delta}\right)\frac{\beta E}{\sigma} - (4F + 2D + G) = 0 \quad (18)$$

Rearranging the above, the test for deviation from $m - type$ to $v - type$ is when the following quantity is positive:

$$\frac{1 + \Phi 1}{2} - \frac{3F + D + G}{4F + 2D + G} \quad (19)$$

We can rearrange this to provide an implicit equation that defines t^{crit2} :

$$\frac{1 + (1 + \alpha\tau^{crit2})^{(1-\sigma)}}{2} - \frac{3 + \delta + \gamma}{4 + 2\delta + \gamma} = 0 \quad (20)$$

For comparison with the figure defining t^{crit1}, we solve this equation graphically for sample parameter values of α, δ, $\gamma$and σ, as in Figure 2 below.

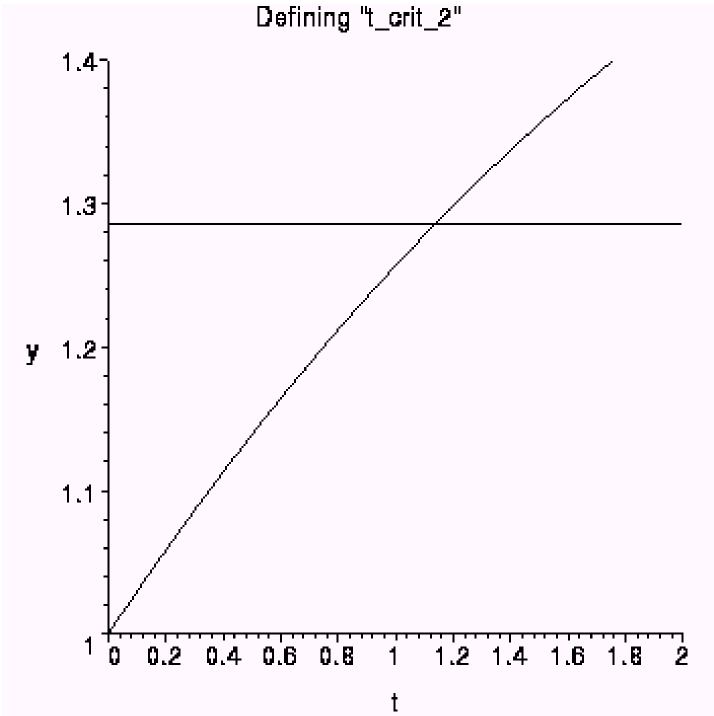

Figure 2: Defining t^{crit2}

The crossing point between the flat line and the curve defines the value of t^{crit2} that solves 20 sample parameter values.[16]

To check that this is a unique definition of t^{crit2} characterising the choice between $m-type$ to $v-type$ firms, imagine an equilibrium in which all firms are $v-types$, and consider under what conditions a single firm would want to deviate to become an $m-type$ whilst all others remain $v-types$. A firm would want to deviate from $v-type$ to $m-type$ if the increase in variable profits from increased foreign sales more than offsets the increase in fixed cost due to the second components plant. The relevant expression is:

$$\frac{1}{\sigma}\left(\frac{2}{\Delta}\right)\frac{\beta E}{\sigma}-(4F+2D+G)\geq\left(\frac{1}{\Delta}+\frac{\Phi 1}{\Delta}\right)\frac{\beta E}{\sigma}-(3F+D+G)=0 \quad (21)$$

Rearranging the above, the test for deviation from $v-type$ to $m-type$ is when the following quantity is positive:

$$\frac{2}{1+\Phi 1}-\frac{4F+2D+G}{3F+D+G}$$

We can rearrange this to provide an implicit equation that defines t^{crit2}.

$$\frac{2}{1+(1+\alpha\tau^{crit2})^{(1-\sigma)}}-\frac{4+2\delta+\gamma}{3+\delta+\gamma}=0 \quad (22)$$

which provides an identical definition of t^{crit2} as the equation above.

To summarise, we can write:

Result 2: when $t > t^{crit2}$, all firms are $m-types$.

[16] Figure 2 uses the following parameter values: $\alpha = 0.3, : \delta = 1$, $\gamma = 3$ and $\sigma = 3$. It is imported from maple file "fdimn_t_crit_1_and_2".

$v-type$ **equilibrium** A, and B are two equations that define t^{crit1} and t^{crit2} respectively. I will now show that these two equations define the parameter space, determining for which combinations of parameters we find $n-type$, $v-type$ and $m-type$ firms.

In Result 1 and Result 2 above, we have characterised the decision choice when t is less than t^{crit1} or greater than t^{crit2}. In demonstrating that these two critical values of t define single switching points between $n-type$ and $v-type$ and $m-type$ and $v-type$ respectively, we have also shown two other things. First, we have shown that if a firm is a $v-type$ and t is a little above t^{crit1}, the firm would choose to remain a $v-type$. Secondly, we have also shown that if a firm is a $v-type$ and t is a little below t^{crit2}, the firm would wish to remain a $v-type$. This suggests that when t lies between t^{crit1} and t^{crit2} the optimal choice for firms is $v-type$. We now need to demonstrate that in general, t^{crit1} is less than t^{crit2}, and so verify the existence of a parameter space in which the optimal choice of firm type is $v-type$. We can show this graphically for sample parameter values by plotting the four curves that are described in 16 and 20 above, for example as in Figure 3.

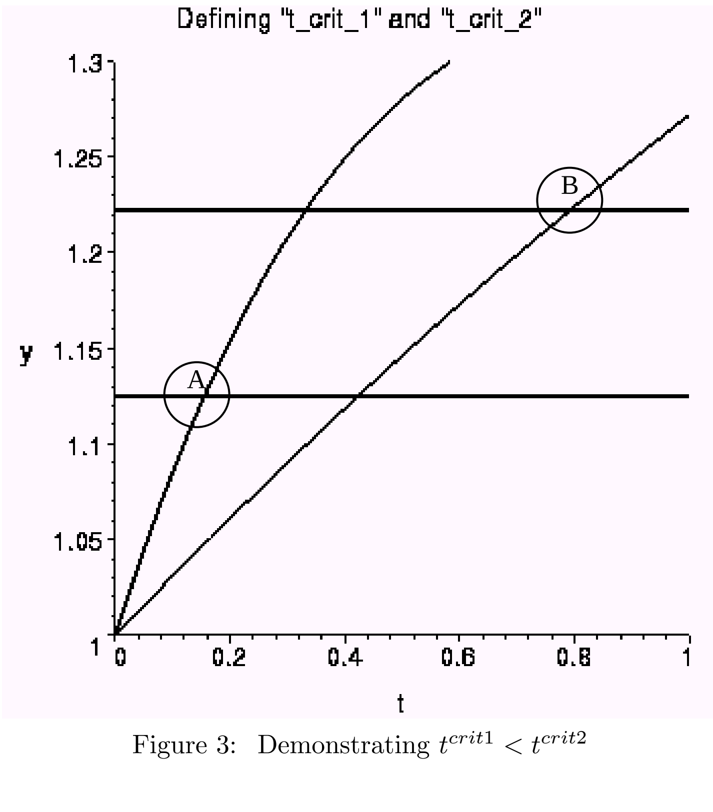

Figure 3: Demonstrating $t^{crit1} < t^{crit2}$

Point A indicates the value of t^{crit1} that solves 16 for the given sample parameters; point B indicates the value of t^{crit2} that solves 20 for the given sample parameters.[17] In the example illustrated by Figure 3, t^{crit1} is clearly less than t^{crit2}.

We can also show numerically, for a series of parameter values, that t^{crit1} is less than t^{crit2}. The table below shows the values of t^{crit1} and t^{crit2} for various combinations of α and σ.[18] In this table I report only the single economically meaningful positive root for each case, where this exists.[19]

t^{crit1}; t^{crit2}	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$	$\sigma = 3$	$\sigma = 4$
$\alpha = 0.1$	1.3 ; 22	0.48 ; 8.0	0.30 ; 4.7	0.21 ; 3.4	0.14 ; 2.2
$\alpha = 0.2$	1.9 ; 11	0.61 ; 4.0	0.36 ; 2.4	0.26 ; 1.7	0.16 ; 1.1
$\alpha = 0.3$	$*$	0.85 ; 2.7	0.47 ; 1.6	0.33 ; 1.1	0.20 ; 0.7
$\alpha = 0.4$	$*$	2.00 ; 2.0	0.70 ; 1.2	0.46 ; 0.8	0.27 ; 0.5
$\alpha = 0.5$	$*$	$*$	$*$	$*$	0.50 ; 0.7

Notice that when $\sigma = 2$ and $\alpha = 0.4$, we find that $t^{crit1} = t^{crit2} (= 2)$. When the two t^{crits} take on the same value, this means there is a value of t, below which $n-type$ firms are the most profitable type, and above which $m-type$ firms are preferred; there is no parameter space in which $v-type$ firms are preferred because α is too high.. The intuition here is that economies of scale are not sufficiently concentrated in components production to warrant the fragmentation of production, given the structure of trade costs and in particular the fraction of trade costs associated with transporting the component.

Next we establish analytically under what conditions $t^{crit1} < t^{crit2}$. Recall we have two implicit equations 16 and 20 that define t^{crit1} and t^{crit2} respectively. Rearranging 20 gives the direct equation:

$$t^{crit2} = \frac{\left(\frac{4+2\delta+\gamma}{2+\gamma}\right)^{\frac{1}{\sigma-1}} - 1}{\alpha} \tag{23}$$

Writing an additional condition $t^{crit1} = t^{crit2}$ and substituting the above expression for t^{crit2} into 16 , we obtain a condition for α :

$$\alpha = \frac{\left[\frac{4+2\delta+\gamma}{2+\gamma}\right]^{\frac{1}{\sigma-1}} - 1}{\left[\frac{4+2\delta+\gamma}{\gamma}\right]^{\frac{1}{\sigma-1}} - 1} \tag{24}$$

[17] The sample parameter value used to plot Figure 3 are $\alpha = 0.25, : \delta = 1$, $\gamma = 5$ and $\sigma = 3.5$. [Plotted in Maple with points added in Powerpoint.)

[18] The values of $t^{crit1}$and t^{crit2} shown in this table are calculated using $\delta = 1$ and $\gamma = 3$.

[19] $*$ indicates there are no real roots to the equation defining $t^{crit1}$for this combination of parameters

The above is a condition on α, such that when this condition holds, t^{crit1} and t^{crit2} are equalised.

Using the numerical results in the table above to guide the inequality, we can write:

Result 3: $t^{crit1} < t^{crit2}$ **iff:**

$$\alpha < \frac{\left[\frac{4+2\delta+\gamma}{2+\gamma}\right]^{\frac{1}{\sigma-1}} - 1}{\left[\frac{4+2\delta+\gamma}{\gamma}\right]^{\frac{1}{\sigma-1}} - 1}$$

Conversely, when α is equal to or greater than the above expression, there is no parameter space in which $v - type$ firms can exist.[20] The model reduces to the Markusen model with a straightforward choice between $n - type$ and $m - type$ firms. The intuition here is that when the trade costs associated with the components reach a certain level, it would never make sense to manufacture the component at home and complete the assembly abroad. This is because the economies of scale are not sufficiently concentrated in component production to justify fragmentation of the type described in section 2 above.

Combining the three results above, we have:

Result 4: when $t^{crit1} < t^{crit2}$ and $t^{crit1} < t < t^{crit2}$, all firms are v-types. This result demonstrates that there is a possibility that a $v - type$

equilibrium occurs at intermediate values of t .[21] The exact breadth of the intermediate values of t that will sustain the $v - type$ equilibrium depends on the remaining parameters, and we explore these below.

[20] The direction of the inequality is established by intuition and verified using numerical results.

[21] The parameters used to draw this figure are: $\sigma = 3.5$, $\gamma = 5$, and $\alpha = 0.35$.

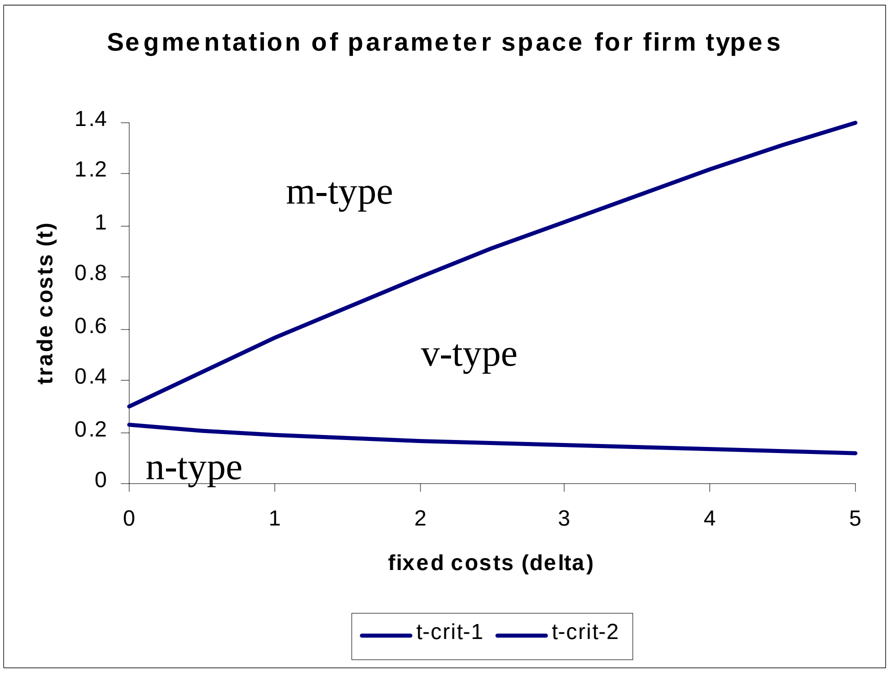

Figure 4: Segmentation of parameter space for firm types

To summarise the equilibrium analysis, we have the result that, depending on the parameter values, there are three possible types of equilibrium: all firms operate a single production facility, servicing both markets from their home base; all firms operate two production facilities,serving their home market from their home-based plant and their foreign market from their FDI facilities; or all firms have operate a single chassis production facility, which supplies the intermediate good to the both the home and foreign assembly plants.

For completeness and to facilitate comparison with other models, I now consider the case where the choices open to firms is either to export the finished product; or to produced the whole product in the export market. In this case there, following the logic from above, at some point a level of trade costs at which an $n - type$ firm would find it profitable to switch to being an $m - type$. We call this level of trade costs t^{crit0}.

The following indirect equation defines t^{crit0}, the level of trade costs at which firms switch from $n - type$ to $m - type$ (or vice versa) when there is a simple choice between two firm types and no option to fragment production.

$$\frac{2}{1 + (1 + t^{crit0})^{(1-\sigma)}} - \frac{4 + 2\delta + \gamma}{2 + \delta + \gamma} = 0 \tag{25}$$

4 Comparative Statics

In this section we consider the effect of key parameters α, δ, γ and σ on the critical values of t^{crit1} and t^{crit1}.

4.1 The influence of α

Recall that the parameter a represents the value of the component as a proportion of the finished product. Consider how a change in α would affect the critical value of t at which firms would want to establishing an assembly plant in the foreign market and thus "switch" from $n - type$ to $v - type$. Taking the total differential of 16, we obtain:

$$\frac{dt^{crit1}}{d\alpha} = \frac{t(t + 1 + (1 + t)^{\sigma})}{1 + (1 + \alpha t)^{\sigma} - \alpha(1 + (1 + t)^{\sigma})}$$

Signing this, we see the numerator is clearly positive, whilst the denominator is ambiguous. Specifically, the denominator is positive provided:

$$\frac{(1 - \alpha)}{\alpha} + \frac{(1 + \alpha t)^{\sigma}}{\alpha} > (1 + t)^{\sigma} \tag{26}$$

The first term in the above expression is the savings in trade costs when assembly is performed locally. The second term is a measure of the premium on the price of foreign goods charged to consumers when the foreign goods are assembled locally. The term on the right-hand side is a measure of the price premium on foreign goods that consumers are charged when the goods are imported. Numerical analysis demonstrates that the denominator is almost always positive provided neither σ nor t nor α are too large. So 26 will be positive provided both σ, t and α are sufficiently small.

We can summarise:

Result 5: As α becomes smaller, for plausible parameter values, the lower is the value of t that makes it worth setting up the assembly plant abroad.

We now consider the impact of α on t^{crit2}. Taking the total differential of 20 and rearranging we obtain:

$$\frac{dt^{crit2}}{d\alpha} = -\frac{t}{\alpha}$$

This expression is clearly negative, so a fall in α will result in a higher t^{crit2}. This gives us:

Result 6: when α is small, trade costs can be relatively high before firms switch from $v - type$ to $m - type$.

Combining Result 5 and Result 6 we have:

Proposition 1: a small α expands the range of parameters which sustain $v - type$ firms.

This proposition is illustrated in the pair of diagrams below.[22] In Figure 5a the range of trade costs over which (for a given d) firms are $v - types$ is narrow, compared with Figure 5b, where all parameters used in the plot are identical except that α is smaller.

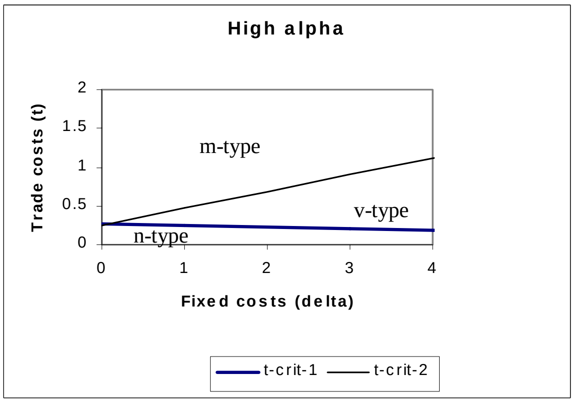

Figure 5a: Equilibrium firm types with high α

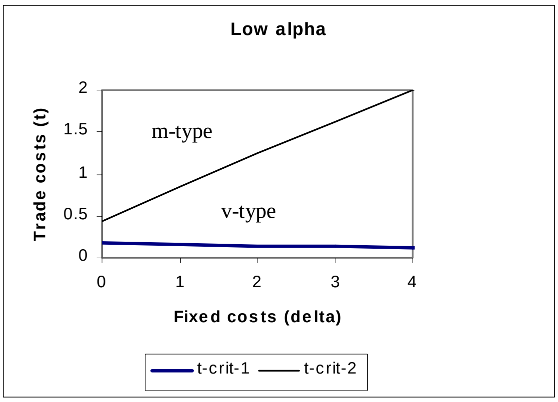

Figure 5b: Equilibrium firm types with low α

[22] Self-reminder in case change required: these diagrams are from excel spreadsheet "calculating t-critical" using numbers generated in maple worksheet "t_crit_numerical".

Another interesting thing about alpha is that when alpha is relatively large, there may be no v-type firms. For example Figure 6 illustrates a two-dimensional segmentation of the parameter space for sample parameters $\sigma = 2$, $\gamma = 3$, and $\alpha = 0.4$. In this case for relatively low values of δ, there is a binary choice between $n - type$ and $m - type$ firms, even though the technology allows fragmentation, and $v - type$ firms only exist for relatively high values of δ. The intuition here is that when delta is small the economies of scale are not more intense in the components production compared with assembly. From this we learn that fragmentation will only occur when the relative economy of scale for production of components compared with the cost of trading components is sufficiently large compared with economies of scale in assembly. Thus that when α is high, and δ is relatively low, there may be no parameter space in which $v - type$ firms exist.

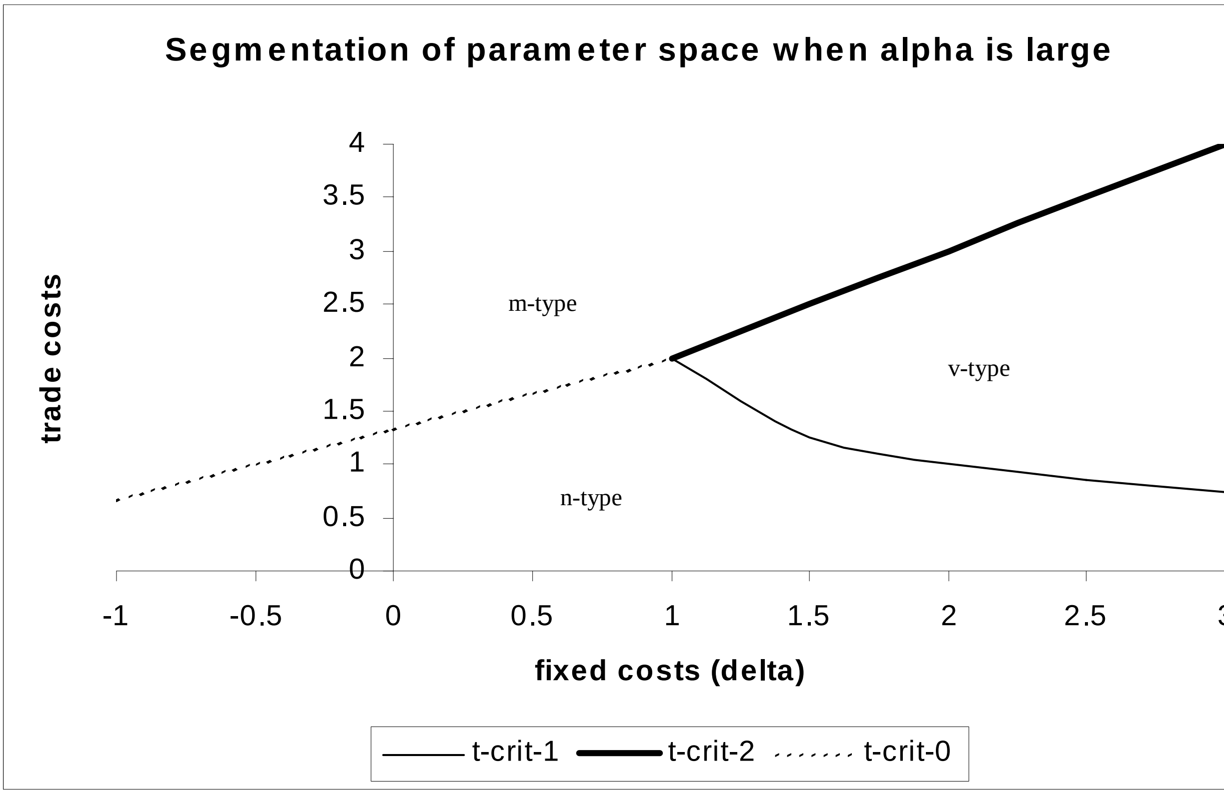

Figure 6

alpha is a fraction that represents the cost of transporting components compared with the cost of transporting the finished good. Hence $(1 - \alpha)$ is the saving in transport costs associated with fragmented production compared exporting the finished good.

4.2 The influence of δ

In this section we consider the influence of δ on the critical values of t. Recall δ is defined as $\frac{D}{F}$, where D is the difference between the fixed cost of components plant and the fixed cost of the assembly plant, and F is the fixed cost of the assembly plant. Hence $\delta = \frac{F_c - F_a}{F_a}$. We can interpret δ as a measure of the relative importance of the fixed costs associated with the importance of producing components, compared with the fixed cost of assemble. A high delta means that the components factory is much more expensive than an assembly plant. When $\delta = 0$, the fixed plant costs are split equally between components and assembly. If delta is negative, the assembly plant entails higher fixed costs than the components factory. Given that the fixed costs of the components factory cannot be negative, (i.e.. $F_c \geq 0$), the definition of δ creates a natural restriction on δ such that $\delta \geq -1$. Note that $\delta = 1$ when $F_c \geq 0$. When the two elements of production require same fixed investment, $D = 0$. When D is zero, the economies of scale is equal between the two stages of production. As a focal case we make a working assumption that economies of scale are greater in the production of the component compared with assembly. In such case, $\delta > 0$. When $\delta = 1$, the fixed costs of associated with producing the component are exactly double the fixed costs associated with assembly. A large delta indicates big economies of scale in component production relative to assembly.

Totally differentiating 16 and rearranging we obtain:

$$\frac{dt^{\underline{crit1}}}{d\delta} = \frac{(1 + (1+t)^{(1-\sigma)})^2(\sigma - 1)}{(2+\delta+\gamma)^2[\alpha(1+\alpha t)^{-\sigma}(1+(1+t)^{(1+\sigma)}) - (1+t)^{-\sigma}(1+(1+\alpha t)^{(1+\sigma)})]}$$

This expression is ambiguous and depends on:

$$\alpha(1+t)(1+t)^{\sigma} \lesseqgtr (1+\alpha t)(1+\alpha t)^{\sigma}$$

but for when numerical examples are examined it turns out that almost all plausible combinations of parameters, the expression is negative. Thus we have:

$$\frac{dt^{\underline{crit1}}}{d\delta} < 0$$

This means that when δ is high, the we expect a relatively low value for t^{crit1}. So we can write:

Result 7: a large δ LOWERS the level of trade costs at which it becomes beneficial for a firm to set up a second assembly plant.

We now examine the impact of δ on t^{crit2}. Totally differentiating 20 and rearranging we obtain:

$$\frac{dt^{\underline{crit2}}}{d\delta} = \frac{[1 + (1+\alpha t)^{-\sigma} + (1+\alpha t)^{-\sigma}\alpha t]^2(1+\alpha t)^{\sigma}(2+\gamma)}{2\alpha(\sigma-1)(3+\delta+\gamma)^2} > 0$$

This is clearly positive, so we can write:

Result 8: a large δ RAISES the level of trade costs at which it becomes beneficial for a firm to set up a second components factory.

Combining Results 7 and 8 we have:

Proposition 2: a large delta expands the range of trade costs over which we find $v - type$ firms.

To summarise, we find that smaller the α (i.e., the LESS the trade costs are concentrated in the component) and the greater the δ (i.e., the MORE the economies of scale are concentrated in the component), the greater is the parameter space for $v - type$ firms.

4.3 Other parameters (σ and γ)

When sigma is very small we expect an $n - type$ equilibrium, because for a small sigma the level of trade costs at which firms would want to deviate from an $n - type$ equilibrium is too high to be meaningful, regardless of the other parameter values. Here σ acts as a measure of the openness of the economy, with a low σ representing a relatively open economy.

As σ increases, the value of both t^{crit1} and t^{crit2} becomes lower, until t^{crit1} becomes low enough that a switch could be optimal. As σ increases further, the range of trade costs over which $v - types$ are found narrows. When sigma is small (less than 2) gamma must be high to get a meaningful result. As sigma approaches 1, the size of gamma necessary for the result to be meaningful becomes very high.

GAMMA - the derivative of the implicit equation defining t^{crit1} with respect to γ is the same as that with respect to δ. (i.e. ambiguous but almost always negative.) Thus a high value of γ implies a relatively low level of trade costs at which a firm would wish to switch from n-type to v-type. But the derivative with resepct to γ for the equation defining t^{crit2} has the opposite sign to that for δ. We find $\frac{dt^{crit2}}{d\gamma} < 0$. Thus a large gamma also reduces the value of trade costs at which a firm would want to switch from $v - type$ to $m - type$.

Note that γ must be positive for the model to be defined. A large γ implies that overheads are large, and intuitively the number of firms will be fewer.

5 Volumes of trade and FDI, number of varieties, total fixed costs and welfare

In this section I establish the volumes of trade and sales from foreign affiliates that occur for varying levels of trade costs; consider the number of varieties yielded by the solution to the model; examine total fixed costs; and determine the levels of welfare that prevail.

5.1 Volume of trade

Here I calculate the volume of trade. The volume of trade (VT) is defined as the sum of worldwide exports. Within the model, this translates into the world consumption of non-local varieties, (or rather, the imported fraction thereof), plus melted resources.[23] The consumption of non-local varieties is:

$$\sum_{i=1}^{N^*} x_i^{fh} + \sum_{i=1}^{N} x_i^{hf}$$

where $N \equiv n + v + m$ and n, v and m are respectively the number of $n - type$, $v - type$ and $m - type$ firms in H; foreign variables are denoted with an asterisk. To obtain the volume of imports, we multiply the consumption of each variety by the fraction of the finished good that is actually imported; and to obtain the volume of exports we multiply the whole lot by $1 + t$ to allow for the amount that "melts" during transit.

Section 3 above establishes that the instantaneous equilibria yields a single type of firm. We therefore need to specify the volume of imports that occurs in each type of equilibrium.

In the $n - type$ equilibrium 100% of the foreign variety consumed is imported. The volume of exports in the $n - type$ equilibrium is therefore:

$$VT^n = (1 + t) \left[n^* x_j^{fh^{n-type}} + n x_j^{hf^{n-type}} \right]$$

which after substituting relevant expressions and simplifying becomes:

$$VT^n = \frac{2\Phi}{1 + \Phi} \beta E$$

and can be simulated using sample parameter values.

[23] As the model specifies symmetric countries, there is no trade in the Z good to worry about.

Specifying volume of trade in the $v - type$ equilibrium is only slightly more tricky. Recall that in this case, only the component crosses an international border. In section 2 above we defined the value of the component as fraction α_1 of the value of the finished good x. This is slightly distinct from α, which is the fraction of trade costs associated with shipping the component alone. In theory it makes little difference whether α is equal to α_1 or not, but we wish to allow for the practical possibility of $\alpha < \alpha_1$, which means that shipping intermediate goods may be proportionally less expensive compared with shipping final goods. For simplicity, to simulate volumes of trade in the $v-type$ equilibrium we assume $\alpha_1 = \alpha$. Hence

$$VT^v = (1+t)\left[v^* \alpha x_j^{fh^{v-type}} + v\alpha x_j^{hf^{v-type}}\right]$$

which becomes:

$$VT^v = \frac{2\alpha(1+t)(1+\alpha t)^{-\sigma}}{1+\Phi 1}\beta E$$

and can be simulated using sample parameter values.

The simplest case to specify relates to the $m - type$ equilibrium, in which horizontal multinationals produce their whole product from start to finish in each market. Thus there is no trade and the fraction imported is zero: $VT^m = 0$.

Figure 7 depicts the volume of trade that would occur at various levels of trade costs for the three types of equilibria.

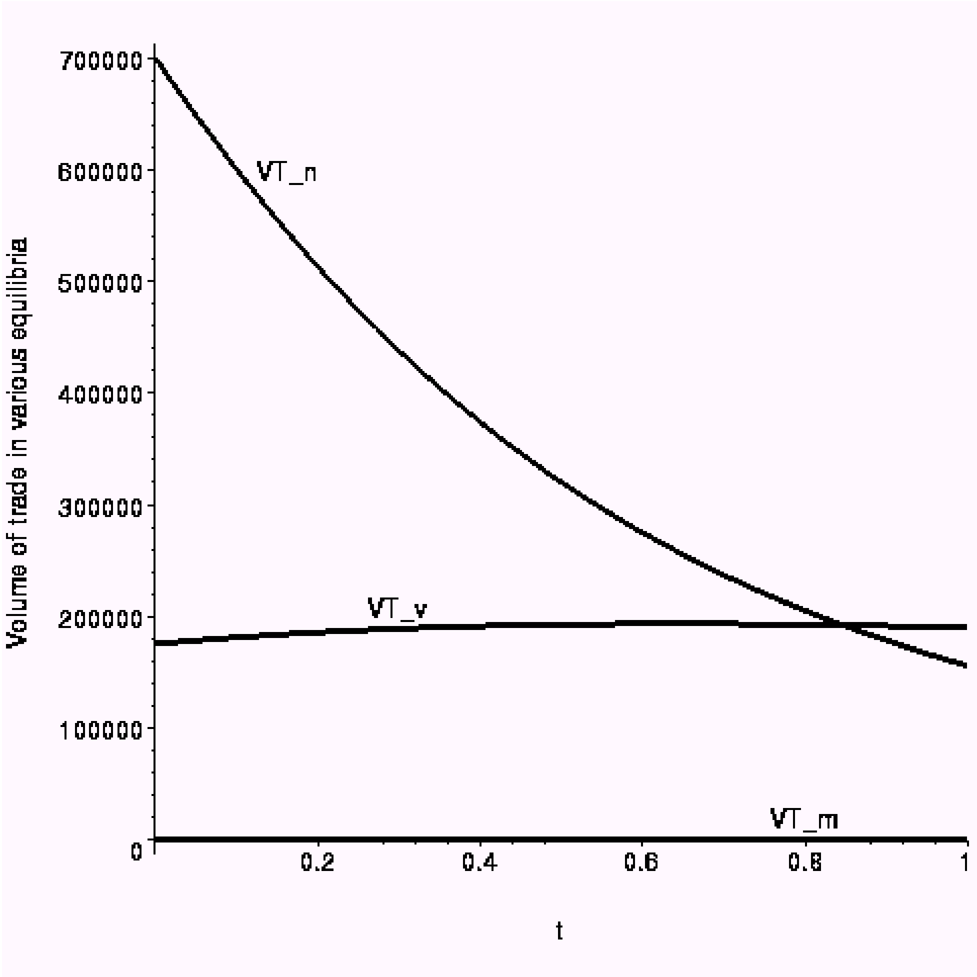

Figure 7: Volume of Trade

5.2 Sales from foreign affiliates

I define sales from foreign affiliates as the volume of consumption of goods of foreign origin that are produced locally. In inward investment fora, this is referred to as local value-added. In this section I derive expressions for the volumes of sales from foreign affiliates that occur in each of the three equilibria. Derivation of these expressions is straightforward following on from the section on volume of trade.

In the $n{-}type$ equilibrium, all foreign goods that are consumed are imported, and there are no sales from foreign affiliates. $(SFA^n = 0)$. In the $v - type$

equilibrium, the component is imported, but the assembly is performed locally. The volume of sales from foreign affiliates therefore equates to $(1-\alpha_1)$ of the volume of foreign goods consumed, which when $\alpha = \alpha_1$ is:

$$SFA^v = \frac{2(1-\alpha)(1+\alpha t)^{-\sigma}}{1+\Phi 1}\beta E$$

which is a declining function of t. In the $m - type$ equilibrium, consumer price of foreign varieties is the same as the consumer price of home varieties, and so consumers consume the same amount of each variety. Hence the volume of sales from foreign affiliates is given by $SFA^m = \beta E$.

Figure 5.2 uses the curves show in Figure 5.1 together with the expressions for sales from foreign affiliates and solutions to t^{crit1} and t^{crit2} for sample parameters of δ and γ to show how the volume of trade and sales from foreign affiliates changes as trade costs increase and the equilibrium moves from one type to another.

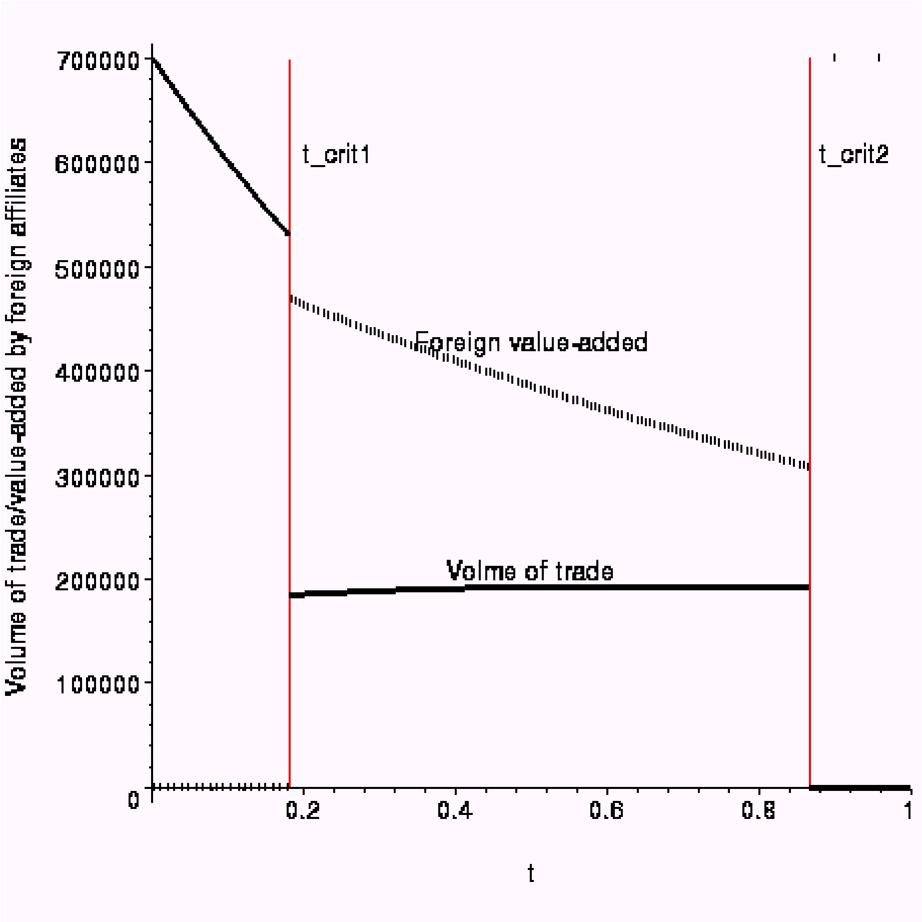

Figure 5.2: Volume of trade and foreign value-added

Figure 5.2 illustrates that, for plausible parameter values and for a wide range of trade costs, we anticipate trade in tandem with sales from foreign affiliates and FDI.

5.3 Number of varieties

In Dixit-Stiglitz models, the number of varieties is determined endogenously and is a function of per-firm fixed costs which we denote as F^{y-type}, where y specifies the type of firm. Using the free entry condition to equate fixed costs and operating profits, and recalling that operating profit equals revenue divided by sigma (refer equation above), for an individual firm we can write:

$$F^{y-type} = \frac{revenue}{\sigma}$$

In the single country Dixit-Stiglitz model, revenue per firm is total revenue for the $X - sector$ divide by the number for firms. In the two-country model, each firm receives revenue from two sources, $Home$ and $Foreign$. And in turn, $Home$ expenditure on X is split between varieties of X produced at $Home$me and varieties of X produced in $Foreign$. However, using the result that we have established in section 3 above, which is that in this model with symmetric countries, all firms are of a similar type and there are no mixed equilibria, all firms must be identical in scale. So revenue per firm equals $\frac{\beta E}{N}$ where N^y is the total number of firms of a single type in one economy. Substituting this expression for revenue into the above, we obtain:

$$N^y = \frac{\beta E}{\sigma F^{y-type}}$$

where F^{y-type} is the per-firm fixed cost for the type of firm that prevails in equilibrium. It follows that we have:

$$n = n^* = \frac{\beta E}{\sigma F^{n-type}} \tag{27}$$

$$v = v^* = \frac{\beta E}{\sigma F^{v-type}} \tag{28}$$

$$m = m^* = \frac{\beta E}{\sigma F^{m-type}} \tag{29}$$

Thus the number of firms in equilibrium will vary with the type of equilibrium that prevails. Given the breakdown of fixed costs and how these vary with type of firm (described in section 2 above), we know that $F^{n-type} < F^{v-type} < F^{m-type}$, so we can determine:

$$n > v > m$$

which means that the number of firms in an $n - txpe$ equilibrium will, ceteris parabus, be greater than the number of firms in a $v - type$ equilibrium, which in turn will be greater than the number of firms in an $m - type$ equilibrium. These expressions for the number of firms are used in the analysis of welfare in sub-section 5.5 below.

5.4 Total fixed costs

Total expenditure in one country on fixed costs is given by fixed costs per firm multiplied by the number of firms.[24] In this model the fixed cost per firm is

[24] For a particular equilbrium, we can wirte, $F^{total} = N^{y-type} * F^{y-type}$, which holds in a situation where all firms are the same type. In a two-country mixed- equilibrium the total fixed cost would be:
$TFC = (n + n^*)(2F + D + G) + (v + v^*)(3F + D + G) + (m + m^*)(4F + 2D + G)$

different for different types of firm. But as we have shown above, the number of firms per country is a function of the per-firm fixed cost.

$$N^{y-type} = \frac{\beta E}{\sigma F^{y-tpye}}$$

where F is the fixed cost per firm. So for a particular equilibrium we can write, $F^{total} = N^{y-type} * F^{y-type}$. Hence the total expenditure on fixed costs can be expressed:

$$F^{total} = \frac{\beta E}{\sigma}$$

Hence economy-wide expenditure on fixed costs is constant, and is independent of the type of equilibrium that prevails.

Thus it turns out that total fixed costs (TFC) as measured on an economy-wide or worldwide basis are constant for all levels of trade costs. This is an interesting result in itself, as it is not obvious that this would be the case in our model. The constancy of total fixed cost occurs despite the fact that, as trade costs rise, the type of firm that prevails in equilibrium changes, entailing different per-firm fixed costs. An intuitive understanding of constant fixed costs despite the changing equilibria rests on the idea that is the number of firms that adjusts endogenously to preserve total economy-wide expenditure on fixed costs. Specifically, when t is lower than t^{crit1}, and an $n-type$ equilibrium prevails, the equilibrium number of firms is higher than the number of firms in a $v-type$ equilibrium that occurs when t is a little higher than $t^{crit1.}$ Similarly, if we consider the change from a $v-type$ equilibrium to an $m-type$ equilibrium, (this happens in our model if t rises from just below t^{crit1} to just above t^{crit2}), the number of firms will decline further.

The per-firm fixed costs varies from one equilibrium to another. But in the shift to a different equilibrium it is the number of firms that adjusts and thus preserves the economy-wide fixed costs at a constant level.

5.5 Welfare

In this section I look at welfare. Welfare is most easily measured by evaluating the indirect utility function. The indirect utility function is obtained by substituting the expressions for quantities consumed into the utility function to obtain an expression for utility in terms of prices of all goods and expenditure. The indirect utility function is:

$$V = \left[\left[\sum_{i=1}^{N}\left(\left(\frac{p_j^{-\sigma}}{\sum_{i=1}^{n} p_i^{1-\sigma}}\beta E\right)^{1-\frac{1}{\sigma}}\right)\right]^{\frac{1}{1-\frac{1}{\sigma}}}\right]^{\beta} * \left[\frac{(1-\beta)E}{P_z}\right]^{(1-\beta)} \tag{30}$$

Recall that due to mill-pricing, the consumer prices of foreign-produced goods vary with location of production.

In the model expounded in this paper, we have established that there are no opportunities for mixed equilibria to occur, and that either an $n - type$, a $v - type$ or an $m - type$ equilibrium occurs. This simplifies the evaluation of welfare. Consumer prices of foreign-produced goods and a price index can be defined for the different types of equilibria.[25] Substituting these into the indirect utility function, together with the expressions for N derived above, we obtain expressions to evaluate welfare levels as a function of t and various parameters under each of the three types of equilibria, expressed in terms of pure parameters and exogenous variables.

Defining W^y to be the welfare achieved under a $y - type$ equilibrium, where $y = n$, v or m , and $B \equiv \left(\frac{(1-\beta)E}{P_z}\right)^{(1-\beta)}$ we have:

$$W^n = \left[\left[\frac{\beta E}{\sigma F^{n-type}}\left(\left(\frac{\sigma F^{n-type}}{1+\Phi}\right)^{1-\frac{1}{\sigma}} + \left(\frac{(1+t)^{-\sigma}\,\sigma F^{n-type}}{1+\Phi}\right)^{1-\frac{1}{\sigma}}\right)\right]^{\frac{1}{1-\frac{1}{\sigma}}}\right]^{\beta} * B$$

$$W^v = \left[\left[\frac{\beta E}{\sigma F^{v-type}}\left(\left(\frac{\sigma F^{v-type}}{1+\Phi 1}\right)^{1-\frac{1}{\sigma}} + \left(\frac{(1+\alpha t)^{-\sigma}\,\sigma F^{v-type}}{1+\Phi 1}\right)^{1-\frac{1}{\sigma}}\right)\right]^{\frac{1}{1-\frac{1}{\sigma}}}\right]^{\beta} * B$$

$$W^m = \left[\left[\frac{2\beta E}{\sigma F^{m-type}}\left(\left(\frac{\sigma F^{m-type}}{2}\right)^{1-\frac{1}{\sigma}}\right)\right]^{\frac{1}{1-\frac{1}{\sigma}}}\right]^{\beta} * B$$

Thus welfare in the $n - type$ and $v - type$ equilibria is a function of t, and depends also on the other exogenous variables such as income and the β from the Cobb-Douglas upper tier, and on parameters such as α, σ and per-firm fixed costs. Welfare in the $m - type$ equilibrium does not depend on t. Figure 8 below plots the three curves as functions of t.

[25] Prices for the different location configurations are calculated above as $P_{hh} = P_{fh}^{m-type} = 1$; $P_{fh}^{m-type} = 1 + t$; and $P_{fh}^{m-type} = 1 + \alpha t$.

Working on the denominator from the sub-utility function, the price index for the home country will be:

$\Delta = (n + v + m)(P_{hh})^{1-\sigma} + n^*(P_{fh})^{1-\sigma} + v^*(P_{fh})^{1-\sigma} + m^*(P_{fh})^{1-\sigma}$

In a symmetric equilibrium, $n = n^*$, $v = v^*$ and $m = m^*$. Expressions for price are calculated in [reference] above. Thus:

$\Delta^n = n(1 + \Phi)$, $\Delta^v = v(1 + \Phi 1)$ and $\Delta^m = 2m$.

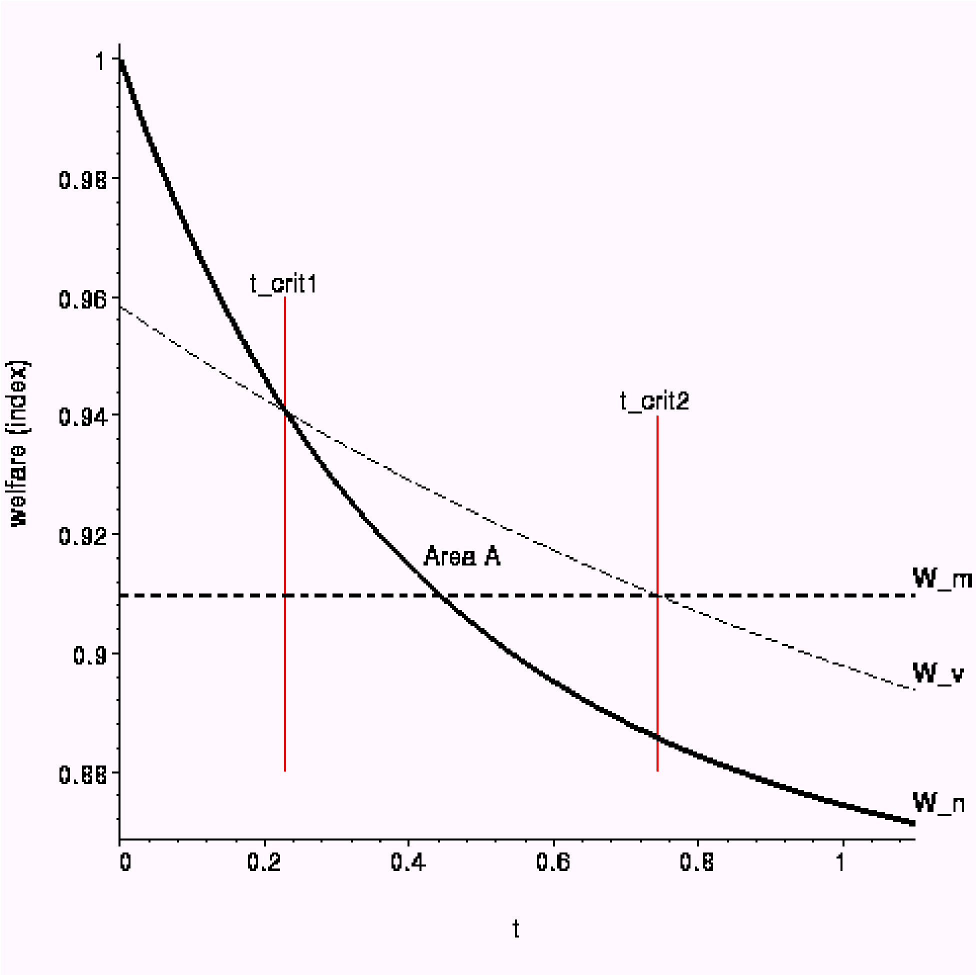

Figure 8: Welfare-maximising location decisions

The graph shows that, when trade costs are low, welfare is highest under the $n - type$ equilibrium. Then as trade costs rise to an intermediate level, welfare is maximised by a $v - type$ equilibrium. And when trade costs are high, the highest level of welfare is achieved by an $m - type$ equilibrium.

It turns out that the crossing points of the welfare contours occur at $t = t^{crit1}$ and $t = t^{cri2}$. The implications of this are most interesting: Figure 5.3 shows that the maximum welfare achievable when $t < t^{crit1}$ occurs for an $n - type$ equilibrium; that the maximum welfare achievable when $t^{crit1} < t < t^{crit2}$ occurs for a $v - type$ equilibrium; and that the maximum welfare achievable when $t > t^{crit2}$ occurs for an $m - type$ equilibrium. This leads to:

Proposition 3:
The location choices made by profit-maximising firms leads to a welfare-maximising outcome from the consumers' perspective.

The intuition is that welfare is highest when trade costs are very low and an $n - type$ equilibrium prevails. As trade costs rise, welfare falls because the amount of resources that "melts" during shipping increases, thus less of the imported varieties are consumed. Then, as trade costs continue rising, at some point there is a switch to the $v - type$ equilibrium. When this switch is made, the number of firms adjusts downwards and welfare is reduced as the number of varieties falls. Welfare continues falling, albeit at a less steep rate, as trade costs continue rising, until there is a final switch to an $m - type$ equilibrium. Again the number of firms adjusts downwards, thus reducing welfare through the loss of variety.

The figure also demonstrates an increase in welfare due to fragmentation. The vertical height of the central triangle, labeled "Area A", shows the gain in welfare effected by fragmentation, compared with a situation where technology were to force a straight choice between $n - type$ and $m - type$ firms.[26]

6 Conclusion

This paper models trade and FDI in a world consisting of two symmetric countries. Using a monopolistic competition model of international trade which includes positive trade costs and endogenous multinational firms, we introduce an intermediate good and allow firms to fragment production internationally. The result is that under certain conditions, identical countries engage in both intra-industry FDI and intra-industry, intra-firm trade. This result provides a theoretical explanation for a well-observed but little explained phenomenon in the overlap between the theory of international trade and the theory of multinational enterprises. Examination of welfare demonstrates that firms make location choices that happen to maximise consumer welfare.

[26] By contrast, in the case where we have an alpha sufficently large as to preclude a $v - type$ equilibrium, this is illustrated on the welfare map as the W^{v-type} curve lying below either or both W^{n-tpye} and W^{m-type} for all levels of trade costs.

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