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Abstract  
Improved consumption risk sharing is one of the fundamental predicted benefits of increased financial integration, yet the empirical evidence concerning this proposition is mixed. Using the novel empirical technique of wavelet analysis, this paper for the first time in the literature uncovers the heterogeneous evolution of consumption and output correlations over the time and frequency dimensions simultaneously. Periods of strong comovement in consumption growth rates not only occur during times of common ( uninsurable) shocks to output, but also to some extent during times of increased financial integration. This evidence adds a new dimension to the consumption output correlation puzzle, which appears to only hold at certain time periods and frequencies.

Keywords: Consumption Output Correlation Puzzle, International Consumption Risk Sharing, Wavelet Analysis  
JEL Classification: F36, F41

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1 Introduction

It is widely known that improved international consumption risk sharing, that is the ability of agents to diversify their portfolios and hence insure themselves against idiosyncratic country specific shocks, is one of the fundamental predicted benefits of increased financial integration. Yet, whereas elementary theory tells us that financial integration should yield benefits in terms of decreased country-specific consumption risk, by most empirical measures there is at best mixed evidence to support this hypothesis. Many explanations have been offered for this apparent lack of risk sharing, provided on one hand by theoretical studies which emphasise for example imperfections in goods and asset markets, to empirical works which stress factors such as home bias in bond and equity holdings (see below for a fuller overview of the literature). However, a more recent branch of the literature, which also tests the risk sharing proposition empirically, has suggested that risk sharing may exist at a more subtle level than previously thought. That is, recent works have analysed whether risk sharing occurs at different frequencies (see Pakko 2004) and the degree to which financial integration has affected consumption risk sharing over time (see Flood, Marion and Matsumoto 2010).

In this vein, the present study, through employing the novel empirical technique of wavelet analysis - a time varying spectral approach more common in astrophysics, meteorology and neuroscience, yet also of use in the economic context and explained in detail below - extends this previous work and asks how risk sharing has evolved over time and frequency dimensions simultaneously? Several dimensions of this question are examined. Firstly, how do the properties of consumption and output correlations vary over time and frequency? Secondly, does this evidence shed new light on the 'consumption-output correlation puzzle'? Finally, does the time varying spectral evidence concerning international consumption risk sharing bear any relation to periods of de jure and de facto international financial integration?

In answering these questions, we emphasise several points. Firstly, the comovement of national consumption and output growth correlations with their respective rest of the world counterparts varies over both time and frequency dimensions, highlighting the importance of using a time varying spectral approach to this question. Secondly, analysis of the consumption growth rate correlations, together with those for output growth, reveals that much of the comovement between national and rest of the world consumption aggregates is attributable to common output shocks between countries. Thirdly, the consumption output correlation puzzle is of relevance only at certain points in time and at certain frequencies. Finally, evidence of international consumption risk sharing - a high consumption correlation accompanied by a low output correlation - is found at varying times and frequencies, corresponding to some extent with periods of increased international financial integration.

Thus the major contribution of this paper is to apply the novel empirical technique of wavelet analysis to uncover heterogeneous international consumption risk sharing experiences at different frequencies and over time simultaneously, and to analyse how these results correspond with periods of financial integration. In terms of the literature concerning international consumption risk sharing, it relates to and extends more recent studies which suggest that agents may share risk differently at different horizons (Pakko 2004, Flood, Marion and Matsumoto 2010) and that international capital flows are a nontrivial source of international risk sharing (Bracke and Schmitz 2008, Imbs and Fratzscher 2007). Furthermore at the methodological level, it highlights the usefulness of applying the empirical technique of wavelet analysis to economic questions in order to explore the potentially rich dynamics of empirical phenomena.

Thus the analysis below is organised as follows. Section II reviews the theoretical and empirical literature concerning consumption risk sharing. Since the present study uses a somewhat non traditional measure of variable comovement, this section also reviews the application of non traditional empirical methods to diverse set of economic questions. Section III introduces and discusses the novel empirical technique of wavelet analysis used in this study to analyse the evolution of international consumption risk sharing over the time and spectral dimensions simultaneously. Section IV presents and discusses the empirical results of the study, contrasting the outcomes of a time series versus wavelet based analysis of consumption growth correlations and the implications of international financial integration. Section V concludes the analysis and develops avenues for further research.
2 Review of the Literature - Financial Integration and Consumption Risk Sharing

There exists a vast literature concerning the benefits of financial integration in terms of the extent of consumption risk sharing. Below both the theoretical and empirical literature concerning financial integration and consumption risk sharing is reviewed.

2.1 Theoretical Studies

The implications of financial liberalisation for consumption risk sharing have been well documented. Simple theory tells us the following. Let an individual maximise the following objective function:

\[ E_o \sum_{s=1}^{\infty} \beta^s u(C_i^s, \theta_i^s) = E_o \sum_{s=1}^{\infty} \beta^s \left[ \frac{(C_i^s)^{1-\gamma}}{1-\gamma} \exp \left( \frac{\theta_i^s}{\gamma} \right) \right] \]

(1)

where \( \theta_i^s \) represents all factors, such as preference shocks or leisure, that affect individual \( i \)'s utility and discount factors are constant across all agents. If there is optimal risk sharing\(^1\), then this implies that

\[ \ln C_i^t - \ln C_j^t = k^{ij} + \frac{\theta_i^t}{\gamma} - \frac{\theta_j^t}{\gamma} \]

(2)

where \( k^{ij} \) and \( k^{ij} = -\frac{1}{\gamma} \ln k^{ij} \) are constants that depend upon initial wealth or the weight a social planner attaches to the utility of the agent in a particular country. Therefore if we assume that the \( \theta_i^t \) are constant, then equation (2) implies that the consumption ratio between two countries is constant over time under perfect risk sharing. Furthermore, it follows that consumption growth rates are equalised under perfect risk sharing (which holds for any country pair and for any one country relative to the rest of the world). Therefore, under perfect risk sharing, a country’s per capita consumption is a fixed share of average world per capital consumption (and the variance of the latter is thus zero as well).

Theoretical studies have however offered explanations for the observed lack of international consumption risk sharing (see below for a review of the empirical evidence concerning the lack of risk sharing). Some have emphasised the role of market imperfections and incompleteness as an explanation, for example Blank (2009) extends the framework of Ghironi and Melitz (2005) to a setting with impediments to international trade in goods and financial markets. In this setup, both of these sorts of imperfections are sources of deviations from perfect risk sharing across countries. However, Pakko (1997) notes that in a two-country, two-good endowment model with certain conditions on substitution elasticities, the complete markets framework outlined above can be associated with low consumption correlations. Others have concentrated on the role of non-tradable and durable goods or the presence of transaction costs (see Kose et. al. 2007 for an overview of these studies). Moreover, Kose et. al (2007) note that the theoretical prediction of a consumption correlation of unity depends upon on the form of the utility function - if consumption was the only argument in the utility function in a complete markets model, then the consumption correlation would be unity. If another argument was included in the utility function, the cross-country consumption correlation would be less than unity, but if would still be very high if markets were complete.

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\(^1\) Achieved if there is a full set of Arrow – Debreu securities between an agent in country \( i \) and one in country \( j \).
2.2 Empirical Literature

2.2.1 Studies Concerning International Consumption Risk Sharing

Empirically speaking, a wide variety of studies have examined the hypothesis that financial liberalisation entails benefits in terms of consumption risk sharing. Following the seminal papers of Backus, Kehoe and Kydland (1992), Obstfeld (1994, 1995), Tesar and Stockman (1995), Lewis (1996), who discovered and deeply explored the well known ‘consumption - output correlation puzzle’, the literature extended the study of the relationship between consumption and income in two main directions.

Firstly, correlation ($\rho$) measures were established by computing correlation coefficients of cross – country consumption aggregates measured either in levels or in growth rates. If consumption risks are insured perfectly, then the ratio of individual country consumption to world consumption is constant and the correlation coefficient is constant. A rejection of perfect risk sharing occurs when the correlation coefficient between two country’s growth rates turns out to be significantly different from unity. Studies that fall into this category are those of for example Canova and Ravn (1997) and Heathcote and Perri (2003). Almost all of these studies find individual country consumption to be poorly correlated with world consumption growth. A related strand of the literature notes the ‘consumption - output correlation puzzle’, that is the observed tendency for cross - country output correlations to be higher than cross - country consumption correlations (in a simple endowment setting, with complete Arrow - Debreu asset markets, consumption correlations should be perfect across countries, regardless of the consumption - output correlation). Whilst a raft of studies exist in this area (see for example Backus et al 1992, Baxter 1995, Maffezzoli 2000, Baxter and Farr 2001, Heathcote and Perri 2002 and Kehoe and Perri 2002 for surveys of this literature), a recent example is that of Pakko (2004), who examines the spectral properties of consumption (and output) correlations. This study finds that low consumption correlations are prevalent in the business cycle range (a finding which is robust to alternative filtering methodologies) for a range of OECD countries, and also notes it is ‘curious’ that there exists more evidence of risk sharing at higher frequencies than within the business cycle range.

The second category of tests, regression ($\beta$) measures, undertake a regression of consumption growth rates on idiosyncratic output growth or other variables such as world consumption growth rates. An example of this style of equation from Obstfeld (1994, 1995) is given by:

$$\Delta \ln C_{it} = \beta_0 + \beta_1 \Delta \ln W_{it} + \beta_2 \Delta \ln GDP_{it} + \varepsilon_{it}$$

In the above equation, under perfect risk sharing, $\beta_1 = 1$ and $\beta_0 = \beta_2 = \sigma_i^2 = 0$, the latter because idiosyncratic shocks should not affect consumption growth rates. A number of studies, such as Kose et al (2007) and Artis and Hoffman (2006) have employed variants of the above regression to analyse consumption risk sharing in this manner, without being able to make a convincing case for the hypothesis of interest. Furthermore, these types of measures have been used to show that microeconomic aspects, such as whether the financial system is market rather than bank based, tend to affect the degree of risk sharing (Leibrecht and Scharler 2009).

Another strand of the empirical literature, emerging from the work of Asdrubali et. al. (1996), purports to analyse the channels of international consumption risk sharing. In examining the level of interstate risk sharing in the United States, the latter study develops a methodology for measuring the extent of risk sharing which is achieved through different channels. This technique essentially involves a quantification of the amount of risk sharing across US states in terms of a decomposition of the cross - sectional variance of state output into different components which represent the different channels of risk sharing. This study spawned a variety of similar studies using the same methodology, such as that of Sorenson and Yoshia (1998), who analysed patterns of international risk sharing amongst European and OECD countries, and Kalemi-Ozcan, Sorenson, and Yoshia (2005) who studied patterns of risk sharing in the European Union using the same methodology.

Recent evidence also tends to suggests a role for financial integration as a channel affecting the extent of consumption risk sharing. Using a panel data framework, Sorenson et. al. (2007) demonstrate an inverse relationship between home bias in bond and equity holdings and international risk sharing, with less of the former implying more of the latter. This correlates with evidence that two aspects related to home bias in asset holdings - namely separability of preferences between traded and non - traded goods and barriers to free flowing international capital - help explain the lack of international risk sharing observed in the data (Lewis 1999). The potential role for the net investment income and net capital gains on international portfolio equity holdings to affect consumption risk sharing is analysed by Bracke and Schmitz (2008). The purpose of their study is to analyse whether risk sharing through international financial markets (defined as the reduction of volatility in investment returns and hence smoothing of variations in income and consumption across countries through international portfolio diversification) operates
in practice and whether this functions through capital gains or through income earned on international securities. They find that whereas the incidence of net capital gains is countercyclical to idiosyncratic output shocks and also significantly impacts upon consumption risk sharing, the results for income flows are insignificant as firms tend to prefer to keep their dividend streams relatively constant in the presence of profit fluctuations. Further in terms of the risk sharing role of international capital flows, Imbs and Fratzscher (2007) analyse the prevalence of multilateral and bilateral risk sharing. In the former case, their approach is to regress the cyclical component of consumption \( (c_{it}) \) on the cyclical component of output \( (y_{it}) \) and an interaction term which allows for the dependence of risk sharing on financial integration \( (\phi_i) \), as follows:

\[
  c_{it} = \alpha_i + \beta_1 y_{it} + \beta_2 y_{it} \phi_i + \varepsilon_{it}
\]

They first estimate equation 4 using the total value of capital held abroad relative to GDP as their measure of \( \phi_i \). As a second measure of \( \phi_i \), they disaggregate into the three observable types of financial asset; portfolio investment, FDI and bank loans. Thus estimating the above equation using both these \textit{de facto} and \textit{de jure} indicators of financial openness, they stress that the extent of multilateral consumption risk sharing is directly related to the intensity and composition of international capital flows, with more cross holdings of capital - especially of portfolio investment - tending to be associated with more insurance. Moreover, they also investigate the issue of bilateral risk sharing, finding that countries that are the biggest recipients of OECD investment all appear to achieve significant risk sharing, regardless of the type of asset used. Another finding of this in depth study is that institutions matter for risk sharing, i.e. it is the conjunction of poor institutions and closedness to international markets that makes risk sharing impossible.

Additional evidence concerning the role of international financial integration in international consumption risk sharing has been uncovered using the most modern econometric techniques. Qiao (2010) estimates a nonstationary panel regression in order to test the degree of longer run risk sharing and to allow for richer data generating (e.g. unit root) processes. Here it is found that for the period of 1950 - 2008, about 14% of long run risks had been shared by OECD countries and in emerging market countries. During the financial integration era of the past two decades, the authors estimate that long run risk sharing in OECD countries increased more than in emerging market countries. Moreover, when investigating the relationship between a selection of measures of financial integration and cross - country risk sharing this study finds that more capital flows are associated with more long - run risk sharing.

Further recent attention has turned to the degree of consumption risk sharing. Flood, Marion and Matsumoto (2010) develop a new welfare based measure which takes into account both necessary and sufficient conditions for risk sharing. The essence of this approach is to measure how close countries come to the ideal benchmark of perfect risk sharing by computing over different time intervals the squared deviations in a country’s share of world consumption from its average over the time period. Their new measure can best be described as the conditional variance of the log ratio of individual - country per capita consumption \( (X_{i,t-j}) \) to world per capita consumption \( (\bar{X}_{i,t}) \) (which they show is a monotonic transformation of a simple social welfare function) as follows:

\[
  \sigma^2_{i,t} = \frac{1}{T} \sum_{j=1}^{T} \sigma^2_{i,t-j} = \frac{1}{T} \sum_{j=1}^{T} (X_{i,t-j} - \bar{X}_{i,t})^2
\]

Under perfect risk sharing, this variance is zero, thus the farther a country is from perfect risk sharing, the bigger the variance, and, \textit{ceteris paribus}, the lower the social welfare. They note several features of this measure. Firstly, it covers all insurable risks - when the measure is zero, the only risks that remain are involve world - wide consumption, which is uninsurable. Secondly, since their measure is a conditional one it does not become badly behaved when applied to potentially trending or non - stationary variables. Thirdly, the measure does not attempt to measure the degree of consumption smoothing, since although consumption risk sharing may help to smooth consumption, the relevant variable for consumption smoothing should be the individual consumption of an agent or a country, rather than a measure of relative consumption. Finally, although it does not distinguish whether a country achieves higher risk sharing intentionally or not, it allows the authors to measure whether improved risk sharing arises from higher business cycle synchronisation (high frequency risk sharing) or growth rate convergence (low frequency risk sharing). Thus their measure analyses a subtly different aspect of risk sharing, namely the way in which financial integration affects the degree of international consumption risk sharing. They also compute the
averages of the standard deviations of relative consumption for each country group, computed in rolling windows of 15 and 20 years in length, finding For the entire length of their sample period that industrial countries share risks best, with ‘More Financially Integrated Countries’ next, followed by ‘Less Financially Integrated Countries’. Moreover, another key finding of this analysis is that for industrial countries, significant low frequency risk sharing occurred in the early part of their sample (the 1950s to 1970s), prior to the most recent period of financial integration, leaving little room for additional low - frequency improvement in risk sharing later in the sample period.

A further tangential issue to that of consumption risk sharing is that of consumption smoothing. In this context, Islamaj (2009) undertakes a broad review of the literature and studies a number of testable implications arising from a general equilibrium model of consumption smoothing. Using a number of measures of financial liberalisation, it is found in this study that significant evidence of nonlinearities exist in the effects of financial liberalisation on consumption smoothing, with the effect being stronger for more open economies.

2.2.2 Other Measures of Variable Comovement

The present analysis touches on a number of other measures of variable comovement, namely dynamic correlation and wavelet approaches. Whilst the details of the respective methodologies are presented in Section 3, here some of the existing studies using dynamic correlation and wavelet measures are outlined.

Firstly, the dynamic correlation methodology (detailed below) was first developed by Croux et. al. (2001) and essentially entails a comovement measure that can vary across frequencies. Applications of this technique in the recent literature include Crone (2005) who analysed the similarity of state level business cycles in the US, Rua and Nunes (2005) who investigated coincident and leading indicators of economic activity in the Euro area, Camacho et. al. (2006) and Eickmeier and Breitung (2006), both of whom examined the harmonisation of European business cycles, and Lemmens et. al. (2007) who studied levels of consumer confidence across European countries.

In terms of wavelet analysis, this is an empirical technique usually used in disciplines such as astrophysics and meteorology, yet it is gaining increasing acceptance as a useful empirical tool in the field of Economics (see Crowley 2007 for a survey). This technique is of specific applicability in assessing any empirical economic phenomena where it is suspected that comovement may potentially vary over both time and spectral dimensions simultaneously. A number of studies have used this technique in an array of contexts, with recent examples including for example the paper of Kim and In (2005) who investigate the relationship between stock market returns and inflation, Gencay et. al. (2005) and Fernandez (2005) who studied the CAPM model, Gallegati et. al (2008), Yogo (2008) and Rua (2010) who used wavelets for business cycle analysis, and Rua and Nunes (2009) who focused on international stock market returns.

However, the literature is yet to apply the technique of wavelet analysis to the question of international consumption risk sharing and the potentially time - varying spectral characteristics of international consumption growth rate correlations. Thus, in order to uncover the potentially heterogeneous effects of financial integration on consumption risk sharing over time and frequency dimensions simultaneously, the present analysis concentrates on correlation measures of risk sharing, i.e. the correlation between own country and rest of the world consumption growth rates. The significant point of departure from and contribution to the previous literature is the use of a novel empirical technique - that of wavelet analysis - in order to analyse the evolution of cross country consumption correlations over time and spectral dimensions simultaneously. This methodology is examined in greater detail below.
3 Empirical Methodology and Data

Before presenting the results of the empirical analysis in more depth, we examine in detail the technique of wavelet analysis and describe the dataset used in the study.

3.1 Methodology

The major contribution of this study is to analyse the evolution of cross-country consumption and output correlations over time and frequency dimensions simultaneously using the empirical technique of wavelet analysis. Furthermore, we analyse how this evidence corresponds to periods of international financial integration. In the context of the previous literature, the present study conceptually falls into the 'correlation measures category', although as mentioned below, we employ an empirical methodology which may be seen as a refinement of previous techniques.

In the context of methodology, the first question to arise is why not use more traditional time series or spectral methods to analyse cross-country consumption correlations? There are several reasons to move beyond the more traditional techniques. Firstly, whereas the well known and popular time series correlation coefficient provides in a single number the degree of comovement between two series over a particular sample, being a synthetic measure it can be limited in investigating the potential time-varying comovement between economic variables. Whereas it is possible that the strength of the comovement between two variables may vary over time, the correlation coefficient is unable to detect this. Moreover, in the specific context of studies concerning international consumption risk sharing (such as those mentioned in the above literature review), this measure has indicated an extremely low degree of correlation between domestic country consumption growth rates and that of the rest of the world, counter to what we expect from the theory. This is also the case for the sample analysed in this study (see below for details). In order to remedy these deficiencies, rolling window correlations, or non-overlapping periods, are usually considered in order to evaluate potential time varying properties of comovement. However, these approaches suffer from the major drawback that the results are quite sensitive to the length of the window of the particular sample period considered (see for example Gayer 2007). Even if one dismisses this critique (since we usually test the robustness of our results to the window length), rolling window correlations are still unable to capture aspects of the correlation between two variables which may simultaneously vary over the frequency dimension.

In terms of spectral methods, the equally well known Fourier transform involves using the sum of sine and cosine base functions that do not fade away (i.e. they have 'infinite energy') and do not change over time (they have 'finite power') at various wavelengths in order to represent a particular given function (Crowley 2007). As such, this technique does not allow for any time dependence of the signal and is unable to provide information concerning the time evolution of spectral characteristics, since it is assumed that the frequency content of a function is stationary along the time axis. In order to overcome this, windowed Fourier analysis applies a short time window to the signal in question and performs the Fourier transform within this window as it slides across the data. By transforming short segments of the signal separately, this technique relaxes the assumption of no variation over time. However, the major drawback of the windowed Fourier approach is that since the window width and hence the time resolution is constant for all frequencies, the fixed time window tends to contain a large number of high frequency cycles and few low frequency cycles (resulting in an overrepresentation of high frequency components and underrepresentation of low frequency components) when a wide range of frequencies is present (Rua 2010).

To remedy the deficiencies of these more traditional methods, some have proposed and employed an alternative spectral based method to analyse the comovement between two series at differing frequencies. Croux et. al. (2001) for example proposed a 'dynamic correlation' measure, which ranges between $-1$ and $1$ and is conceptually similar to the standard time series correlation measure, but offers the advantage that this measure can vary across frequencies. However, the major drawback of this measure is that since it is defined in the frequency domain, it disregards the time dependence of comovement i.e. it provides a snapshot of the comovement at the frequency level and is unable to capture time varying features (Rua 2010).

In contrast to this, wavelet analysis offers the ability to analyse the comovement of a series over both the time and frequency dimensions simultaneously. In this sense, wavelet analysis can be intuitively seen as a time varying spectral approach and hence a further refinement of Fourier analysis. That is, whereas using the latter approach one can try to mimic a signal with a complex combination of sine and cosine waves, the signal in question is still assumed to be homogeneous over time. In contrast to this, wavelets fade away and only last for a short period of time (i.e. they have 'finite energy'). Thus wavelets are said to be heterogeneous over time (i.e. they have 'compact...
support’), such that in order to approximate a series that continues over a long period, wavelet functions, each indexed by location, are strung together (Crowley 2007).

Whilst the technical aspects of wavelet analysis are relegated to the Appendix (see also the survey of Crowley 2007 for an overview), a further basic intuition behind wavelet analysis is as follows. As already mentioned, in basic Fourier analysis, any variable \( x(t) \) can be represented as the sum of projections onto sine and cosine functions (over the range \((0, 2\pi)\)). Similarly in wavelet analysis, any series \( x(t) \) can be built up as a sequence of projections onto base wavelet functions\(^2\). The desirable property of these wavelet functions is that they can be stretched and dilated in order to analyse the behaviour of a time series at different levels (called ‘scales’, for the purpose of this analysis these scales are the different frequencies of the data) and at each period in time. Thus, in order to overcome the disadvantages of the windowed Fourier analysis - the inability to allow for an adequate assessment of the high and low frequency components of a time series, even in the windowed Fourier approach - the wavelet approach uses base functions which can be stretched and translated in both frequency and time dimensions, allowing the window width to narrow when focusing on high frequencies and to widen when assessing low frequencies, thus allowing the time evolution of low and high frequency components of a data series over time to be fully captured (Rua 2010).

Therefore, in terms of the drawbacks of alternative methodologies and the interest of this paper, namely to assess the evolution of international consumption risk sharing over time and frequency dimensions and to relate this to periods of financial integration, it seems natural to employ the wavelet analysis technique. Thus the application of this technique to the question of international consumption risk sharing represents a further refinement and extension of preceding time series and spectral approaches in the literature. As noted in the Appendix, a wavelet based measure of comovement in the time frequency space (which indicates at which times and frequencies the comovement between two variables is higher) is known as the ‘wavelet coherence’ (denoted by \( \rho_{xy}(\tau, s) \)). This quantity intuitively plays the role of a contemporaneous correlation coefficient around each moment in time and for each frequency and ranges between the values \(-1\) and \(1\), with the commonplace representation of this measure is as a contour plot (Rua 2010).

For demonstrative purposes I present below the evolution of the time - frequency correlations between both the consumption growth rate of the United States and that of the rest of the world (left hand pane) and the output growth rate of the United States and that of the rest of the world (right hand pane - see Appendix for results for the full sample)\(^3\):

**Figure 1 - Evolution of Consumption Correlation (left hand pane) and Output Correlation (right hand pane Over Time and Frequency**

![Figure 1](image-url)

In the above graph, frequency (in years) is plotted on the vertical axis (as one moves from the top to the bottom of the graph one moves from highest to lowest frequency), with the timescale indicated on the horizontal axis. The graph is read in a manner akin to a contour plot, with increasing shades of darkness indicating a higher comovement between the two variables. Thus one can identify the frequency bands (on the vertical axis), the time intervals (on the horizontal axis) and the strength with which the variables move together and therefore analyse the extent to which the strength of the comovement varies over frequency and time.

\(^2\)Note that there are in practice a number of admissible wavelet functions. Rua (2010) notes that the most frequent choice of wavelet function is the ‘Morlet Wavelet’, which is akin to a complex sine wave and provides a good balance between time and frequency localisation.

\(^3\)Whilst it may be desirable to present a clear colour gradation of the degree of variable comovement, this scheme is that used in the previous wavelet literature (Rua 2010)
Thus the above graph of the time frequency consumption correlations (left hand pane) and the time frequency output correlations (right hand pane) for the United States reveal some interesting results. Firstly, there is evidence of common trend between the consumption growth rate of the US and that of the rest of the world. This is represented by the darker shade at the bottom of the left hand pane of the above Figure. Stronger comovement between consumption growth rates is also evident at various other points in time at various frequencies. Turning to the right hand pane, the national output growth rates of the US and that of the rest of the world are also highly intertwined, especially at the common trend level. It is highly intuitive that the longer term growth rates of the US and that of the rest of the world should be highly correlated. Comparing the two panes, it is evident that much of the increased consumption comovement corresponds with periods of comovement between domestic and rest of the world output growth rates.

This highlights that in order to isolate the effects of increased financial integration from that of common output shocks, and therefore to properly analyse the extent of international consumption risk sharing, we need to compare the wavelet analysis of consumption growth correlations with that of output growth correlations (see Kose et al. 2007 for a further discussion of isolating the risk sharing implications of common output shocks from those international financial integration). Under perfect (or very high) risk sharing, a very high consumption correlation would be accompanied by a low correlation of the national output growth with the corresponding rest of the world growth rate. Otherwise, the high consumption growth rate would be driven by common shocks to output (which are by definition uninsurable), not by the increase in cross border asset holdings and the associated income insurance benefits. By comparing the time frequency evolution of both consumption and output growth correlation, we shed new light on the consumption - output correlation puzzle and assess at which times / frequencies improved risk sharing holds. Moreover, by comparing these periods with indicators of international financial integration, we are also able to assess the contribution of financial integration to this process. Before undertaking this analysis below, we first comment on the data used for this study.

3.2 Data

In order to employ the wavelet technique as described above, relatively high frequency data as well as a large number of observations concerning the variables of interest is needed. Whereas past practice in studies of consumption risk sharing has been to use data from the Summers Heston (2006) dataset, this data is only available at the annual frequency and is hence unsuitable for use in wavelet analysis. Two options are generally available in this circumstance - one could either use the Summers Heston (2006) dataset and interpolate the data to a quarterly frequency, or use an entirely different data source.

The former approach, that of interpolating the Summers - Heston (2006) dataset, tends to pose a problem, since although interpolating a sluggishly moving variable such as population is generally acceptable, the same may not be said of a jump variable such as consumption. This study therefore opts for the latter approach of using a different data source altogether, although some interpolation of population data is still required. All quarterly data concerning private consumption, and annual observations on population are taken from the International Financial Statistics (IFS) of the IMF (see the Appendix for more details concerning the data sources). In order to obtain quarterly estimates of population, the yearly values are linearly interpolated to quarterly ones. All data concerning consumption and output are first deflated into real values using the respective CPI indices for each country, and then converted into US dollars using period average exchange rates (both of these series are available from the IFS).

Moreover, the empirical analysis compares the time - frequency correlation of own country consumption growth rates with the consumption growth rate of 'rest of the world' consumption. This 'rest of the world' consumption variable for the ith country is simply constructed as the sum of each individual country consumption in each period for which quarterly data is available, minus the consumption of country i. The countries in the 'world' for the purposes of this study are are listed in the Appendix. The exact composition of the 'world' in each period will not matter as long as the major industrialised countries are included, which is the case in the present study. This is because the major industrialised nations are responsible for the majority of consumption risk sharing (Flood et. al. 2010).4

3.2.1 Capital Account Openness

One of the interests of this paper is to identify how consumption risk sharing at different frequencies relates to periods of capital account openness. To identify such periods, both a de jure and a de facto indicator of financial integration is used. In terms of the former, in practice there are many such indices available, such as the Chinn

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4 Since the consumption aggregates for industrialised countries are the biggest components of the 'world' in each quarter, they receive the biggest weight in the world aggregate in each quarter.
The different databases used in this study are each available over different sample periods. Previous studies have used a wide variety of sample periods, for the most part recent studies have employed data commencing in 1950 and extending into the early 2000s. Presently, in order to harmonise the sample coverage of the different datasets employed by the current study, the sample period runs from 1973:1 to 2005:4 (note for some countries however, the sample period is slightly shorter due to data availability, with the exact sample period coverage by country is restricted sample coverage).

Moreover, the main drawback of this database is that it can often be difficult to isolate the actual source of and reason for a particular deviation from openness. Moreover, for the purposes of this study, the Chinn-Ito (2008) index also contains information concerning current account transactions, whereas presently in the context of international consumption risk sharing we are mainly interested in capital account transactions (complete capital markets) and asset trading only. The Quinn (1997) index tabulates information on the intensity of controls for 94 countries during 1950–99 (a recent update extends this through to 2005) by ranking different control instruments by their economic importance in a continuous variable ranging between zero and one. Schindler (2009) presents an index of financial integration which is distinguished by its high level of disaggregation by asset, direction of flow, residency, and intensity of control for a sample of 91 countries from 1995 to 2005, again based on information from the AREAER. The main drawback of this latter index for the purposes of the current study is its relatively recent restricted sample coverage.

Since the interest of this study is in an overall de jure measure of integration which also covers a reasonable timespan, we therefore utilise the financial reform index of Abiad et. al. (2010), which highlights several aspects of financial sector reform in a cross-country context over the period 1973–2005, importantly for the present analysis capturing information on the international financial integration of various countries in an international capital account subindex. This capital account subindex of the financial reform index of Abiad et. al. (2010) is constructed as follows. Original IMF country reports for each country in each year from 1973–2005 are used to answer three questions: is the exchange rate system unified (coded as 0 when a special exchange rate regime for either capital or current account transactions exists, and 1 when the exchange rate system is unified)?; does a country set restrictions on capital inflow (coded as 0 when significant restrictions exist on capital inflows and 1 when banks are allowed to borrow freely from abroad without restrictions and there are no tight restrictions on capital inflows)?; and does a country set restrictions on capital outflows (coded as 0 when restrictions exist on capital outflows and 1 when capital outflows are allowed to flow freely or with minimal approval restrictions)? The score on each of these three aspects is then aggregated and an openness score is given for each year, with a total of 3 meaning that a country is 'fully liberalised'; a score of 2 implies that a country is 'largely liberalised'; the capital account is 'partially repressed' if the total equals one, and 'fully repressed' if the total is equal to zero.

Moreover, the use of both kinds of indicators of financial integration allows a fuller identification of periods of financial integration and permits an analysis of how these periods relate to international consumption risk sharing.

3.2.2 Sample Period and Coverage

The different databases used in this study are each available over different sample periods. Previous studies have used a wide variety of sample periods, for the most part recent studies have employed data commencing in 1950 and extending into the early 2000s. Presently, in order to harmonise the sample coverage of the different datasets employed by the current study, the sample period runs from 1973:1 to 2005:4 (note for some countries however, the sample period is slightly shorter due to data availability, with the exact sample period coverage by country is listed in the Appendix)\textsuperscript{5}. Thus the empirical study covers the two important periods of recent economic history a

\textsuperscript{5}It may be desirable to chart the evolution of international consumption risk sharing over a longer timespan, since it has for example been noted elsewhere that industrial countries achieved much of their risk sharing since by the 1970s (Flood et. al 2010). However, we choose our sample within the constraints of the available data.
la Kose et. al. (2007), namely the 'common shocks' era - associated with sharp fluctuations in the price of oil and contractionary monetary policies in major industrial countries - from 1973 - 1986, and the more recent financial globalisation era. Thus we expect a high degree of output growth rate comovement, at least amongst industrialised countries, during the former era, with some evidence of increased consumption risk sharing in the latter era due to increased asset trading. Moreover, due to the fact that sufficient quarterly data on consumption aggregates is only available for a subset of countries in the IFS, the empirical analysis is largely restricted to the 'Industrial' and 'More Financially Integrated' countries for which data is available, as per the categorisation of Flood et. al. (2010). The sample countries are listed in the Appendix.

4 Results

4.1 Main Results

4.1.1 Time Series Correlations

In order to establish some benchmark results concerning consumption risk sharing for the current sample, the Table A2 in the Appendix presents the results for the traditional time series correlations between the own country growth rate of consumption and rest of the world consumption growth rates, along with the time series correlations between own country and rest of the world output growth, for each respective country in the sample (the \( p \)-values of the correlations are given in brackets). The first column of the table presents the correlations over the whole sample, whilst the second and third column present the results for the 'pre integration' (1973:1 to 1989:4) and post integration (1990:1 to 2005:4) eras respectively. In interpreting these results, we remember that according to the theory, perfect risk sharing implies that the correlation between own country and rest of the world consumption should be one (or slightly less than one in the case of a utility function including other arguments, including for example leisure). Moreover, this 'consumption correlation' should be much higher than the corresponding correlation between own country and rest of the world output growth rates, with evidence from previous studies indicating that this is generally not the case (the 'consumption - output correlation puzzle').

Thus the key results of the time series correlations are as follows. Theory tells us that perfect risk sharing implies that the correlation between own country and rest of the world consumption should be one (or slightly less than one in the case of a utility function including other examples, including for example leisure). Moreover, this consumption correlation’ should be much higher than the corresponding correlation between own country and rest of the world output growth rates, with evidence from previous studies indicating that this is generally not the case (the 'consumption - output correlation puzzle').

Thus the key results of the time series correlations are as follows. Firstly, considering the whole sample period, no country reaches the ideal perfect risk sharing benchmark of a correlation of unity between its own consumption growth rate and that of the rest of the world. For all countries with the exception of Belgium, the consumption correlations are insignificant.

Secondly, concerning the output correlations, we note that the theory tells us that under financial integration, the consumption correlation should be much higher than the respective output correlation. However in this case, all countries exhibit the consumption output correlation puzzle, with output correlations that are much higher than the respective consumption correlation. In the cases of Canada, Hong Kong and the USA, both the consumption and output correlations are insignificant.

The picture is even more mixed in the comparison of the pre financial integration (1973:2 to 1989:4) and financial integration eras (1990:1 to 2005:4). In the pre integration era, with some exceptions (Korea, USA, South Africa, Canada, Hong Kong, Israel) all countries exhibit the consumption output correlation puzzle. For all countries with the exception of Hong Kong, this puzzle persists into the financial integration era. For the Philippines, the output correlation becomes insignificant in the post 1990 period.

Thus in terms of the theory concerning the risk sharing benefits of financial integration, it is puzzling that the 'consumption - output correlation puzzle’ tends to persist even into the post 1990 financial integration era. What is happening here? As mentioned above, time series correlations tend to be quite sensitive to the sample period chosen for the analysis. However, this aspect could be ameliorated to some extent by employing rolling windows or rolling regressions (with the caveat that these are also sensitive to the length of the window chosen). Most importantly for this analysis, the ability of purely time series correlations to detect consumption risk sharing is quite limited in that

\[ \text{The magnitude of these correlations generally corresponds with that given in Kose et. al. (2007) for private consumption.} \]
it fails to take account of aspects of the dynamic correlation which vary over the frequency dimension at each point in time. That is, at each point in time the correlation between domestic country consumption growth rates and output growth rates with their rest of the world counterparts could be stronger at say the business cycle frequency than at the common trend frequency (or vice versa), in turn implying that the incidence of the consumption-output correlation puzzle may differ by frequency as well.

Thus this latter possibility, along with the time series correlation results presented above, imply that we need to move beyond traditional methods to a time varying spectral approach such as wavelet analysis, in order to highlight the comovement of consumption growth rates and output growth rates over time and frequency dimensions simultaneously and provide a richer description of international consumption risk sharing. This analysis is presented below.

4.1.2 Wavelet (Time Varying Spectral) Analysis

Since the pure time series correlation evidence for the present sample is quite mixed, and moreover cannot pick up the evolution of risk sharing over different frequencies, we move now to the results of the wavelet (time varying spectral) approach. The full graphical results are available in the Appendix and provide a rich information set concerning the evolution of the strength of consumption and output correlations over time and frequency. In order to interpret these results, the analysis proceeds as follows. Firstly, we investigate how the comovement between consumption growth rates of each country and that of the rest of the world varies over time and frequency dimensions. Secondly, we undertake a similar analysis of the correlations between the growth rates of national output and that of the rest of the world for each country in the sample.

Thirdly, since by definition common shocks are uninsurable, a comparison of the time varying spectral properties of consumption and output growth rates permits an assessment of whether a higher degree of consumption growth rate comovement is driven by risk sharing or by common output shocks. That is, periods of high comovement in consumption growth rates, without a correspondingly high output growth rate, indicate periods of enhanced international consumption risk sharing. High comovement of consumption growth rates along with high comovement in output growth rates indicates periods of common shocks. Thus we shed new light on the well known ‘consumption-output correlation puzzle’, finding that this holds at certain frequencies and at certain times only.

Finally, we analyse the potential role for international financial integration and increased cross border asset holdings in international consumption risk sharing, using both de jure and de facto measures of financial integration. The intuition here is that increased cross border asset holdings should lead to income transfers (or valuation gains a la Bracke and Schmitz 2008) that act as insurance against country specific factors. The complete results of the analysis are presented below.

Consumption Correlations

The strength of consumption growth rate correlations over time and frequency is illustrated in the left hand pane of the Figures in the Appendix. Firstly, it is noticeable that the consumption growth rate comovements vary over both time and frequency dimensions, highlighting the importance of the time varying spectral approach. Whereas previous approaches in the literature were unable to capture aspects of the data spectrum which varied over the sample period, wavelet analysis enables us to discern the degree to which the cross country consumption growth rates move together at each point in time and at each frequency.

Secondly, especially for the industrial countries in the sample, a high degree of correlation between consumption growth rates, represented by darker shades, is found at the 8 years plus frequency since the 1970s over the whole sample. This represents common trends in consumption growth rates between each country and the rest of the world. Yet even amongst industrial countries, the strength of this common trend in consumption growth varies at each point in time. For some countries, such as Sweden and Switzerland, it is evident that this common trend is quite strong over the whole sample period, whereas for others such as Australia and United Kingdom this is not the case.

Thirdly, the evidence presented here for industrialised countries tends to support previous findings of low consumption correlations at the business cycle frequency (see Pakko 2004). This is given by the lighter shades in the middle of the left hand panes of the Figures in the Appendix. Whilst the timing of these periods, where the domestic country growth rate of consumption move counter to the world aggregate, generally occur from the mid 1980s to the mid 1990s at the 2 to 4 year frequency, the length of these periods differs by country. So for example...
Austria experiences a long counter cyclical spell from 1980 to 1995, while Japan experienced a comparatively short period of countercyclical movement in 1990.

Finally, the results of for the 'more financially integrated' countries diverge from those of the industrial countries. The evidence suggests that although these countries exhibit longer term consumption growth rate convergence, less harmonisation at the business cycle frequency is present. Thus whilst some (Israel, South Africa, Philippines, Hong Kong, Korea) exhibit some evidence of common trend frequency comovement with the rest of the world, the degree of business cycle frequency comovement of consumption growth rates with the rest of the world tends to be lower than for industrialised countries.

How are these results of time frequency correlations to be interpreted? An initial assessment would be that the higher degree of consumption comovement, represented by the darker shades in the graphs, is indicative of higher international consumption risk sharing. However, this approach would ignore the possibility that the correlation of consumption growth rates is driven by common output shocks. That is, the pre 1990s section of the current sample period was a period characterised by a set of common output shocks associated with sharp fluctuations in the price of oil and contractionary monetary policy in major industrial countries (Kose et. al. 2007). Thus the consumption growth rate comovement could be driven by common uninsurable output shocks, rather than representing international consumption risk sharing. Moreover, consumption growth rate convergence could be driven by output growth rate convergence associated with the diffusion of technology, ideas and institutions. These possibilities are explicitly addressed below in the analysis of output growth rate correlations.

Output Correlations

How do the correlations of output growth rates of each country with that of the rest of the world vary over time and frequency? To answer this question we now focus our attention on the middle pane of the figures in the Appendix, which present the evolution of the time frequency correlations between own country output growth and rest of the world output growth over time and frequency (these plots are read in a similar fashion as those for consumption growth rates).

Analysis of these graphs reveals several results. Firstly, as with the time frequency consumption growth rate correlations, those for output also vary over both time and frequency dimension, again highlighting the value added of the time varying spectral approach. The common trend comovement (the darker shades at the bottom of the graphs) between output growth rates of industrialised countries and the rest of the world is quite high, indicating a high degree of output growth rate convergence at lower frequencies. A typical explanation for this common trend integration would be output convergence associated with the trade in ideas and technology transfer.

Secondly, consistent with the notion of a 'common shocks' era, a large group of industrialised countries (Austria, France, Finland, Italy, Norway, Spain, Sweden, Switzerland, United Kingdom, Belgium, Denmark, Netherlands) exhibit a high degree of output growth synchronisation at the business cycle frequency during the 1980s and into the early 1990s. Many of these countries had already achieved longer term (common trend) output growth rate convergence since the beginning of the sample in the early 1970s.

Finally, many countries experience periods in which their output growth moves in a countercyclical manner to that of the rest of the world. Country experience differs widely in this regard, with countercyclical growth rates tending to occur at business cycle and shorter frequencies. However, a common factor amongst industrialised countries is that these periods of weaker (or negative) output comovement occur at the start of the sample period (in 1973), coinciding not only with the first oil shock, but also with the instability associated with the collapse of the Bretton Woods system of fixed exchange rates. Furthermore, it is clear in the case of all European countries that a high degree of output growth convergence vis-a-vis the rest of the world had been achieved at all frequencies. It is noticeable however that the members of the Eurozone in the sample (Austria, Finland, France, Italy, Spain, Belgium, Netherlands, Portugal), exhibit a brief period of negative comovement of output at the business cycle frequency at the time of the adoption of the single currency at the start of the 2000s8.

The main implication to arise from this result is that much of the ‘international consumption risk sharing’, which was previously observed in the data as high consumption growth correlations, may in fact be driven by common (uninsurable) shocks to output. Secondly, since the strength of consumption and output comovements varies by time and by frequency, it then follows that the ‘consumption - output correlation puzzle’ may only hold at certain time periods and at certain frequencies. These possibilities are examined further below.

The Consumption - Output Correlation Puzzle, Common Output Shocks and International Consumption Risk Sharing  Given the preceding results concerning the time and frequency evolution of consumption

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8Causation is elusive however - the output shock also occurs for Denmark, which of course did not adopt the Euro at this time.
growth rate and output growth rate correlations, we are now in a position to revisit the 'consumption output correlation puzzle', as well as assessing the extent of international consumption risk sharing, now from the time varying spectral perspective. This is achieved by comparing the left hand and middle panes of the figures in the Appendix. A light shade at a particular point in time and frequency in the left hand pane (for consumption correlations), accompanied by a darker shade at the same point in time in the middle pane (for output correlations) indicates the presence of the consumption - output correlation puzzle. Conversely, a darker shade in the left hand pane accompanied by a lighter shade in the right hand pane indicates improved international consumption risk sharing. A third possibility is that if the two graphs exhibit the same pattern, it can be concluded that the consumption growth rate correlations are primarily driven by output shocks.

The key results to emerge from this comparison are threefold. Firstly, the consumption - output correlation puzzle for each country holds only at certain frequencies and at certain points in time. (at other times, enhanced international consumption risk sharing is present or the increased consumption comovement is driven by output convergence). Table 3 below summarises some prominent examples of countries which display the consumption output correlation puzzle, and the period of time and frequency (from highest to lowest ordering these are 'shorter business cycle', 'business cycle' and 'common trend' frequencies) at which this occurs:

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>1973 - 1995</td>
<td>business cycle and common trend</td>
</tr>
<tr>
<td>Korea</td>
<td>1985 - 2005</td>
<td>common trend</td>
</tr>
<tr>
<td>Belgium</td>
<td>1973 - 2005</td>
<td>all frequencies</td>
</tr>
<tr>
<td>Philippines</td>
<td>1973 - 2005, 1995</td>
<td>common trend, business cycle</td>
</tr>
<tr>
<td>Portugal</td>
<td>1977 - 1995</td>
<td>business cycle</td>
</tr>
</tbody>
</table>

For example in the case of Norway, the consumption - output correlation puzzle is prevalent at the common trend frequency from over the whole sample period, as well as from the early 1980s until the mid 1990s at the business cycle horizon. This is a result which is not only confined to the industrial countries in the sample, so for example the Philippines exhibits the puzzle at the common trend frequency over the whole sample, as well as in 1995 over the business cycle horizon, yet not at other times. Overall, the evidence tends to suggest that most of the sample countries exhibit signs of the consumption output correlation puzzle at the common trend level. This in turn implies that that the most prominent explanations for the consumption output correlation puzzle - non tradable and durable goods consumption, market incompleteness and transaction costs - are pertinent for inhibiting consumption insurance at different frequencies. For example, the incidence of transaction costs may be relevant for preventing insurance against certain kinds of risks, or it may be that incomplete asset markets are more relevant for insurance at other frequencies.

Secondly, it is also evident for some that the time frequency correlations of consumption and output growth rates are quite similar (see for example the United States, Israel, Japan). For these countries, the high co movement of consumption is not driven by improved international risk sharing, rather by longer term output growth rate convergence. In the case of the United States, much of this convergence had already been achieved by the start of the sample period in the early 1970s...Moreover, the evidence presented here suggests that for many industrialised countries, much longer term (common trend) convergence was already apparent in the early 1970s, a phenomenon which is accentuated during the 'common shocks' era of the 1970s and 1980s at the business cycle frequency. As noted earlier, this period was associated with sharp fluctuations in the price of oil and contractionary monetary policies in major industrial countries. According to the data it was these common shocks to output drove a high correlation between domestic country and rest of the world consumption.

Thirdly, the comparison of the wavelet graphs for consumption and output growth correlations also reveals enhanced international consumption risk sharing for a smaller set of countries in the sample, at a range of different times and frequencies. The following table 3 presents some prominent examples of countries experiencing improved international consumption risk sharing, in the sense of a high consumption comovement accompanied by a low output comovement, listed by the time period and frequencies (from highest to lowest ordering these are 'shorter business cycle', 'business cycle' and 'common trend' frequencies) at which this occurs:
Table 3 - International Consumption Risk Sharing By Country

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Frequency of Risk sharing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>late 1980s</td>
<td>business cycle</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2000</td>
<td>shorter business cycle</td>
</tr>
<tr>
<td>United States</td>
<td>1975 - 2003</td>
<td>common trend</td>
</tr>
</tbody>
</table>

The above table demonstrates that international consumption risk sharing, as opposed to consumption growth rate comovement driven by common output shocks - is present for a selected group of countries, and at selected times and frequencies. It appears that country experiences are divergent and there are no readily discernible patterns. Moreover, it is important to realise that at certain time periods there may be country (or region) specific shocks to output which - if not discounted as the effect of a shock - may lead to misinterpretation as being indicative of risk sharing. Two such instances - 1973 - 1975 (the breakdown of Bretton Woods and the first oil shock) and the late 1990s / early 2000s (the adoption of the Euro for eurozone members) have already been mentioned.

The question then arises as to what might explain these periods of enhanced international consumption risk sharing? Whilst the role of output growth convergence has already been noted above as an explanation for common trend comovement, we now examine below the role of financial integration and cross border asset holdings as an explanation for improved consumption risk sharing at shorter frequencies.

The Role of Financial Integration In Enhanced Risk Sharing  What role does financial integration, in the sense of increased cross border asset holdings, play in international consumption risk sharing? Intuition tells us that the increased cross border asset holdings should provide improved insurance against idiosyncratic country specific risks. But to what extent and at which frequencies does this operate?

To analyse this question, we present for each country in the sample the evolution of the de jure measure of financial integration (dashed line), the capital account subindex of Abiad et. al. (2010), against the de facto measure of financial integration (thick bars), given by the ratio of external assets GDP + external liabilities in the right hand pane of the figures in the Appendix. The object of this analysis is to compare periods of high financial integration in both the de facto and de jure senses with the consumption and output correlations to infer the role of increased cross border asset holdings in insuring against idiosyncratic country specific factors. Thus the use of the de jure measure is indicative of the official policy stance regarding the capital account and enables us to analyse whether discrete policy liberalisation events tend to affect risk sharing, whereas the de facto measure gives a sense of the actual cross border asset position (the actual extent to which agents can yield income transfers arising from writing insurance contracts and trading cross border assets) and its relation to international consumption risk sharing.

However, in answering this question, it should be borne in mind that since the empirical comparison of correlations generated by the wavelet analysis with levels of financial integration does not explicitly control for covariates, one cannot be too ambitious (in a formal statistical sense) in making statements concerning causal relationships between periods of increasing integration and the stronger comovement of consumption growth rates. However, as indicated by the previous literature reviewed above, which itself does control for the effect of covariates, cross border capital flows do play a significant role in international consumption risk sharing, both in terms of effects attributable to both income and valuation gain effects\(^9\).

A second challenge in the current analysis is to disentangle the effects of output comovement and increased cross border asset holdings on consumption comovement. However, as indicated above, we can restrict the analysis to the periods of enhanced international consumption risk sharing, that is the times and the frequencies which are characterised by high consumption growth correlations, but lower output growth correlations. This is because we are interested in the effects of increased financial integration during times and at frequencies characterised by international consumption risk sharing, rather than those periods where output growth rate comovement is driving the consumption correlations.

The first key result concerns the usefulness of examining both de jure and de facto measures of financial integration. This is because there are (sometimes large) changes in de facto integration, even during times of de jure capital account repression (even though the latter measure is derived from IMF staff country assessments of capital control regimes). Moreover, a country is integrated in a de facto sense, even though the de jure measure

\(^{9}\)It would of course be of interest to control for covariate effects in order to isolate the effect of financial openness on international consumption risk sharing. In the current context, this would necessitate the use of wavelet regression techniques (which is intuitively speaking a nonparametric regression method) a la Ramsey and Lampart (1998) in order to analyse the conditional effects of financial openness on risk sharing at different frequencies. Yet since the focus of this paper is on the consumption output correlation puzzle, we leave this aspect for future research
indicates a period of capital account repression. This is in line with the hypothesis that de jure measures - no matter how sophisticated - cannot capture the enforcement and effectiveness of capital controls (see Kose, Prasad, Rogoff and Wei 2006). Secondly, focusing on the de jure measure (dashed line), there is little evidence that policy measures to liberalise the capital account are associated with periods of enhanced international consumption risk sharing. That is, if we focus on periods when there is an upward movement in the de jure financial integration measure, these periods do not appear to coincide with risk sharing. For example, the capital account liberalisation of Italy in the early 1980s was not associated with improved risk sharing, in fact the business cycle co movement of domestic consumption with its rest of the world counterpart is negative during this time. On the other side of the coin, periods of capital account repression - even those due to exogenous events - do not appear have any significant affect either. The case of South Africa during the period 1985 - 1990, a period of acute capital account repression under the regime of multilateral financial sanctions - is particularly instructive, since there appears to be little significant change in the degree of consumption growth rate correlation during this period. Thus it is unlikely that periods of international consumption risk sharing bear much relation to official regulations governing the capital account.

How does the degree of de facto integration, which signals the degree of cross border asset holdings and hence the ability of countries to insure themselves with income from asset trading, impact upon risk sharing? Some countries experience risk sharing (in the sense that the consumption comovement is higher than the respective one for output) at times of higher integration. Examples are Australia and United States, which in the year 2005 engage in risk sharing at the 1 year frequency, while their de facto integration ratios were in excess of 150% of GDP. As noted by Flood et. al. (2010), such short - term risk sharing may be brought about by insurance contracts or trading country - risk specific securities. However, this is not the case for many countries in the sample, which at high levels of integration do not exhibit signs of risk sharing. Moreover, for some (e.g. Philippines) risk sharing also seems to be prevalent at medium levels of integration, for example in the early 1990s at the one year frequency, when the integration ratio stood at 100% of GDP. Thus whilst there is some suggestion for a role for increased cross border asset holdings and financial integration in improving shorter term risk sharing, a firmer conclusion concerning this awaits further investigation.

5 Conclusion

Improved consumption risk sharing is one of the fundamental predicted benefits of financial integration. However, despite clear theoretical predictions, a plethora of empirical studies has revealed mixed evidence concerning this issue, with recent efforts focusing on analysing the degree of financial integration. In this vein, the current analysis has analysed several aspects. Firstly, in contrast to previous studies, the key contribution of this paper was to investigate how the comovement between consumption growth rates and output growth rates of each country with their rest of the world counterparts varies over time and frequency dimensions simultaneously.

Since by definition common shocks are uninsurable, a comparison of the time frequency correlations properties of consumption and output growth rates permitted an assessment of whether a higher degree of consumption growth rate comovement is driven by risk sharing or by common output shocks. That is, periods of high comovement in consumption growth rates, in the absence of high output growth rate correlation, indicated periods of enhanced international consumption risk sharing. High comovement of consumption growth rates along with high comovement in output growth rates indicated periods of common shocks. Finally, the potential role for international financial integration and increased cross border asset holdings in international consumption risk sharing, using both de jure and de facto measures of financial integration, was analysed.

We stress several results of our study. Firstly, the comovements of national consumption and output growth rates with their respective rest of the world counterparts varies over both time and frequency dimensions, highlighting the importance of employing a time varying spectral approach to this question. These results differ whether one considers industrialised or 'more financially integrated' countries, although due to data considerations our sample is dominated by the former.

Secondly, analysis of the consumption growth rate correlations, together with those for output growth, reveals that much of the comovement between national and rest of the world consumption aggregates is attributable to common output shocks between countries. Whilst longer term output growth rate integration is symptomatic of trade in technologies and ideas, the comovement of output growth rates at the business cycle frequency during the 1980s illustrates the prevalence of common output shocks experienced by major industrialised countries during this period. Moreover, it was also found that the time frequency correlations of consumption and output in some cases similar, highlighting that in these instances common shocks tend to drive consumption growth rate comovements.

Thirdly, it was found that for certain countries in our sample, the consumption output correlation puzzle is of
relevance only at certain points in time and at certain frequencies. Whereas previous studies found this puzzle to be present over either time or frequency dimensions in isolation, the contribution of this paper has been to combine both these dimensions to highlight how the Backus-Kehoe Kydland puzzle, which may be driven by a number of factors (such as incomplete asset markets), is of relevance at certain times and frequencies only.

Finally, considering recent periods of financial integration as denoted by *de facto* and *de jure* measures, there is little evidence that specific policy measures to liberalise the capital account are associated with periods of enhanced international consumption risk sharing. Furthermore, capital account reversals, imposed either exogenously or driven by a repressive capital account policy stance, also do not seem to affect risk sharing. In contrast to this, for some countries, large cross border asset holdings (*de facto* integration) do to some extent appear to correlate with periods of high frequency risk sharing. However, for others risk sharing is achieved at lower levels of integration. Thus the true nature of the role of financial integration in international consumption risk sharing awaits future research.

In these terms, the future research avenues arising from this study are manyfold. Firstly, since it was found that the consumption output correlation puzzle is of relevance at certain frequencies and time periods only, future research could investigate which specific factors contribute towards this phenomenon. Secondly, given that previous literature found an important role for portfolio investment in international consumption risk sharing (Fratzscher and Imbs 2007), it may prove fruitful to disentangle the extent to which different forms of international capital flows (debt, equity and FDI) are associated with risk sharing at different frequencies, and to assess the extent to which these effects are associated with developments in the asset or liability side of the external balance sheet. Thirdly, wavelet regression techniques could be employed in order to further investigate the effects of financial integration on international consumption risk sharing conditional on other covariate effects (by extending Equation 3). Finally, wavelet analysis could be applied to a plethora of other interesting risk sharing questions, for example to trace at which frequencies (business cycle or common trend) European integration has been more successful at promoting the harmonisation of its diverse regional economies (this would be subject to the availability of data of sufficient length). Furthermore, one could investigate whether certain economic regions - for example Latin America and the United States - share risks better with each other and over what kind of horizon this risk sharing this entails. These issue are left for future research.
6 References


International Monetary Fund, International Financial Statistics, Washington DC.


7 Appendix

7.1 Description of the Data

The data for this study were taken from the following sources:

Table A1 - Sources of the Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>intcapital</td>
<td>Summary Index Measure of Capital Account Openness</td>
<td>Abiad et. al (2010)</td>
</tr>
<tr>
<td>integration</td>
<td>$\frac{(external \ assets + external \ liabilities)}{GDP}$</td>
<td>Lane and Milesi - Ferretti (2007), Author’s Calculations</td>
</tr>
<tr>
<td>Rest of the World Consumption</td>
<td>Consumption of the ‘Rest of the World’</td>
<td>Author’s Calculations</td>
</tr>
<tr>
<td>Population</td>
<td>Country Population</td>
<td>IMF International Financial Statistics , Author’s Calculations</td>
</tr>
<tr>
<td>Output</td>
<td>Country Output</td>
<td>IMF International Financial Statistics, line 99b</td>
</tr>
<tr>
<td>Rest of the World Output</td>
<td>Output of the ‘Rest of the World’</td>
<td>Author’s Calculations</td>
</tr>
</tbody>
</table>

See the main text for a description of the construction of the dataset used in this study. The countries that comprise the ‘rest of the world’ for the purposes of this study are; Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Brunei Darussalam, Bulgaria, Cambodia, Canada, Chile, Hong Kong, Colombia, Costa Rica, Croatia, Cyprus, Czech Republic, Denmark, Ecuador, Egypt, Estonia, Finland, France, Georgia, Germany, Greece, Guatemala, Hungary, Iceland, India, Indonesia, Iran, Israel, Italy, Japan, Kazakhstan, Korea, Kyrgyz Republic, Latvia, Lithuania, Luxembourg, Malaysia, Malta, Mauritius, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, Poland, Portugal, Romania, Russian Federation, Slovak Republic, Slovenia, South Africa, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United States. The exact composition of the ‘world’ in each period will not matter as long as the major industrialised countries are included, which is the case in the present study. This is because the major industrialised nations are responsible for the majority of consumption risk sharing (Flood et. al. 2010).
7.2 Time Series Correlations

In order to establish some benchmark results concerning consumption risk sharing for the current sample, the Table A1 on the following page presents the results for the traditional time series correlations between the own country growth rate of consumption and rest of the world consumption growth rates, as well as the growth rate of own country output and rest of the world output growth for each respective country in the sample (with \( p - \)values in brackets). Both ‘Industrialised’ and ‘More Financially Integrated Countries’ as per the categorisation of Flood et. al. (2010) are analysed. The industrialised countries analysed are Australia, Austria, Belgium, Hong Kong, Denmark, Israel, Mexico, Netherlands, Philippines, Portugal, Canada, Finland, France, Italy, Japan, Korea, Norway, South Africa, Spain, Sweden, Switzerland, United Kingdom, United States. The ‘More Financially Integrated’ countries comprise Hong Kong, Israel, Korea, Mexico, Philippines and South Africa. * indicates significance of the correlation at the 10% level. ** indicates significance of the correlation at the 5% level. *** indicates significance of the correlation at the 1% level, a bold highlight indicates the greater of the two correlations in each country case. The first column of the table presents the correlations over the whole sample, whilst the second and third column present the results for the ‘pre integration’ (1973:1 to 1989:4) and post integration (1990:1 to 2005:4) eras respectively.
Table A2 - Time Series Correlation Correlation Coefficients Of Own Country Growth Rate With The 'Rest of the World' Growth Rate (real private consumption vs real output), Full Sample

<table>
<thead>
<tr>
<th>Country</th>
<th>Correlation Coefficients Of Own Country Growth Rate With The 'Rest of the World' Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Sample</td>
</tr>
<tr>
<td></td>
<td>'Pre integration' 1973:2 to 1989:4</td>
</tr>
<tr>
<td></td>
<td>'Financial Globalisation' 1990:1 to 2005:4</td>
</tr>
<tr>
<td></td>
<td>$c$</td>
</tr>
<tr>
<td>Australia</td>
<td>0.127 (0.150)</td>
</tr>
<tr>
<td>Austria</td>
<td>-0.068 (0.438)</td>
</tr>
<tr>
<td>Finland</td>
<td>0.030 (0.731)</td>
</tr>
<tr>
<td>France</td>
<td>0.079 (0.372)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.025 (0.774)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.045 (0.613)</td>
</tr>
<tr>
<td>Korea</td>
<td>-0.029 (0.740)</td>
</tr>
<tr>
<td>Norway</td>
<td>-0.006 (0.949)</td>
</tr>
<tr>
<td>South Africa</td>
<td>0.102 (0.246)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.008 (0.932)</td>
</tr>
<tr>
<td>Sweden</td>
<td>-0.002 (0.981)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-0.038 (0.665)</td>
</tr>
<tr>
<td>UK</td>
<td>-0.017 (0.849)</td>
</tr>
<tr>
<td>USA</td>
<td>-0.005 (0.960)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.161 (0.066)*</td>
</tr>
<tr>
<td>Canada</td>
<td>0.096 (0.278)</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>0.099 (0.261)</td>
</tr>
<tr>
<td>Denmark</td>
<td>-0.098 (0.267)</td>
</tr>
<tr>
<td>Israel</td>
<td>0.007 (0.939)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.017 (0.846)</td>
</tr>
<tr>
<td>Philippines</td>
<td>-0.210 (0.016)**</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.069 (0.431)</td>
</tr>
</tbody>
</table>
Theory tells us that perfect risk sharing implies that the correlation between own country and rest of the world consumption should be one (or slightly less than one in the case of a utility function including other examples, including for example leisure). Moreover, this consumption correlation’ should be much higher than the corresponding correlation between own country and rest of the world output growth rates, with evidence from previous studies indicating that this is generally not the case (the ‘consumption-output correlation puzzle’).

Thus the key results of the time series correlations are as follows. Firstly, considering the whole sample period, no country reaches the ideal perfect risk sharing benchmark of a correlation of unity between its own consumption growth rate and that of the rest of the world. For all countries with the exception of Belgium, the consumption correlations are insignificant.

Secondly, concerning the output correlations, we note that the theory tells us that under financial integration, the consumption correlation should be much higher than the respective output correlation. However in this case, all countries exhibit the consumption-output correlation puzzle, with output correlations that are much higher than the respective consumption correlation. In the cases of Canada, Hong Kong and the USA, both the consumption and output correlations are insignificant.

The picture is even more mixed in the comparison of the pre-financial integration (1973:2 to 1989:4) and financial integration eras (1990:1 to 2005:4). In the pre-integration era, with some exceptions (Korea, USA, South Africa, Canada, Hong Kong, Israel) all countries exhibit the consumption-output correlation puzzle. For all countries with the exception of Hong Kong, this puzzle persists into the financial integration era. For the Philippines, the output correlation becomes insignificant in the post-1990 period.

Thus in terms of the theory concerning the risk sharing benefits of financial integration, it is puzzling that the ‘consumption-output correlation puzzle’ tends to persist even into the post 1990 financial integration era. However, these simple time series correlations are unable to detect any risk sharing that is occurring at different frequencies, at different points in time. This underscores the need to move to a time varying spectral approach, such as wavelet analysis.
7.3 Sample Period

The following table presents the sample period for the wavelet analysis of real private consumption data for each country, with the number of usable observations in brackets:

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample Period (# Of Observations)</th>
<th>Output Growth Rate</th>
<th>Private Consumption Growth Rate</th>
</tr>
</thead>
</table>
Wavelet analysis is a technique more common to signal processing in disciplines such as meteorology and physics, yet also of increasing applicability in economics (see for example Rua 2010). The intuition is to examine comovement in the time frequency space i.e. to assess in which frequencies contemporaneous comovement is higher. As noted by Rua (2010), the wavelet transform decomposes a time series in terms of some elementary functions, \( \varphi_{\tau,s}(t) \), which are derived from a time - localised mother wavelet, \( \varphi(t) \), by translation and dilation. Wavelets have a finite energy and compact support, i.e. they grow and decay in a limited time period and are defined as

\[
\varphi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \varphi\left(\frac{t - \tau}{s}\right)
\]

where \( \tau \) is the time position (translation parameter) \( s \) is the scale (dilation) parameter related to the frequency, and \( \frac{1}{\sqrt{s}} \) is a normalisation parameter to ensure that wavelet transforms are comparable across scales and time series.

To be a mother wavelet i.e. a function admissible for wavelet analysis, \( \varphi(t) \), must fulfil certain criteria: it must have a zero mean, \( \int_{-\infty}^{+\infty} \varphi(t)dt = 0 \); its square must integrate to unity to ensure that \( \varphi(t) \) is limited to an interval of time; and it should satisfy an admissibility condition i.e.

\[
0 < C_\varphi = \int_{0}^{+\infty} \frac{|\hat{\varphi}(\omega)|^2}{\omega} d\omega < +\infty
\]

where \( \hat{\varphi}(\omega) \) is the Fourier transform of \( \varphi(t) \), that is \( \hat{\varphi}(\omega) = \int_{-\infty}^{+\infty} \varphi(t)e^{i\omega t} dt \).

The continuous wavelet transformation of a time series \( x(t) \) with respect to \( \varphi(t) \) is given by the following equation:

\[
W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\varphi^*_{\tau,s}(t)dt
\]

\[
= \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t)\varphi^*\left(\frac{t - \tau}{s}\right)dt
\]

where * denotes the complex conjugate. As with its Fourier counterpart, there exists an inverse wavelet transform, defined as:

\[
x(t) = \frac{1}{C_\varphi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varphi_{\tau,s}(t)W_x(\tau, s) \frac{d\tau ds}{s^2}
\]

Thus one can recover the original series \( x(t) \) from its wavelet transform by integrating over all scales and time positions.

Similar to Fourier analysis, one can define several quantities in the wavelet domain. For example, one is able to define the wavelet power spectrum as \( |W_x(\tau, s)|^2 \), which measures the contribution at each time and scale to the variance of the time series. Moreover, one can also identify the cross - wavelet spectrum, which captures the covariance between two series in the time frequency space. Given two time series \( x(t) \) and \( y(t) \), with wavelet transforms \( W_x(\tau, s) \) and \( W_y(\tau, s) \), one defines the cross wavelet spectrum as \( W_{xy}(\tau, s) = W_x(\tau, s)*W_y(\tau, s) \). As the
mother wavelet is in general complex, the cross - wavelet spectrum is also complex valued and can be decomposed into real and imaginary parts.

As noted by Rua (2010), one can also obtain the following measure, known as the wavelet coherence:

$$\rho_{xy}(\tau, s) = \frac{\Re (W_{xy}(\tau, s))}{\sqrt{|W_x(\tau, s)|^2 |W_y(\tau, s)|^2}}$$  \hspace{1cm} (A5)

where $\Re$ denotes the real part of the cross - wavelet spectrum which measures the contemporaneous covariance. This wavelet based measure $\rho_{xy}(\tau, s)$ allows one to quantify comovement in the time - frequency space and analyse over which periods of time comovement is higher. Intuitively, it plays the role of a contemporaneous correlation coefficient around each moment in time and for each frequency. This is important since the strength of a comovement, for example between measures of international risk sharing such as consumption growth rates, may vary over time. Thus by inspecting contour plots of this measure, one can identify the regions in the time frequency space where the two time series comove and assess both time and frequency varying aspects of this comovement, which is undoubtably a richer measure than more traditional correlation measures.
7.5 Wavelet Analysis - Comparing Common Shocks vs International Consumption Risk Sharing With Financial Integration

The following graphs present the main results of this study, the correlation of own country and rest of the world consumption growth rates over time and frequency dimensions, which are yielded by the wavelet analysis. The way to read the wavelet graphs is as follows. Frequency (in years) is plotted on the vertical (from highest to lowest frequency as one moves from top to bottom of the graph) and the timescale on the horizontal axes respectively. The graph is read in a manner akin to a contour plot, with increasing shades of darkness indicating a higher comovement between the two variables. Thus one can identify the frequency bands (on the vertical axis) and the time intervals (on the horizontal axis) where the variables move together and the extent to which the strength of the comovement varies over frequency and time.

Below, the correlations between own country and rest of the world consumption growth rates are displayed in the left hand pane (for real private consumption data), while the correlation between own country and rest of the world output growth is featured in the middle pane. Finally, the right hand most pane displays the measures of international financial integration. For each country in the sample, the measure of de jure financial integration is the international capital flows subindex of Abiad et. al. (2010) (dashed line, right hand axis) whilst the measure of de facto integration is the ratio \( \frac{\text{external assets} + \text{external liabilities}}{\text{GDP}} \), calculated from Lane and Milesi-Ferretti (2007) (thick bars, left hand axis).

Figure 2 - Wavelet Analysis of Consumption Growth Rate Correlations (left hand pane), Output Growth Rate Correlations (middle pane) Compared With The Role of Financial Integration (right hand pane)
Thus one can identify whether the increased comovement of consumption growth rates is driven by common shocks or stems from improved international consumption risk sharing. In the former case, common shocks would be indicated by simultaneously (at the same time and frequency) high comovements of consumption growth rates and output growth rates. In the case of improved international consumption risk sharing, increased consumption growth rate comovement would occur even in the absence of high comovement of output measures. Moreover, comparison of the above figures also facilitates an understanding of how consumption growth rate comovement evolves over different periods of de jure and de facto financial integration.