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Characteristics of Tunisian Business Cycle and International Synchronisation

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CHARACTERITISTICS OF THE TUNISIAN BUSINESS CYCLE AND ITS INTERNATIONAL SYNCHRONIZATION

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Abstract

In this paper, we have applied spectral and cross spectral analysis techniques as an alternative approach to characterize the Tunisian business cycle and measure the degree of its international synchronization. As a robustness check, we have applied these techniques to the industrial production (overall and manufacturing IPI) as well as two synthetic indexes: a dynamic factor and a diffusion index. We found the presence of two types of cycles: a minor cycle of 12.5 quarters (3.1 years) detected in Tunisia and in all its European trade partners and a major cycle of 33.3 quarters (8.3 years) observed in the majority of the cycles studied. The cross–spectral analysis provides a strong evidence of synchronization of the Business cycle in Tunisia and its European partners, particularly at high frequencies. The volatility of the Tunisian business cycle is generally lower than that of the European cycle. It is even lower for longer cycles. The transmission of cyclical shocks from the Euro Area to Tunisia is instantaneous for short cycles. The delays are much longer for the major cycles. They can reach 5 to 6 quarters.

JEL classification:

Keywords: Business cycle, spectral analysis, international synchronization…

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1. Introduction

The paper analyses business cycle in Tunisia and its synchronization with those in the country's main trade partners. Nowadays, there is a renewed interest in business cycle analyses because of a growing sensitivity of cycles of national economies to international markets fluctuations. Greater interdependencies between economies, creation of regional economic groupings, diversification of financial instruments as well as development of New Technologies in Information and Communications are key drivers of this change.

Growing integration of the Tunisian industry into the global value chain and the implementation of the free trade agreement with the European Union led to a greater concentration of the Tunisian foreign trade on a few European countries. Such evolution should imply that the Tunisian business cycles are strongly correlated with the European counterparts. However, the recent findings do not provide clear results. On one hand, Elachehab (2010) and Chebbi and Knani (2013) find that, while the correlation of the Tunisian cycle with the French business cycle is strong, it is weaker with respect to Germany and Italy. On the other hand, Medhioub and Mraïhi (2011) point that the degree of synchronization with the European partners becomes statistically significant when taking into account transmission delays. Ambiguity of results is also related to the basic characterization of the Tunisian Business cycle. Indeed, while Elachehab (2007, 2008), and Chebbi and Knani (2013) conclude that the Tunisian business cycles are shorter than those in the advanced economies, Male (2010) shows that they are on par.

When studying the characteristics of Tunisian business cycle, the existing literature use the Bry-Boschan algorithm or Markov switching regimes models to detect the turning points and to measure the average length of the cycles. The Harding and Pagan (2002, 2006) methodology is used to assess the international synchronization. To overcome the ambiguity of results found in the literature, this paper exploits spectral analysis to understand the Tunisian business cycle characteristics and measure its synchronization with cycles of its European partners. To our knowledge, no previous study has applied this technique to study the Tunisian Business Cycle. Korotayev and Tsirel (2010) use it to verify the existence of Kondratieff waves in the world GDP dynamics. Krupa and Skrzypczyński (2012) use it also to study the synchronization between the US and emerging markets business cycles.

In the paper we apply a univariate spectral analysis to a set of measures of the business cycle to detect which frequency corresponds better to the average length of the Tunisian and European business cycles (Euro Area, France, Germany and Italy). Among the measures of the Business cycle, we selected the Industrial Production Indexes series as well as two synthetic indexes: a diffusion index and a dynamic factor. Then we use the cross-spectral analysis to check the overall correlation between the business cycles, compare their respective amplitudes, and transmission delays frequency by frequency. We find the empirical support for the existence of both minor and major cycles in Tunisia. We detect a significant synchronization between Tunisian and European business cycles, especially between the minor ones.

Applying the same approach, the findings of this paper suggest the presence of both minor and major cycles. We found also a significant degree of synchronization between the Tunisian and the European business cycles especially between the minor cycles.

This paper is organized as follows: section 2 presents a review of the related literature on the business cycle. Section 3 deals with data and the statistical tools to extract their cyclical components. Section 4, discuss the length and amplitude of the business cycle in Tunisia and its synchronization with the business cycles in the Tunisia's European main trade partners using the spectral and cross-spectral analysis. Section 5 concludes.

2. Review of the related literature

"The crises are nothing but turning points from prosperity into depression, and it is the alternation between prosperity and depression which is the really interesting phenomenon" [Schumpeter (1931)]. Valuing the contribution of Clément Juglar to the development of the business cycle theory, the Austrian economist Joseph Schumpeter focused on the focal point of Juglar's work namely to have turned the economic debate from studying the crisis, considered by the time as an exogenous phenomenon, and the reasons of their emergence to the observation and analysis of cyclical economic fluctuations in which crises are only a turning point. This was the starting point of modern theories of economic cycles.

The main believe at that time was the existence of different types of cycles (Kondratieff (1935), Juglar (1862), and Kitchin (1923)) differing in their duration (50 years, 7 to 11 years and 3 years) and their explanations. Schumpeter proposed a very systematic vision of "nesting" different types of cycle: each Kondratieff cycle contains 6 Juglar cycles of 9-10 years, and every Juglar cycle would consist in 3 Kitchin cycles of a slightly over 3 years each. Whenever all types of cycle pass through the same phase, this phase would be of an exceptional intensity (eg 1929 crisis).

However, the idea of the existence of long cycles is not confirmed[1] by modern statistical methods of extraction of cyclical components of time series, neither is the idea of nesting different types of cycle. Burns and Mitchell (1946) from the National Bureau of Economic Research (NBER) focused on the Business cycle (major/trade cycle) and gave in their reference book "Measuring Business Cycles" a definition that denies the idea of nesting different cycles. According to them, *"Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own".* [2]

This definition became a cornerstone of the applied research on Business cycle measurement and analysis. Since Burns and Mitchell, academic research followed two completely different paths. On one hand, a theoretical debate on the exogeneity versus endogeneity of cycles to general equilibrium models pitted the freshwater economists (new-classicals) against the saltwater economists (new-Keynesians…)[3]. On the other hand, great advances have been made in terms of statistical techniques of extraction and analysis of business cycles. These

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[1] Only a recent research employing spectral analysis has confirmed the presence of Kondratieff waves in the world GDP dynamics at an acceptable level of statistical significance – see Korotayev A. V., and Tsirel S. V. (2010)

[2] Burns A. F. and Mitchell W. C. (1946), "Measuring Business Cycles" p 3 – National Bureau of Economic Research

[3] The NBER Working Paper n°2982 (1989) of Robert Barro "*NEW CLASSICALS AND KEYNESIANS, OR THE GOOD GUYS AND THE BAD GUYS*" and the Editorial of Paul Krugman in the New York Times of February 17th, 2014 (in the Opinion Page) "*The Trouble With Being Abstruse (Slightly Wonkish)*", are the most famous among papers and articles dealing with opposition between these school of thought when explaining the drivers of the business cycle.

techniques accompanied the awareness by the economists of the non-linearity of the trend of economic activity and the observed asymmetry of cycles and gave place to new concepts and methods.

On the empirical side, the concept of "growth cycle", opposed to "classical cycle", appeared in the post-war period which was marked by a long sustainable growth (thirty glorious) and where the low phases of the cycle were not characterized by a decrease in the absolute level of activity (depression) but rather by relatively slow rates of growth and a slowdown in economic activity (recession). Therefore, the Burns-Mitchell definition of business cycles has been adjusted for growth cycles (Mintz 1972) as follows: "*Growth cycles are fluctuations in aggregate economic activity. A growth cycle consists of a period of relatively high growth rates occurring at about the same time in many economic activities, followed by a period of similarly widespread low growth rates which merges into the high-growth phase of the next cycle*". The OECD also defines the growth cycle as the "*fluctuations in the economic activity around the long-run potential level, or fluctuations in the output-gap*".

Allowing for fluctuations around growth rates rather than only around the absolute level as in the definition of Burns and Mitchell, this new definition of Business cycle, raised the question of de-trending the economic series to extract the phases of the cycle fluctuating around a non-linear trend. Literature proposes a wide range of statistical techniques of decomposing the time series into cyclical and trend components[4]. The most used technique is the Hodrick-Prescott filter (HP 1997). It is a high-pass filter which minimizes the difference between actual time series and their trend component. Other filters operate in the frequency domain and isolate the cyclical component of a time series by specifying its duration e.g. Band-Pass filters (Baxter-King (1999) and Christiano-Fitzgerald (2003)).

Harding and Pagan (2002, 2006) propose a special methodology for characterizing business cycles in developing and emerging countries and assessing their synchronization with those in industrial economies. They apply Bry-Boschan (1971) algorithm to detect the turning points of a business cycle and measure their mean duration and amplitude. Then, they create binary variables for each phase of the business cycle (expansion and recession) in the emerging countries as well as in some industrial countries. These variables are used to create a concordance statistic to measure the degree of coordination between the business cycles (i.e. measure of the proportion of time when two business cycles were in the same phase).

Recently, spectral analysis has been used in the business cycle analysis. In particular, Korotayev and Tsirel (2010) confirm the presence of Kondratieff waves in the world GDP dynamics at an acceptable level of statistical significance after a long denial of this thesis for decades. Skrzypczynski (2010) uses it to study business cycle characteristics in Poland.

The diffusion of these new techniques allowed for a revival of empirical research on the stylized facts about business cycles and their impact on the economic aggregates and the stabilizing policies mainly in industrial economies. However, less attention has been paid for the study of Business cycles in emerging and developing countries.

Recent literature on business cycles in the middle-income emerging countries shows that although there exist some similarities of business cycles in developing and developed countries, there are also significant differences and that the stylized facts of business cycles are rather country-specific (or intra-group specific).

For a group of 12 middle-income developing countries Agénor, McDermott and Prasad (2000) using HP and band pass (Baxter-King) filters show that output volatility is much higher than that observed in the industrial countries. The business cycle in these economies is positively influenced by those in the industrial countries. A fiscal impulse in the developing countries is negatively correlated with the business cycle, what means that fiscal policy tends to be counter-cyclical. Real wages are procyclical. There is no robust evidence of

[4] For a survey of these techniques see Canova F. (1998).

a consistent relationship between output and prices. There is a weak positive correlation between domestic credit and industrial output. Moreover, the cyclical movements in the terms of trade are strongly and positively correlated with the output. The authors conclude that the supply-side shocks are most important in driving business cycles in developing countries although the results are not uniform across countries[5].

Rand and Tarp (2002) confirm the role of aggregate supply shocks for the cyclical behavior of emerging markets and demonstrate that business cycles in these countries are generally shorter[6] and more volatile than in developed countries, consumption and investment are procyclical. However, in contrast to Agenor, McDermott and Prasad, Rand and Tarp do not find government consumption as countercyclical.

Using Harding and Pagan method for 6 emerging economies Du Plassis (2006) does not find any similarity in duration between their business cycles. Fluctuations seem to have greater amplitude than those in the industrial countries and there is little evidence that output cycles move jointly across emerging and developed economies. There is some evidence of consistency of counter-cyclical monetary policy in some emerging markets similar to that in the USA. He supports the conventional view that *monetary policy is a more flexible stabilization tool than discretionary fiscal policy*.

Using the same method Male (2010) studies the "classical" industrial business cycles of 32 developing countries and finds that their business cycles are not significantly shorter than those in the developed countries. However, there are distinct patterns between regional groups. While the North African and Eastern European countries business cycles are on a par with the developed countries, those of the African countries are significantly shorter while in Asian countries substantially longer. Furthermore, the amplitude of both contraction and expansion is significantly greater in developing than in the developed countries but there is no clear pattern of synchronization between their business cycles.

There are a few studies of Tunisia. For example, Elachhab (2007), (2008), (2010) using Bry-Boschan algorithm (BBQ) to extract the cyclical component of the Industrial Production Index (IPI) and a structural vector autoregression model (SVAR) to verify the role of domestic and foreign supply and demand shocks, detects three minor cycles (of a 3 years average duration) and four major cycles (of a 5 years average duration). Domestic supply shocks seem to explain 54% of the Tunisian business cycle on a four years horizon; domestic demand shocks 35%, whereas external shocks only 18%. Contractions are mainly influenced by the supply shocks and foreign demand shocks. Tunisian business cycle is found to be positively correlated with this in Europe, and especially in France, but much less with that in Italy and Germany. The latter result has been also supported by Chebbi and Knani (2013). Moreover, the correlation between the cycle in Tunisia and France has been growing due to the intensity of bilateral trade, similarity of economic policies. Decrease of the bilateral trade and the dissimilarity between the economic structures are the main reasons of the decreasing correlation between the Tunisian and the German cycles while the dissimilarity of the industrial structure and of the goods and labor markets functioning were the most important determinants of the weaker synchronization between Tunisia and Italy.

Medhioub and Mraïhi (2011) identify turning points and extract the states of the cycle through a Markov switching regime model and use them in a modified version of the Harding and Pagan (2004, 2006) concordance index. They show that when the transmission delays of shocks are taken into account, the industrial cycle concordance is statistically strong and significant; otherwise the synchronization between Tunisia and its main euro-Mediterranean partners is weak.

[5] This conclusion is very important for the economic authorities when designing their stabilization policies.

[6] The average duration found varies from 7.7 to 12 quarters (1.9 to 3 years) compared to duration of 24 to 32 quarters (6 to 8 years) in industrialized countries.

Although this literature on the Tunisian Business tried to help understand its main characteristics and its degree of synchronization with the European business cycle, the findings are still ambiguous and do not permit to draw a clear conclusion. Through assessing different measures of the cycle (IPI and synthetic indexes) and using the spectral analysis method, this paper provides further advance in understanding these issues.

3. Extracting the Business cycle

In this section we discuss possible indicators, especially from a perspective of a genuine middle-income-developing country, as well as a choice of the filtering technique.

3.1. Methodological issues: Which indicator for the Business cycle?

The early literature used GDP or GNP as a general indicator of economic activity to extract its cyclical components, but it was criticized in the 1970s due to perceived uncertainties in the measurement of GNP and very frequent revisions increasing the likelihood of selecting wrong turning points. Moreover, GNP data are not available monthly, whereas a monthly reference chronology is required.[7]

In a developing country context the use of GDP data for measuring business cycle activity can be problematic because agriculture, which still accounts for a large share of aggregate output in many cases, is more influenced by weather conditions than by cyclical factors. Poor measurement of services and informal sector activities may also impart significant biases Agénor, McDermott and Prasad (2000). If we add to these argument the fact that Tunisia quarterly GDP series are only available for the last 15 years (they start in 2000), we can reject the use of GDP as the main economic indicator for measuring the business cycle.

On the other hand, industrial production index captures business cycles even in countries with a moderate contribution of industry to overall GDP. In Tunisia it accounts for 29% of the total value added, nonetheless it remains the main contributor to the variance of the GDP of the market activities (64% in Tunisia[8]) which is generally linked to the business cycle fluctuations. Industrial Production index is also available for a longer period (it starts in 1990, with an acceptable quality) and on a monthly basis. It seems to be a better candidate for our purpose.

However, the IPI is calculated as a weighted average of individual production indexes in many sub-sectors and the weights reflect the structure of the economy for a specific basis year. Since, the economic structure evolves rapidly, especially in developing countries, the cyclical component extracted from this indicator could be amplified or reduced according to the diffusion of the upswings and downswings of the cycle to more or less weighted subsectors. To address this problem and to check the robustness of the cyclical components of the Industrial production Index, we add two different synthetic measures of the cycle: a dynamic factor and a diffusion index.

To calculate these indexes, we used the most detailed industrial production indexes for the sub-sectors and we added to these index some indicators of economic activity in other sectors such as tourism (nights-by), transport (number of flights, number of airways and railways fret and/or passengers, road-fuel consumption), and construction (local sale of cement).

The dynamic factor is generally used in forecasting and studying the monetary policy transmission channels. This technique makes it possible to separate the common components of a large number of time series from their idiosyncratic components using a state-space

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[7] Mintz I. (1972), "Dating American Growth Cycles" p 41 – National Bureau of Economic Research [Economic Research: Retrospect and Prospect, Vol 1, The Business Cycle Today – Chapter 2]

[8] For More details, see appendix 1

model. To extract this dynamic factor, we used a two steps method: in the first step, we extract a common factor out of a balanced dataset (the largest possible sub-sample) through a principal component analysis (PCA) that we use in the second step to calculate the dynamic factor all along the sample by applying the Kalman filter and smoother.[9]

The first dynamic factor is considered as the unobservable component that explains the most common behavior of the total dataset used. Normally, it should include cyclical and irregular behavior. Since by definition, the business cycle is a kind of common behavior of the growth rates of a large number of sectors, all we have to do is the eliminate the irregular component to get a business cycle approximation using this technique.

The diffusion index shows deepness of the diffusion of cyclical shocks. It is constructed in the following way:

1- For each index or variable, we calculate the growth rate (monthly or quarterly) in each period

2- In each period and for each variable, if the growth rate is strictly negative and lower or equal to the mean negative growth rate, we attribute a score (0). If it is negative but higher than the mean negative growth rate, we attribute a score (0.25). If it is approximately null, we attribute the score (0.5). For each positive growth rates, if they are less than their mean positive level, a score of (0.75) is attributed. Otherwise, the score will be (1). By doing so, we substitute each value of each variable by a score varying between 0 and 1.

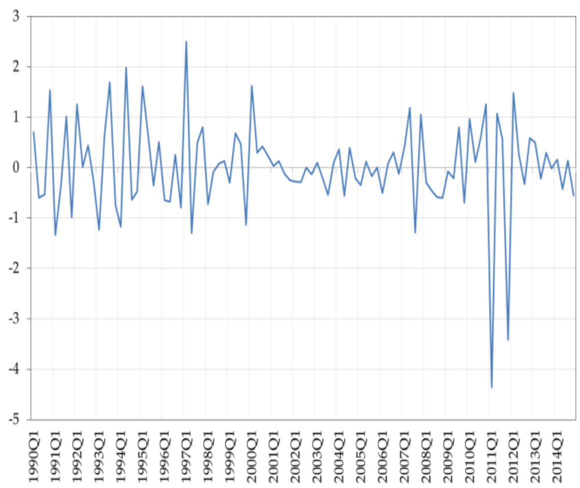
3- Finally, we calculate a simple average of the scores for each period and multiply it by 100 treating all the variables equally: all what we need to know is how much the cyclical movement is spread over the sectors. A value of 50 means that there is as much of positive growth rates as negative. A value of "0" means that the activity of all sectors decreased, while a value of 100 means the opposite.

Having five levels of score allows us to take into account the amplitude of the global movement in addition to the diffusion degree.

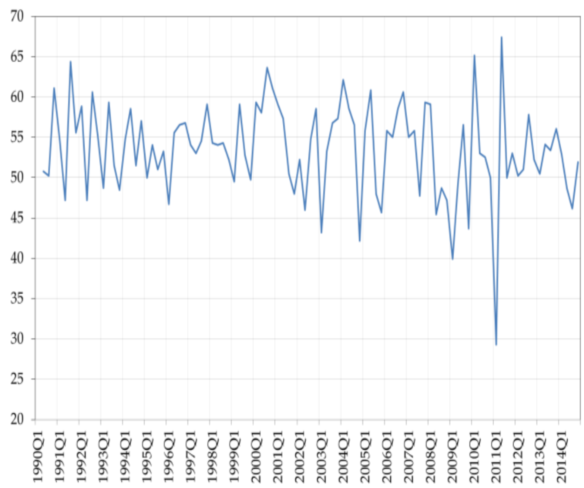
Like the dynamic factor, the diffusion index is calculated starting from growth rates. Therefore, it includes the cyclical movement as well as an irregular component. We need, then, to filter it to extract the cyclical part.

Applying these techniques to the quarterly values of the series in the dataset, we obtained the following indexes that appear to be stationary:

Graph 1. ***Dynamic Factor extracted from the quarterly growth rates of series in the dataset***



Graph 2. ***Diffusion Index extracted from the quarterly growth rates***



[9] For more details about the Dynamic Factor modelling and the functioning of the Kalman filter and smoother, see Appendix 2

3.2. Methodological issues: Which filtering technique for the Business cycle?

The mostly used techniques are statistical filters such as Hodrick-Prescott filter (HP), Beveridge-Nelson decomposition (BN) and Band-Pass filters (Baxter-King (BK), Christiano-Fitzgerald (CF)). Choosing the optimal filter for extracting the business cycle is not easy because each technique can extract different types of information from the data with different statistical moments. Canova (1998), points that data should be passed through a variety of detrending filters which emphasize different business cycle concepts in order to check the implications of theoretical models over a wide range of cyclical frequencies. However, some methods could extract trends which have undesirable features (e.g. BN trends are in some cases more volatile than the series themselves).

In this paper we use both the adjusted HP filter and Christiano-Fitzgerald filter: the former because it is the mostly used filter and the latter because it operates in the frequency domain just like the spectral analysis we conduct.

In this paper, we chose to use both an adjusted HP filter and the Christiano-Fitzgerald filter: the first because it is the mostly used filter and the second because it operates in the frequency domain just like the spectral analysis we intend to conduct.

For the HP filter, we used both simple HP and double HP filters depending on the quarterly time series selected. The double HP filter has been applied to the IPI series. We used, at first, a parameter $\lambda_1 \approx 1$ to eliminate the irregular component corresponding to a 6-quarters frequency. The obtained series includes the trend and the cycle components. These series are filtered again by choosing $\lambda_2 \approx 1600$. The values of λ are obtained through the following formula of Iacobucci and Noullez (2004):

$$\lambda = [2 \sin(\pi \,.\, \nu_c \,.\, \Delta_t)]^{-4}$$

where ν_c stands for the cutoff frequency calculated as inverse of the suggested maximum number of years per cycle (e.g. "1/10" for a 10 years cycle and "1/1.5" for a 6 quarters cycle") and Δ_t the number of observations per year (e.g. "1/4" for quarterly data, "1/12" for monthly data…). For λ_1, $\nu_c = 1/1.5$. It is equal to 1/10 for λ_2. By applying the double HP filter, this technique is transformed from a high-pass filter to a band-pass filter.

To extract the business cycle from the diffusion index and the dynamic factor, we do not need to double filter the series since they do not include a trend component. All we need is to eliminate the irregular component through a simple HP filter using $\lambda_1 \approx 1$.

Considering the Christiano-Fitzgerald filter, we chose the asymmetric version to avoid the truncation at the extremity of the series and chose the band frequency 6 - 40 corresponding to a minimum of 1.5 year and a maximum of 10 years of the Business cycle assumed duration in accordance with the definition of Burns and Mitchell.[10]

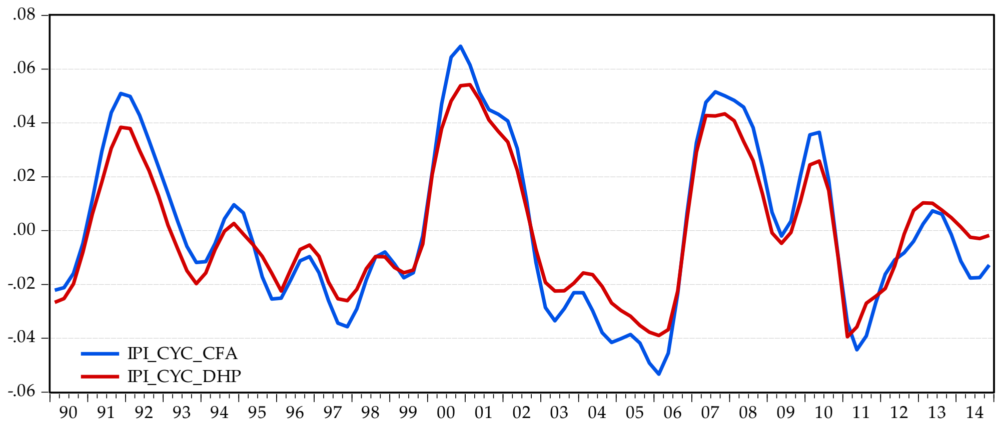
3.3. Estimation results

The results show some differences in the amplitude and volatility between the cyclical components of the IPI series (overall and manufacturing [1990-2014]) and those of the diffusion index and the dynamic factor. The IPI series appears to be smoother and with higher amplitude at low frequencies and contain small amplitude variation at higher frequencies (cf. graph3). Nevertheless, the synthetic indexes are more volatile and show higher amplitude at higher frequencies.

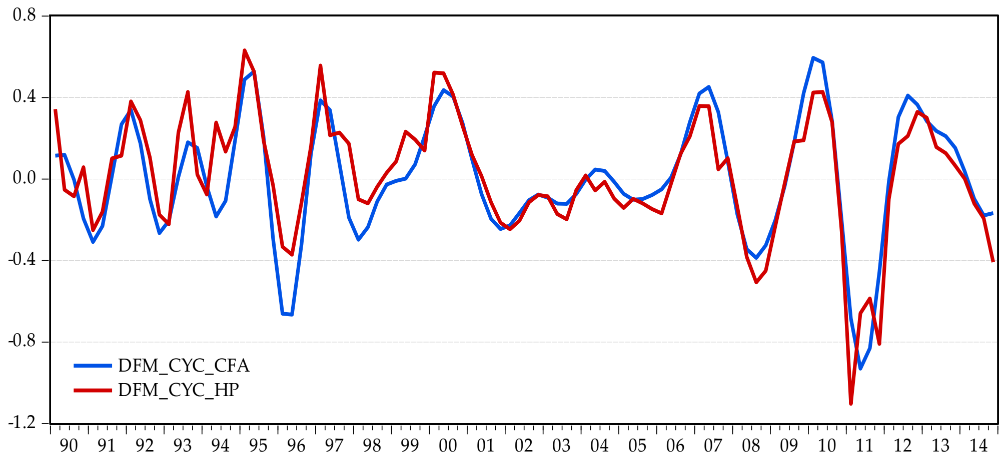
These results confirm the limits of using IPI for extracting Business cycles since the considerable variation in its cycle does not seem to be widely spread over sub-sectors of the economy as measured by the synthetic indexes. This could be due to the aforementioned over-weighting bias.

[10] When applying this method, we took into account the difference of the order of integration of the series: I(1) for the IPI series and I(0) for the Diffusion Index and the Dynamic Factor.

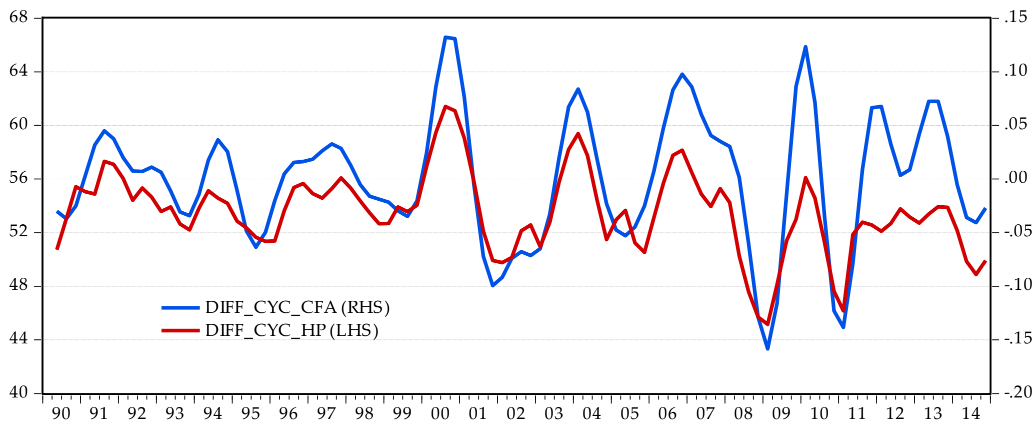
Graph 3. ***Cyclical components of the Industrial production Index***



Graph 4. ***Cyclical components of the Dynamic Factor***



Graph 5. ***Cyclical components of the Diffusion Index***



We notice also that both detrending methods give approximately the same features except for the Christiano-Fitzgerald filter when applied to the diffusion index which amplifies the cyclical movements at high frequencies (cf. graph 5).

4. Characterizing the Tunisian Business Cycle and its international synchronization through spectral analysis :

This section discusses the length and amplitude of the business cycle in Tunisia and its synchronization with the business cycles in the Tunisia's European main trade partners: Euro Area (19 countries), France, Germany, Italy and Spain.

4.1. Methodological issues:

Spectral analysis is a statistical technique that makes it possible to look at the variance of the time series as a distribution over the frequency domain. In business cycle research it can provide useful information on the length and amplitude of the Business Cycle.

The basic tool of spectral analysis is the spectrogram.[11] It is a Fourier Transform of the Auto-Correlation Generating Function of a time series. The basic principle of Fourier analysis is that any function defined over a finite interval of R can be written as weighted sum of sine and cosine functions. This approximation allows recognizing all periodic components - i.e. cyclical - presenting in a curve.

The spectrogram is used to locate the periodicity in the data. A high value of the spectrogram (spectral density) suggests that the series has a periodic component to the corresponding frequency.

The peaks in the spectrogram help identify the frequencies that best explain the dynamics of a series. The peaks between 1.5 and 10 years are considered here as signs of cycles.

In order to conduct the spectral analysis, a time series needs to be stationary, which is the case of our selected cyclical components of the economic activity measures.

To study the international synchronization of the Tunisian business cycle, we have first investigated characteristics of the business cycles of the Euro Area (19 countries), and separately France, Italy and Germany being the main foreign partners of the Tunisian industry. In this aim we have applied the same detrending methods and the spectral analysis to their Industrial Production Indexes and their Business Confidence Indicators.[12]

The cross-spectral analysis is an alternative to Harding and Pagan (2002, 2006) to measure the degree of synchronization between two business cycles. It offers a set of statistics to gauge intensity of the correlation of the business cycles frequency by frequency. These tools are the "Coherence", the "Gain" and the "Phase-shift".

"Coherence" provides an overall measure of association between two series at each frequency. Ranging from 0 to 1, it is a measure of the overall goodness of fit between two time series frequency by frequency just like the R^2 is in the time series domain regressions. If the coherence is high at a specific frequency "ω", then the two series have cycles in common with the frequency "ω". We use this tool at first to check for the existence of any relationship between the different cycles studied.

The "Gain" measures the relation between the amplitudes of the series y_t and x_t frequency by frequency. If the "Gain" value is less than unity at a given frequency, then the series x_t has higher amplitude than the series y_t at that frequency. Thus, "Gain" can be interpreted as the ratio of cycle amplitudes in two economies in our study.

The "Phase shift" shows the lead/lag relation between y_t and x_t at a given frequency. If it is negative at a specific frequency, then x_t leads y_t at that frequency and vice-versa. A null value means a synchronous relation.

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[11] More details on the univariate and cross spectral analysis tools are in Appendix 3

[12] We chose the Business Cycle Indicators published by the OECD organization.

4.2. Estimation results: business cycle length

Inspection of the spectral density values generated by the spectrogram[13] obtained from the Tunisian business cycle measures gives us the following observations. Firstly, there is a strong evidence of a superposition of at least two types of cycles at different frequencies: a major cycle of an average duration varying between 24 and 33 quarters (6.3 and 8.3 years) and a minor cycle of an average duration of 12.5 quarters (3.1 years) across different measures and detrending methods. This can explain divergent results of the previous literature on the Tunisian business cycle where Male (2010) estimated an average duration of the Tunisian business cycle of around 7.6 years while Elachhab (2007) detected shorter cycles: three minor cycles (of a 3 years average duration) and four major cycles (of a 5 years average duration).

Secondly, the amplitude of these types of cycle differs from one measure of the economic activity to another. When using the IPI or the IPI in the manufacturing sector, we can notice that the major cycle has greater amplitude and is clearly identifiable. However, when using the synthetic indexes such as the dynamic factor and the diffusion index, the major cycle is still visible but with a much lower amplitude than the shorter one. The greatest amplitude is that at the 12.5 quarters (3.1 years) duration cycles. We notice also the presence of an even shorter type of cycles for which duration ranges from 6.6 to 10 quarters but this observation is not confirmed by other measures of the business cycle.

Since the synthetic indexes are supposed to reflect better the co-movement of the activity of all economic sectors than the weighted-indexes such as the IPI, we can assume that the minor cycles of an average duration of 12.5 quarters captured by these indexes are more widespread over the economy, probably, reflecting a change in the demand. However, the major cycle is better captured by the IPI. This could probably reflect more the structural inter-sectorial change in the economy.

The results of the spectral analysis of the European business cycles are in some way similar to what we found for the Tunisian case since they show a significant presence of minor cycles in all European economies and across all measures at the same frequency as in Tunisia (i.e. 12.5 quarters or 3.1 years). We find also a strong evidence for the presence of at least one longer type of cycles but at different frequencies across the countries.

<u>Tab.1</u> Average durations detected by the spectral density for the European business cycles

	Industrial Production Index		*Business Confidence Indicator*	
	Double HP filter	*Christiano-Fitzgerald filter*	*HP filter*	*Christiano-Fitzgerald filter*
Euro Area (19 countries)	- 33.3 q (8.3 y) - 16. 7 → 20 q (4.2 → 5 y) - **12.5 q (3.1 y)**	- 33.3 q (8.3 y) - 16.7 → 20 q (4.2 → 5y) - **12.5 q (3.1 y)**	- 33 q (8.3 y) - 16.7 q (4.2 y) - **12.5 q (3.1 y)**	- 33 q (8.3 y) - 16.7 q (4.2 y) - **12.5 q (3.1 y)**
France	- 33.3 q (8.3 y) - 16. 7 → 20 q (4.2 → 5 y) - **12.5 q (3.1 y)**	- 33.3 q (8.3 y) - 16.7 → 20 q (4.2 → 5y) - **12.5 q (3.1 y)**	- 16.7 q (4.2 y) - **12.5 q (3.1 y)**	- 16.7 q (4.2 y) - **12.5 q (3.1 y)**
Germany	- 33.3 q (8.3 y) - 20 q (5 y) - **12.5 q (3.1 y)**	- 33.3 q (8.3 y) - 20 q (5 y) - **12.5 q (3.1 y)**	- 33.3 q (8.3 y) - 16.7 q (4.2 y) - **12.5 q (3.1 y)**	- 33.3 q (8.3 y) - 16.7 q (4.2 y) - **12.5 q (3.1 y)**
Italy	- 33.3 q (8.3 y) - 16.7 q (4.2 y) - **12.5 q (3.1 y)**	- 33.3 q (8.3 y) - 16.7 q (4.2 y) - **12.5 q (3.1 y)**	- 16.7 q (4.2 y) - **12.5 q (3.1 y)**	- 16.7 q (4.2 y) - **12.5 q (3.1 y)**

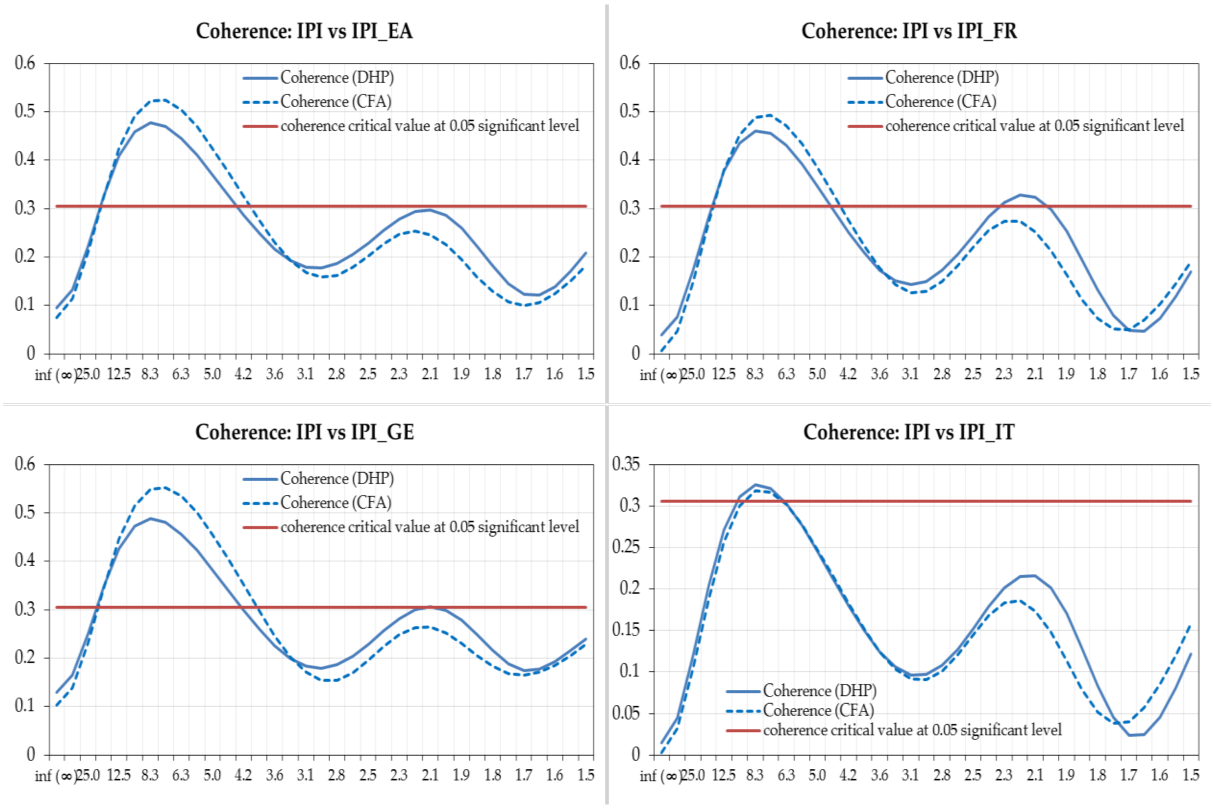
[13] See the graphs of the Spectral density in the Appendix 4

From these results, we may suspect that the minor cycle of 12.5 quarters (3.1 years) detected in Tunisia could be entirely driven by the cyclical behavior of the European demand to the manufacturing sector, especially the export-oriented industries (Clothing and textile, electrical and mechanical industries …) and the tourism and transport sectors. We can also think of the presence of some correlation between the Tunisian and the European business cycles at the lower frequency around 33.3 quarters (8.3 years). In the next section, we conduct a cross–spectral analysis to check this hypothesis.

4.3. Estimation results: international synchronization of the Tunisian Business Cycle

We have applied cross spectral analysis to various measures of the Tunisian business cycle (y_t), on one side, and selected European business cycles on the other side (x_t). When dealing with the cycles extracted from the IPI series, we have found evidence of the presence of a significant and moderate correlation (coherence values around 0.5) between the Tunisian and the European business cycles at low frequencies (around 8.3 years), especially in France and Germany while the correlation with the Italian cycle is less evident.

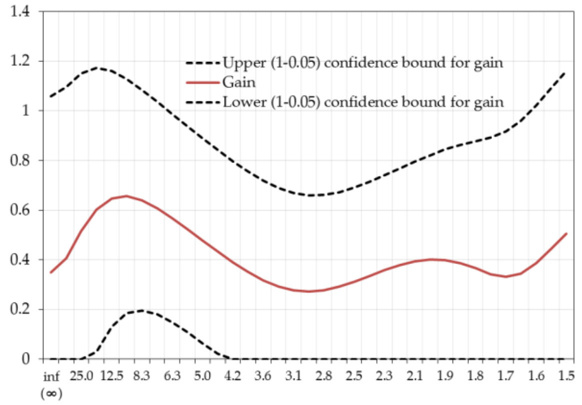
Graph 6. ***Coherence (Tunisian IPI vs European IPIs)***



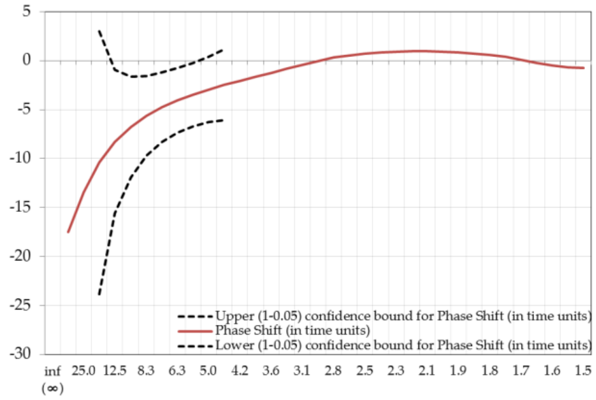
At this frequency, the central "Gain" value is between 0.6 and 0.7 which suggests that the Tunisian business cycle at the low frequency of 8.3 years is less volatile than this in Europe. This result is contradictory to what is generally observed in developing countries where the business cycle is more volatile than in the advanced economies. This could be explained by the fact that the major crisis that can be identified at this frequency (European Monetary system crisis 1993, World financial crisis 2008-2009) hit directly the Eurozone while the Tunisian economy was less open and therefore more immune to the effects of these crises. It was just hit by the spillover effect of the slowdown of the European demand to the rest of the world, with certain lag. The Tunisian economy was also hit by an idiosyncratic shock related to the political revolution. But, at this stage, we cannot confirm that this shock affected the cycle or the trend component of the economic activity.

Indeed, when analyzing the "Phase shift" statistics, we observe that the European business cycle leads the Tunisian cycle at the 8.3 years frequency by 5 to 6 quarters. This confirms the conclusion of the delay of transmission of the spillover effects of the slowdown of the European demand to the Tunisian economy.

Graph 7. ***Gain (Tunisian IPI vs European IPI) – DHP detrending method***



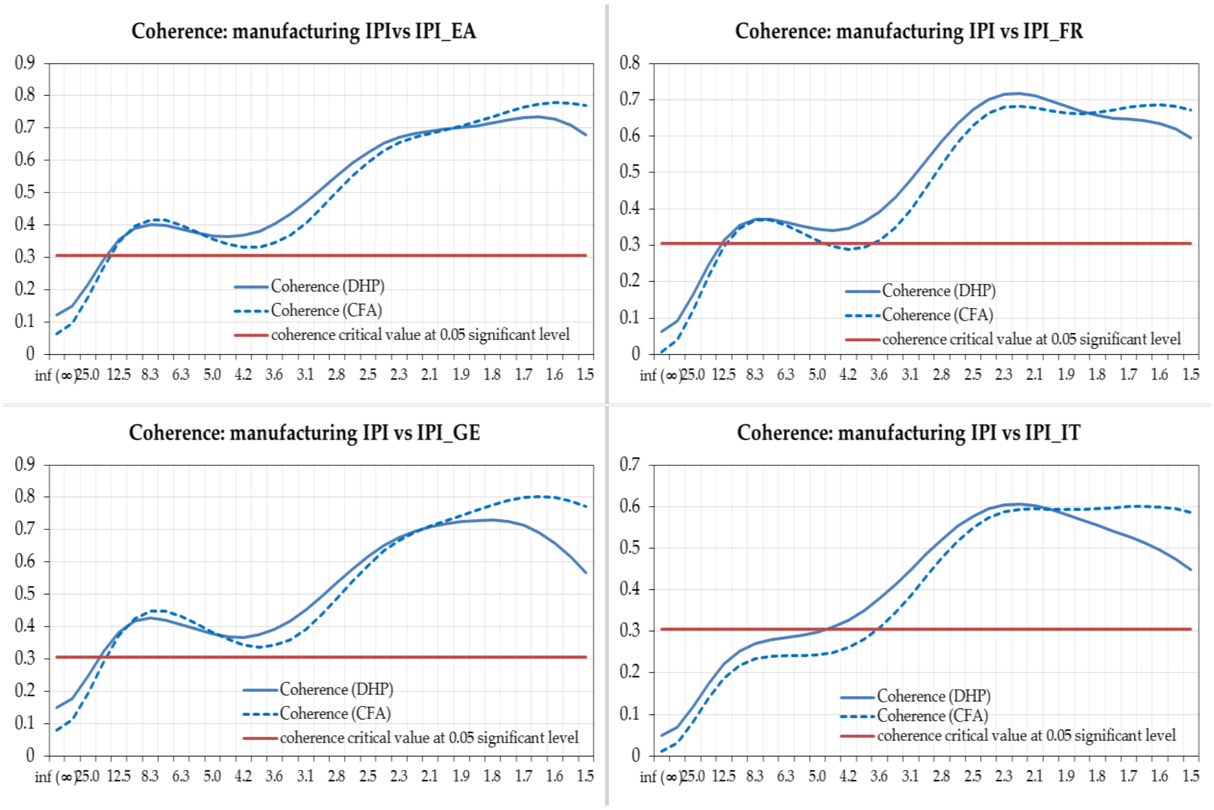
Graph 8. ***Phase shift (Tunisian IPI vs European IPI) – DHP detrending method***



These results answer partly to the hypothesis we draw in section 4.2., where we assumed the presence of some correlation between the Tunisian and the European business cycles at the low frequency of 33.3 quarters. But till now, we have not observed any relation between the cycles at higher frequencies. For deepening the analysis and since we suspect that the main driver of the correlation comes from the European demand for the Tunisian manufacturing, we have conducted the same exercise using the IPI in the manufacturing sectors and the results are more clear but at higher frequencies.

The new coherence values support previous results and indicate a stronger and more significant correlation between the Tunisian and European business cycles. The coherence value exceeds 0.5 for the frequencies higher than 3.1 years and reaches even 0.7 with certain measures of business cycles. The correlation by country shows that the Tunisian minor cycles are correlated with those in the three European partners studied. It is higher with Germany (0.8) and lower with Italy (0.6).

Graph 9. ***Coherence (Tunisian manufacturing IPI vs European IPIs)***

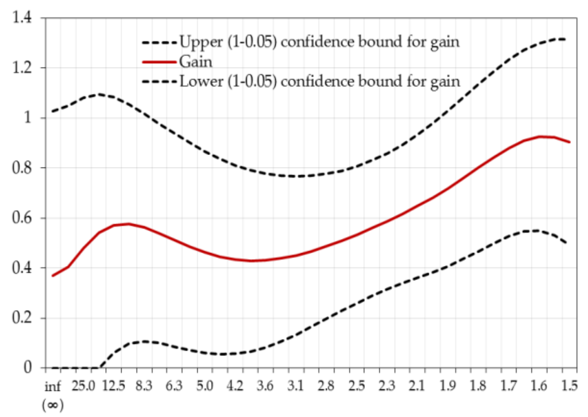


At these frequencies, the "Gain" statistics confirms that the Tunisian business cycle is less volatile than European cycles, but the difference of amplitude tends to be reduced when the cycles become shorter with "Gain values" approaching to 0.8 in Germany and Italy. It is close to unity when comparing to the French business cycle volatility at very short cycles.

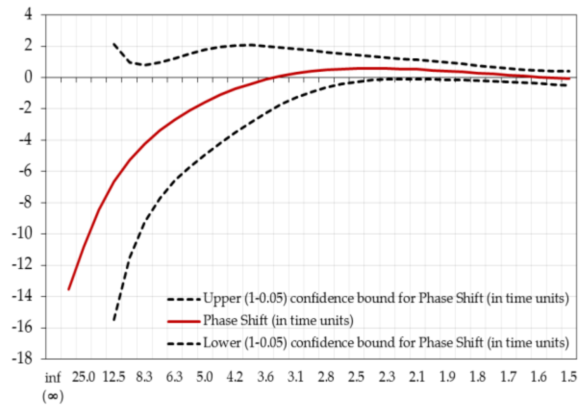
Regarding the transmission delays, the results confirm the relatively long delay of the major cycles (8.3 years) which reaches 5 to 6 quarters obtained when using the global IPI. But the new values of the "Phase shift" indicate that the transmission of the minor cycle shocks from Europe to Tunisia is almost instantaneous. The business cycles at high frequencies are synchronous at both sides of the Mediterranean.

Finally, we have done the same exercise using synthetic indexes of the Business cycle but we did not find any significant correlation.

Graph 10. ***Gain (Tunisian manufacturing IPI vs European IPI) – DHP detrending method***



Graph 11. ***Phase shift (Tunisian manufacturing IPI vs European IPI) – DHP detrending method***



As a final conclusion from this section, we can state that both Tunisian and European data bring some evidence of synchronization of the Business cycle. Even if the major cycle has generally greater amplitude in both economies than the minor cycle, the volatility of the Tunisian business cycle is much lower especially for longer cycle. The transmission of cyclical shocks from the Euro Area to Tunisia is instantaneous for short cycles probably caused by the short term adjustment of stock in the European industry (supply-demand mismatch). The delays are much longer for the major cycles. They could reach 5 to 6 quarters.

5. Conclusion

Understanding the characteristics of a country's business cycle is the first step to design efficient countercyclical economic policies. In particular, the assessment of the degree of international synchronization and its transmission delays could be helpful for the decision-makers in the small open economies.

The previous literature on the characterization of the Tunisian business cycle used either the approach of Bry-Boschan or the Markov switching regimes models to detect the turning points and to measure the average length of the cycles and calculated some measures of correlation between the phases of the cycle in Tunisia and in its main European countries using the method suggested by Harding and Pagan (2002, 2006). The findings are not uniform and did not lead to a common understanding of the Tunisian business cycle characteristics. They need to be confirmed and checked using alternative methods.

In the paper, we have applied spectral and cross spectral analysis techniques as an alternative approach to characterize the Tunisian business cycle and to measure the degree of its international synchronization in terms of correlation between the overall cycles (Coherence), their amplitudes (Gain) and their transmission delays (Phase shift) frequency by frequency.

As a robustness check, we have applied these techniques to the industrial production (IPI) as well as two synthetic indexes that we have constructed using a large disaggregated database: a dynamic factor and a diffusion index. Furthermore, we have used two detrending methods to extract their cyclical components: a modified HP filter and the asymmetric version of the Christiano-Fitzgerald filter. The comparison of the cyclical components shows differences in the amplitude and volatility between these of the IPI series and those of the diffusion index and the dynamic factor. The IPI series appear to be smoother and with higher amplitude at low frequencies and contain small amplitude variation at higher frequencies. Nevertheless, the synthetic indexes are more volatile and show higher amplitude at higher frequencies. We notice also that both detrending methods give approximately the same features except for the Christiano-Fitzgerald filter when applied to the diffusion index which amplifies the cyclical movements of the higher frequency cyclical movements.

The application of a univariate spectral analysis to each of these series and to those in the Euro Area, France, Germany and Italy has allowed us to observe two types of cycles: a minor cycle of 12.5 quarters (3.1 years) detected in Tunisia and in all its European trade partners and a major cycle of 33.3 quarters (8.3 years) observed in the majority of the cycles studied but not with all measures of the business cycle.

Furthermore, we have conducted a cross–spectral analysis to deepen the study of the synchronization of business cycles in these partner countries. We have found that both Tunisian and European data provide a strong evidence of synchronization of the Business cycle. Even if the major cycle has generally greater amplitude in both economies, the volatility of the Tunisian business cycle is generally lower than that of the European cycle. It is even lower for longer cycles. The transmission of cyclical shocks from the Euro Area to Tunisia is instantaneous for short cycles probably caused by the quick adjustment of stock in the European industry (supply-demand mismatch). The delays are much longer for the major cycles. They can reach 5 to 6 quarters.

The findings of this paper shed new light on the Tunisian business cycle. They should however be extended by further studies of the transmission channels of the European business cycles to the Tunisian economy and their impact on investment and employment. Further understanding of these elements will permit to better define the most appropriate stabilizing policies by the economic authorities.

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Appendix 1. Some economic features: contribution to economic growth

Source : author's calculation

Mean Contribution [1997 T1 - 2014 T4]		***Industry excluding construction***
	to the value of the market GDP	30%
	to the growth (in points of annualized growth rate)	0.3%
to the variance	***direct contribution***	53%
	indirect contribution	11%
	total contribution	**64%**

Appendix 2. The state space model for the DFM and the Kalman filtering

This appendix describes the state space model, then the Kalman filter which allows estimating the unknown variables (factors in this application) conditionally to observed variables up to time t. Smoothing algorithm is used - in a second step - to refine the estimate by taking into account the whole observations.

1- The state space model

A state space model for a multivariate process Y_t is represented by the following equation system:

$$\begin{cases} Z_{t+1} = AZ_t + \varepsilon_t & (1) \\ Y_t = CZ_t + \tau_t & (2) \end{cases} \qquad \begin{pmatrix} \varepsilon_t \\ \tau_t \end{pmatrix} \approx N(0, \begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix})$$

Equation (1) which describes the dynamics of the unobserved variables over time is the transition equation. Equation (2) is the measurement equation and it describes the relationship between observed and unobserved variables. We denote by:

Y_t the measurement variable at time t
Z_t the state variable at time t
ε_t the innovation vector at time t
τ_t the measurement error vector at time t
A the transition matrix
C the measurement matrix

It is assumed that the measurement equation errors are independent of the transition equation error:

$$E(\varepsilon_t \tau_s) = 0 \quad \forall t, s = 1, \dots, T$$

Moreover, the state space representation is completed by specifying the behavior of the initial state (Z_0):

$$Z_0 \sim N(\mu, P)$$
$$E(\varepsilon_t Z_0) = E(\tau_t Z_0) = 0 \quad \forall t, s = 1, \dots, T$$

It is also assumed that matrices A and C are independent of time (the system is called a time-invariant system).

2- Kalman Filter

The Kalman filter is a recursive algorithm that allows estimating – at each time t – the optimal state vector given information available up to time t;

$$Z^*_{t|t} = E(Z_t | Y_0, \dots, Y_t) \quad ; t = 1, \dots, T$$

This filter consists of iterating the following five steps for $= 1, \dots, T$:

$$\begin{cases} (1) & Z^*_{t|t} = Z^*_{t|t-1} + K_t\left(Y_t - CZ^*_{t|t-1}\right) \\ (2) & \Sigma_{t|t} = (I - K_tC)\Sigma_{t|t-1} \\ (3) & Z^*_{t+1|t} = AZ^*_{t|t} \\ (4) & \Sigma_{t+1|t} = A\Sigma_{t|t}A' + Q \\ (5) & K_t = \Sigma_{t|t-1}C'(C\,\Sigma_{t|t-1}C' + R)^{-1} \end{cases}$$

Where $Z^*_{t|t} = E(Z_t|Y_0, \dots, Y_t)$ is the current estimate of the state vector, $Z^*_{t+1|t} = E(Z_{t+1}|Y_0, \dots, Y_t)$ is the optimal predictor of Z_t given information at time t and $\Sigma_{t+1|t}$=V$(Z_t - Z_{t+1|t})$ is the prediction error variance. The matrix K_t is called the Kalman Gain.

The first two equations (1) and (2) update the Z_t estimate using information contained in the prediction error. If we look carefully at equation (1), we find that the update term take into account the new information contained in observed measurement Y_t. It's simply the measurement residual weighted by the kalman gain which is defined by equation (5). Equation (2) updates the state estimate error covariance matrix $\Sigma_{t|t}$. Equations (3) and (4) update the state estimate at (t+1) using the transition equation.

Note that the recursive algorithm of Kalman Filter requires an initialization of the mean and covariance of Z_0 since $Z^*_{1|0} = \mu$ and $\Sigma_{1|0} = P$.

3- Smoothing Algorithm

The smoothing algorithm is used to refine the states estimate by using not only the current and previous information but also the future information. This time, we iterate back the following calculations for $t = T-1, \dots, 1$

$$\begin{cases} Z^*_{t|T} = Z^*_{t|t} + F_t\left(Z^*_{t+1|T} - Z^*_{t+1|t}\right) \\ \Sigma_{t|T} = \Sigma_{t|t} + F_t\left(\Sigma_{t+1|T} - \Sigma_{t+1|t}\right)F_t' \end{cases} \qquad \text{where} \qquad F_t = \Sigma_{t|t} A' \Sigma^{-1}_{t+1|t}$$

Note that the starting point is the output of Kalman Filter at time T.

Up to now, we have assumed that parameters A, C, Q and R of the state space are known. If this is not the case, the log-likelihood of the model can be constructed over the filter iterations and unknown parameters could be estimated by different methods. Across the literature, we can identify 4 possible methods to estimate the dynamic factor model:

- Spectral density estimation (Formi & al. (2000));
- Principal Component Analysis (Stock & Watson (2002 a));
- Pseudo-maximum Likelihood Estimation (Doz, Giannone & Reichlin (2006,2007));
- The two-steps methods (Doz, Giannone & Reichlin (2006,2007)).

Remember that in order to estimate the dynamic factors in this application, we have followed a two-steps method (Doz, Giannone & Reichlin (2006,2007)). The first one is based on the Principal Component Analysis which allowed us to estimate the state initial values ($Z^*_{1|0} = \mu$, $\Sigma_{1|0} = P$) and also parameters A, C, Q and R. Then, in a second step, we have applied the Kalman Filter and smoother and we have obtained as a result a refined measure of the dynamic of factors estimated over the whole period.

Appendix 3. Main statistics of the Spectral Analysis

The spectrogram:

Is called spectrogram the following I_T function:

$$I_T(\omega) = \frac{1}{T}\left|\sum_{t=1}^{T} X_t e^{-it\omega}\right|^2 ; \qquad \omega \in [-\pi, \pi]$$

If the process $X_t \in Z$ admits a spectral density, so $\frac{1}{2\pi} I_T(\omega)$ is an unbiased estimator of the spectral density.

We call spectral density (or spectrum) of X the discrete Fourier transform, S,of following autocovariances (when available):

$$S(\omega) = \frac{1}{2\pi} \sum_{h=-\infty}^{+\infty} \gamma(h) e^{-ih\omega}$$

The cross-spectrogram:

In the same manner as the spectral density of a univariate series is given by the Fourier transform of its autocovariance function, the cross-spectrogram between two series Y_{jt} and Y_{mt} will be calculated as the Fourier Transform of their cross-covariance function:

$$S_{jm}(\omega) = \frac{1}{2\pi} \sum_{h=-\infty}^{+\infty} \gamma_{jm}(h) e^{-ih\omega}$$

The cross spectrogram is generally with complex values. The real part is called cospectrum $[co_{jm}(\omega)]$ between Y_{jt} and Y_{mt}, while the imaginary part is called quadrature $[qu_{jm}(\omega)]$.

$$S_{jm}(\omega) = co_{jm}(\omega) + i\ qu_{jm}(\omega)$$

Be 2 processes u_t and v_t. S, the spectral density of the matrix at the frequency ω is written:

$$S_y(\omega) = \begin{pmatrix} S_{uu}(\omega) & S_{uv}(\omega) \\ S_{vu}(\omega) & S_{vv}(\omega) \end{pmatrix}$$

The diagonal elements $S_{uu}(\omega)$ and $S_{vv}(\omega)$ are the univariate spectrum of the processes u_t and v_t. They are real valued. However, the non-diagonal elements are complex valued since, in general, $\gamma_{uv}(h) \neq \gamma_{vu}(-h)$. $S_{uv}(\omega)$ is the cross-spectrum from v to u :

$$S_{uv}(\omega) = co_{uv}(\omega) + i\ qu_{uv}(\omega)$$

The coherence:

"Coherence" provides an overall measure of association between the two series v_t and u_t at a given frequency (ω). Such a measure is defined as:

$$h_{uv}(\omega) = \frac{[co_{uv}(\omega)]^2 + [qu_{uv}(\omega)]^2}{S_{uu}(\omega)\ S_{vv}(\omega)}$$

<u>The "Gain":</u>

The "Gain" is a statistics that measures the relation between amplitudes frequency by frequency. It takes values between 0 and $+\infty$.

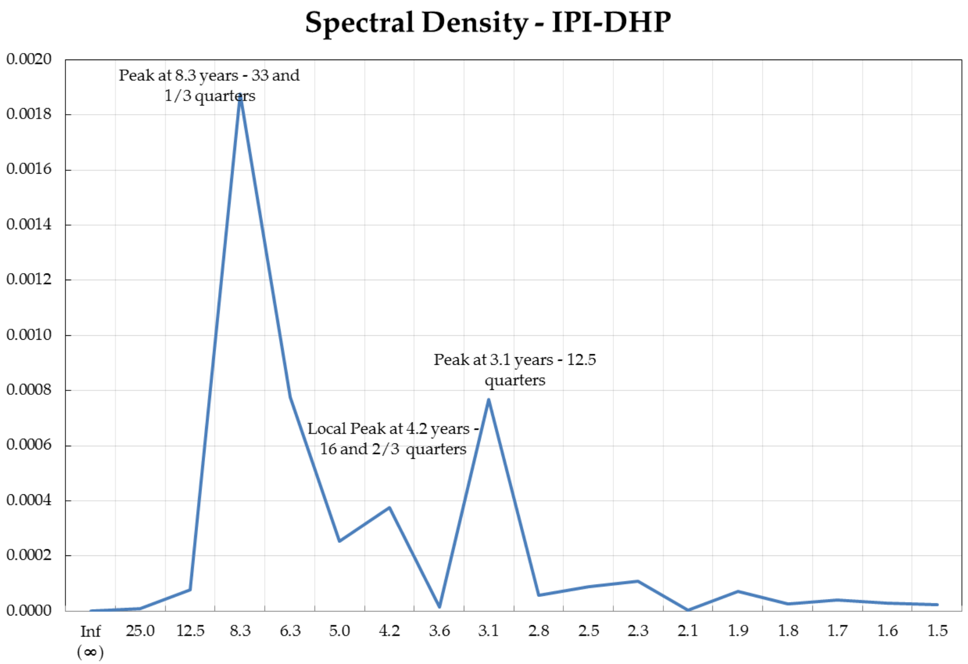
$$G_{uv}(\omega) = \frac{\left([co_{uv}(\omega)]^2 + [qu_{uv}(\omega)]^2\right)^{1/2}}{S_v(\omega)}$$

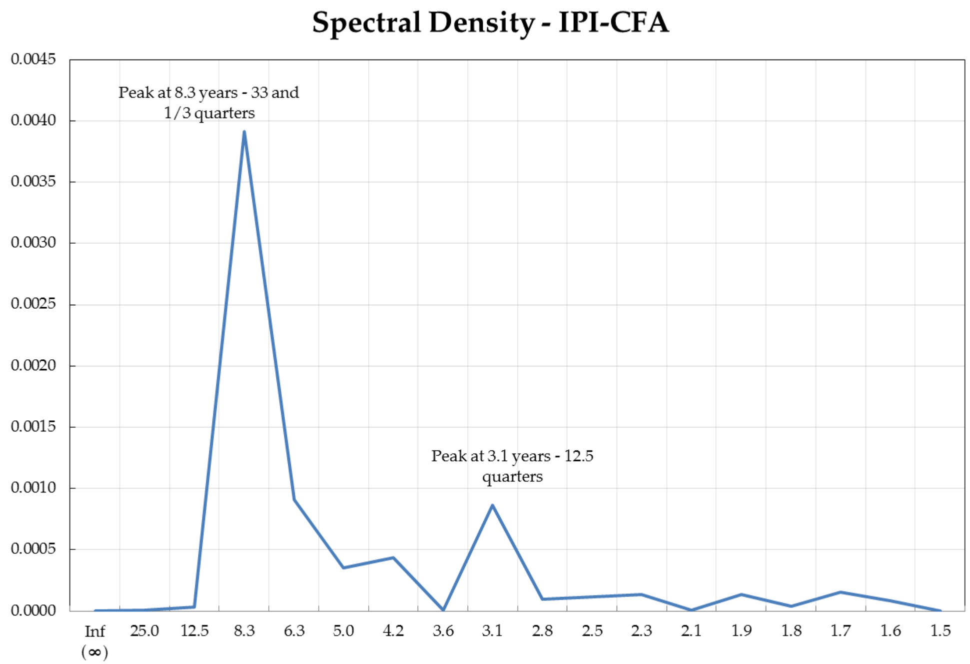
<u>The "Phase shift":</u>

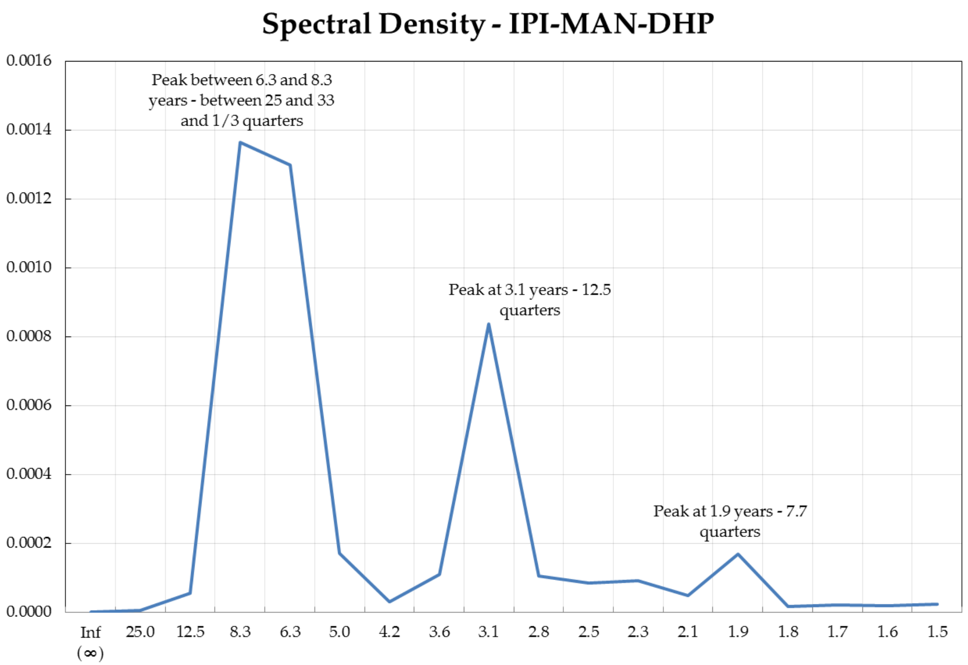
This statistics measure the relation between the turning points of two series in terms of leads and lags frequency by frequency.

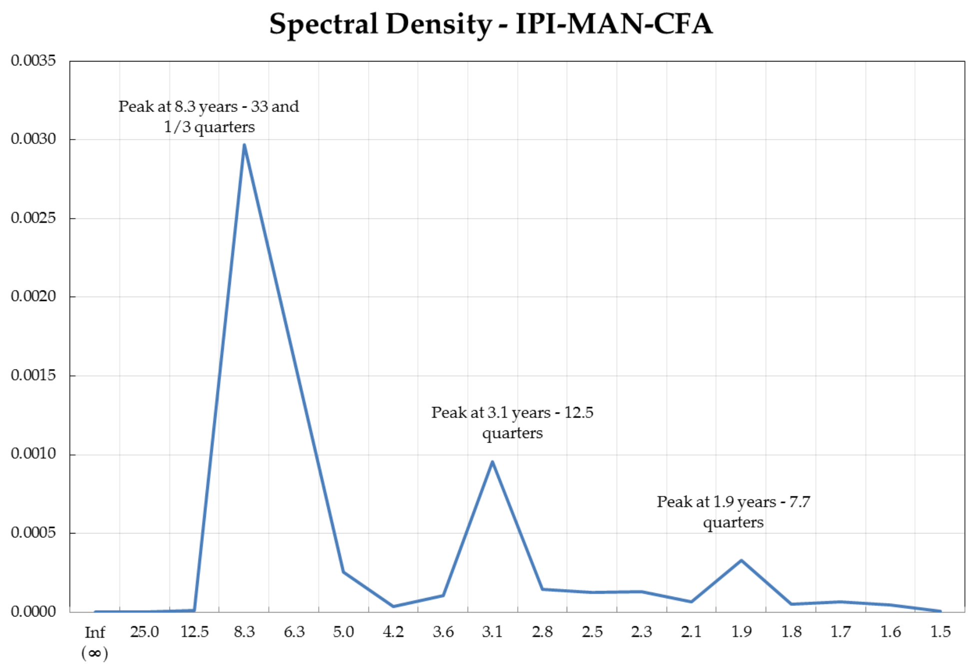
$$\phi_{uv}(\omega) = tan^{-1}\left(\frac{-qu_{uv}(\omega)}{co_{uv}(\omega)}\right)$$

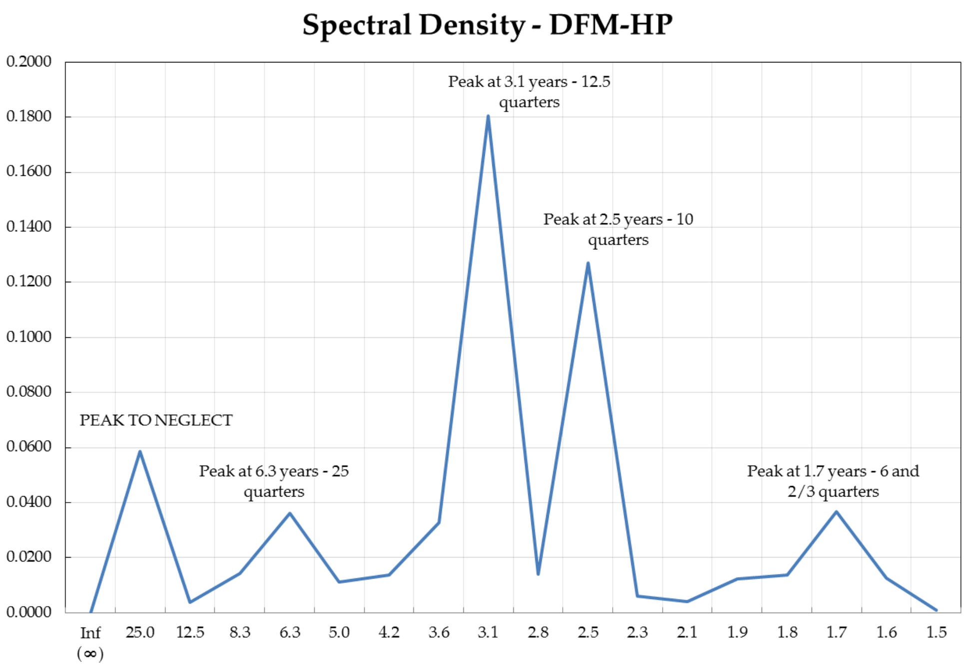
Appendix 4. Univariate Spectral Analysis results

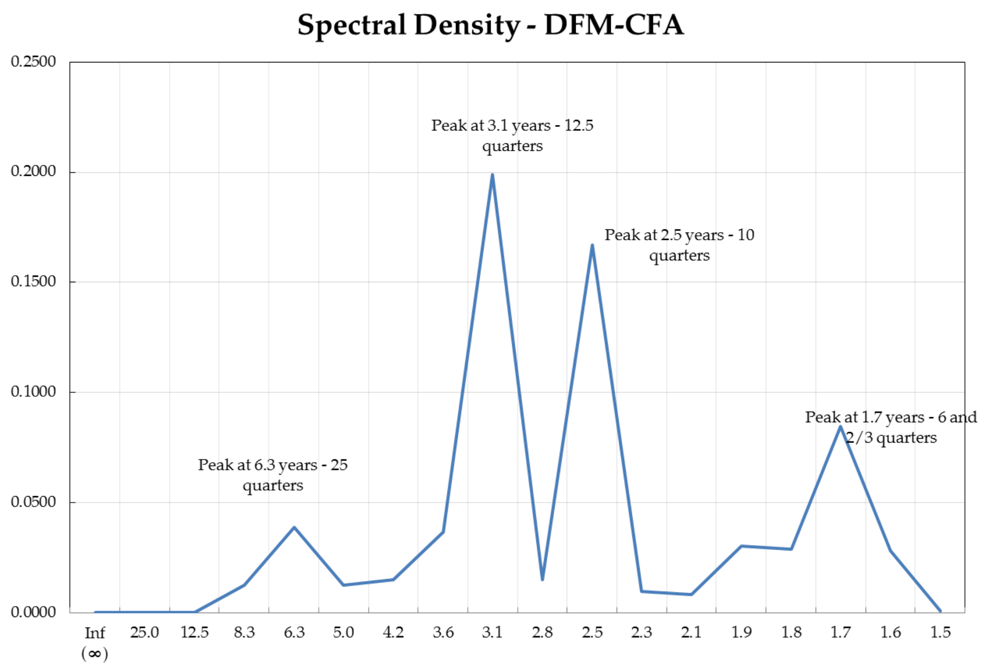


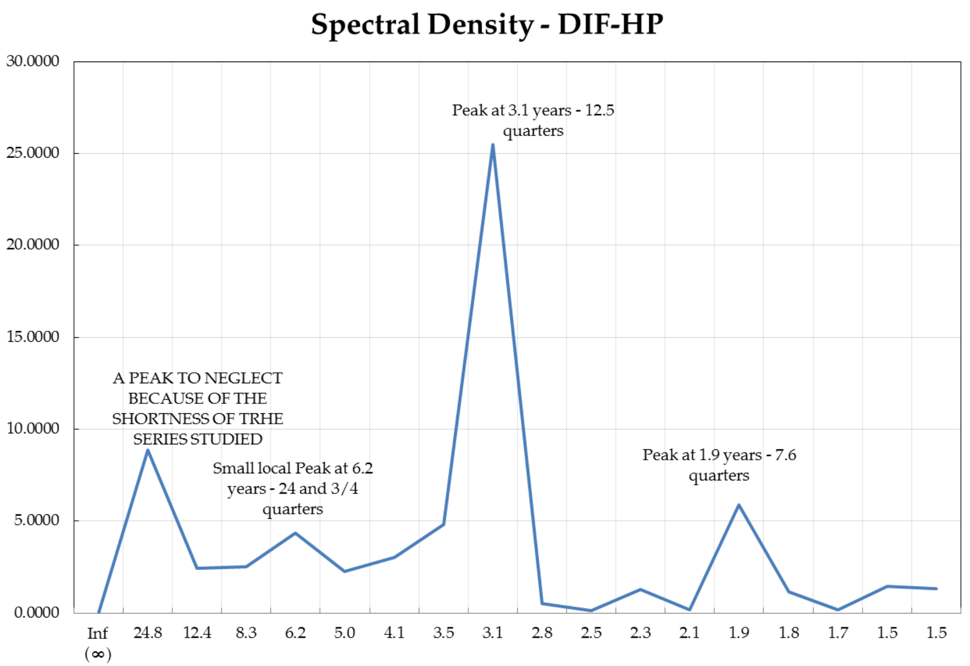


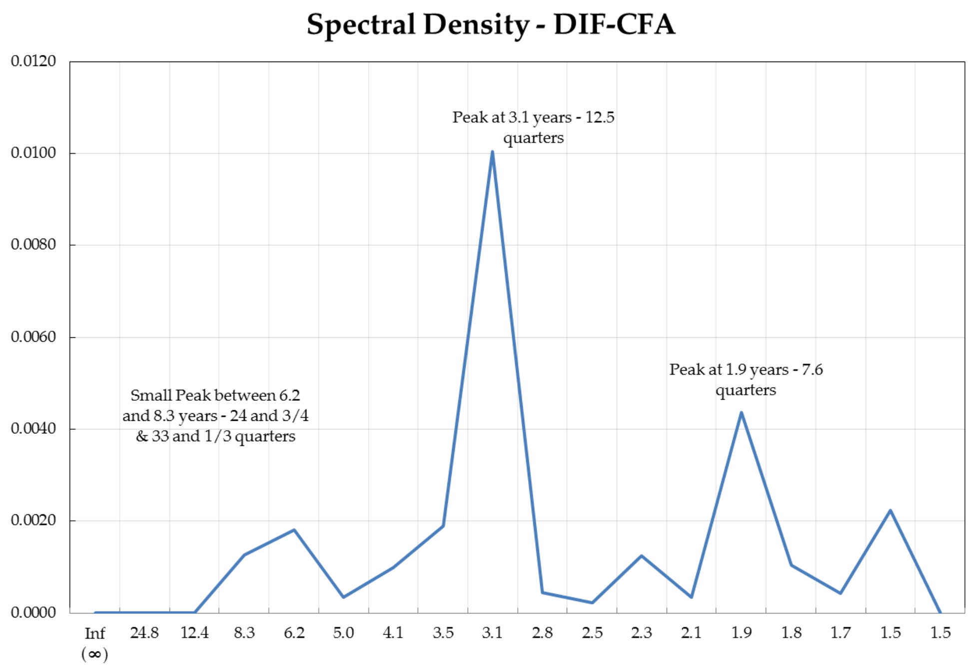




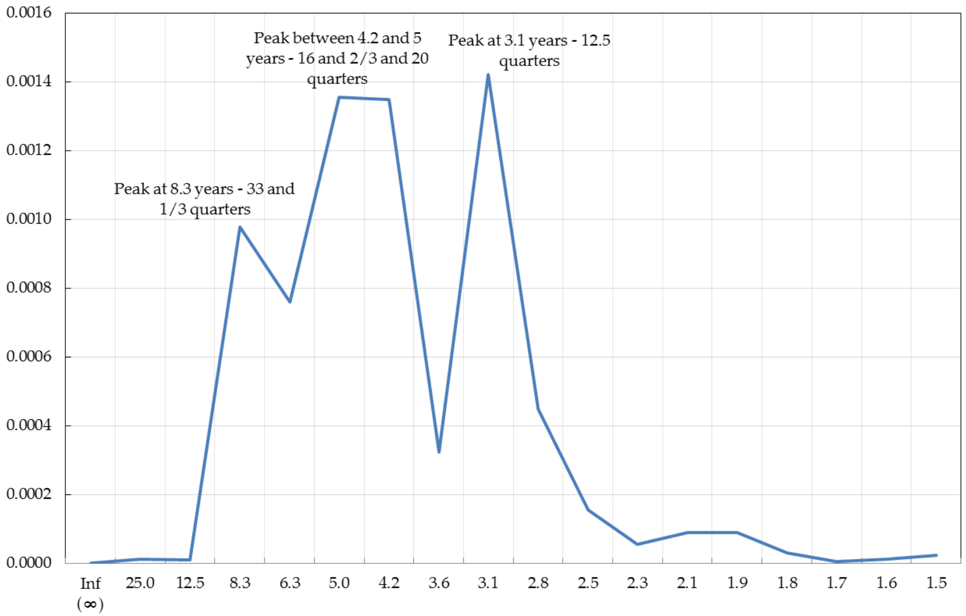

Spectral Density - DFM-HP
0.2000
0.1800
0.1600
0.1400
0.1200
0.1000
0.0800
0.0600
0.0400
0.0200
0.0000
PEAK TO NEGLECT
Peak at 6.3 years - 25 quarters
Peak at 3.1 years - 12.5 quarters
Peak at 2.5 years - 10 quarters
Peak at 1.7 years - 6 and 2/3 quarters
Inf (∞)
25.0
12.5
8.3
6.3
5.0
4.2
3.6
3.1
2.8
2.5
2.3
2.1
1.9
1.8
1.7
1.6
1.5


Spectral Density - DFM-CFA
0.2500
0.2000
0.1500
0.1000
0.0500
0.0000
Peak at 3.1 years - 12.5 quarters
Peak at 2.5 years - 10 quarters
Peak at 6.3 years - 25 quarters
Peak at 1.7 years - 6 and 2/3 quarters
Inf (∞)
25.0
12.5
8.3
6.3
5.0
4.2
3.6
3.1
2.8
2.5
2.3
2.1
1.9
1.8
1.7
1.6
1.5

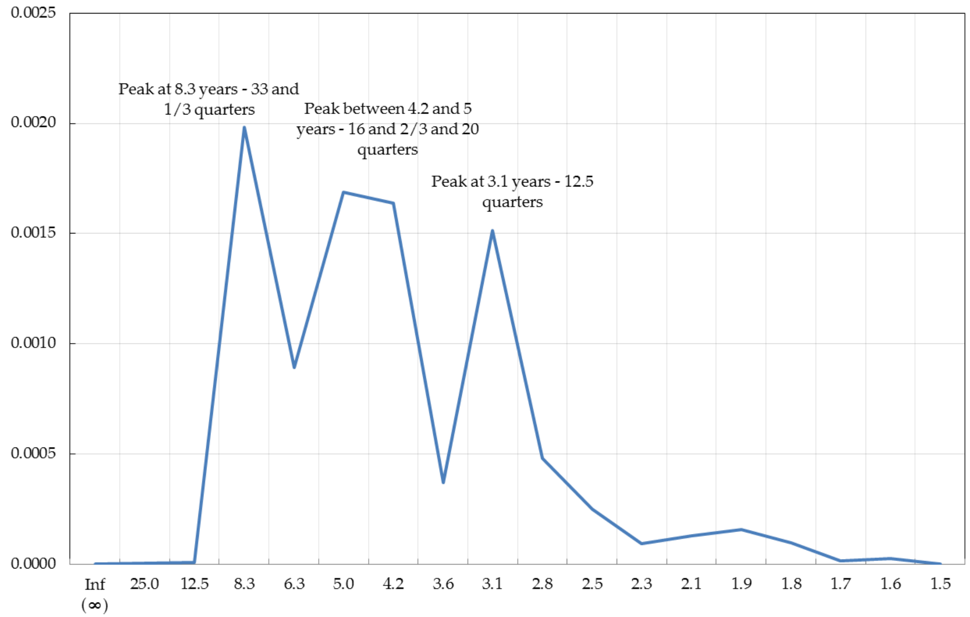

Spectral Density - DIF-HP
30.0000
25.0000
20.0000
15.0000
10.0000
5.0000
0.0000
Peak at 3.1 years - 12.5 quarters
A PEAK TO NEGLECT BECAUSE OF THE SHORTNESS OF TRHE SERIES STUDIED
Small local Peak at 6.2 years - 24 and 3/4 quarters
Peak at 1.9 years - 7.6 quarters
Inf (∞)
24.8
12.4
8.3
6.2
5.0
4.1
3.5
3.1
2.8
2.5
2.3
2.1
1.9
1.8
1.7
1.5
1.5


Spectral Density - DIF-CFA
0.0120
0.0100
0.0080
0.0060
0.0040
0.0020
0.0000
Peak at 3.1 years - 12.5 quarters
Peak at 1.9 years - 7.6 quarters
Small Peak between 6.2 and 8.3 years - 24 and 3/4 & 33 and 1/3 quarters
Inf (∞)
24.8
12.4
8.3
6.2
5.0
4.1
3.5
3.1
2.8
2.5
2.3
2.1
1.9
1.8
1.7
1.5
1.5

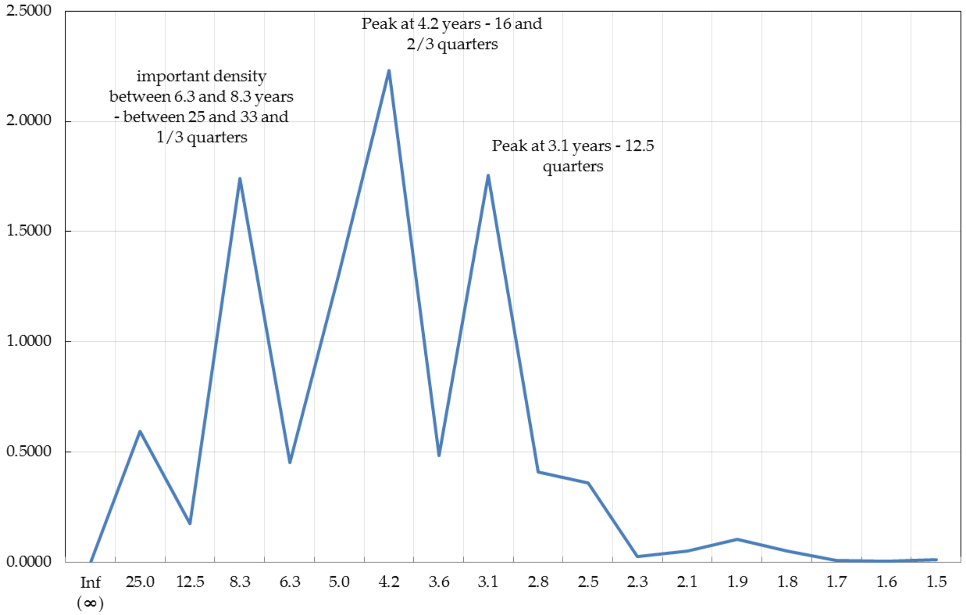
Spectral Density - EA-IPI-DHP



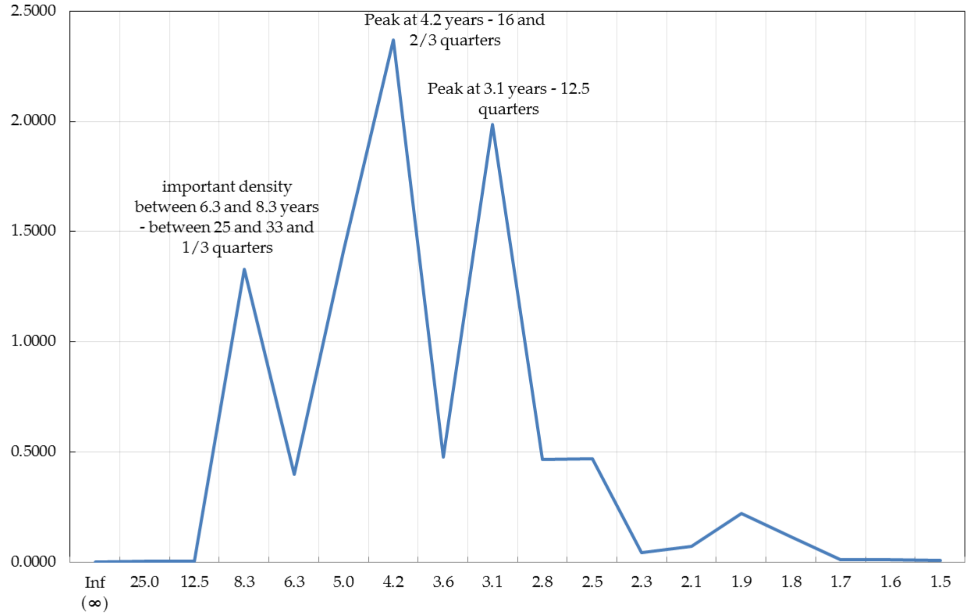
Spectral Density - EA-IPI-CFA



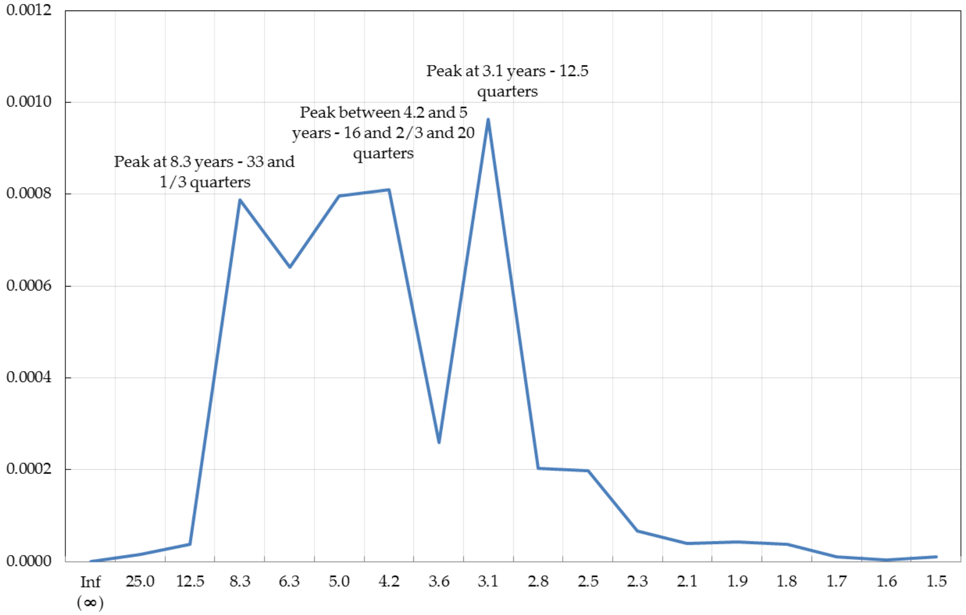
Spectral Density - BCI-EA-HP



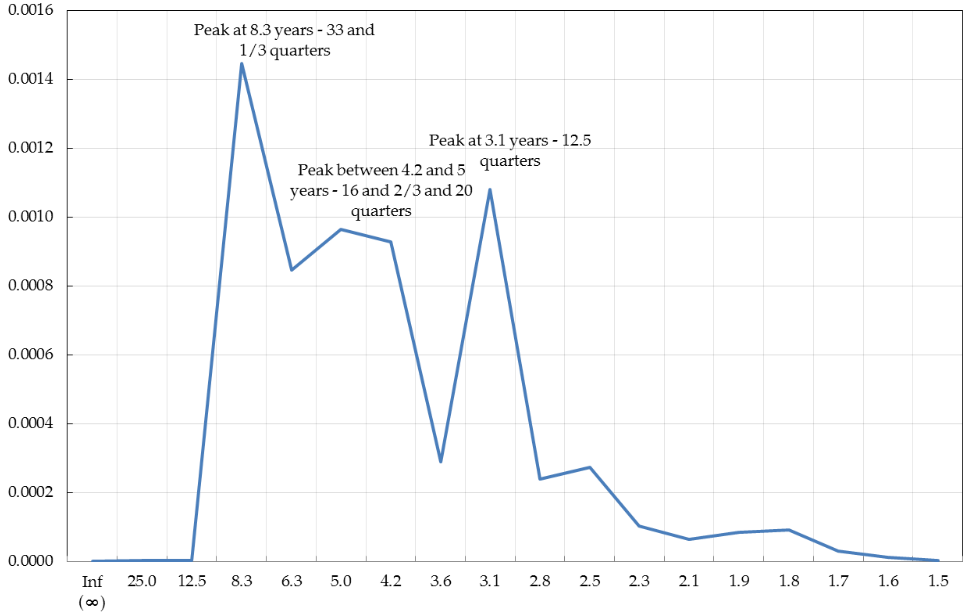
Spectral Density - BCI-EA-CFA



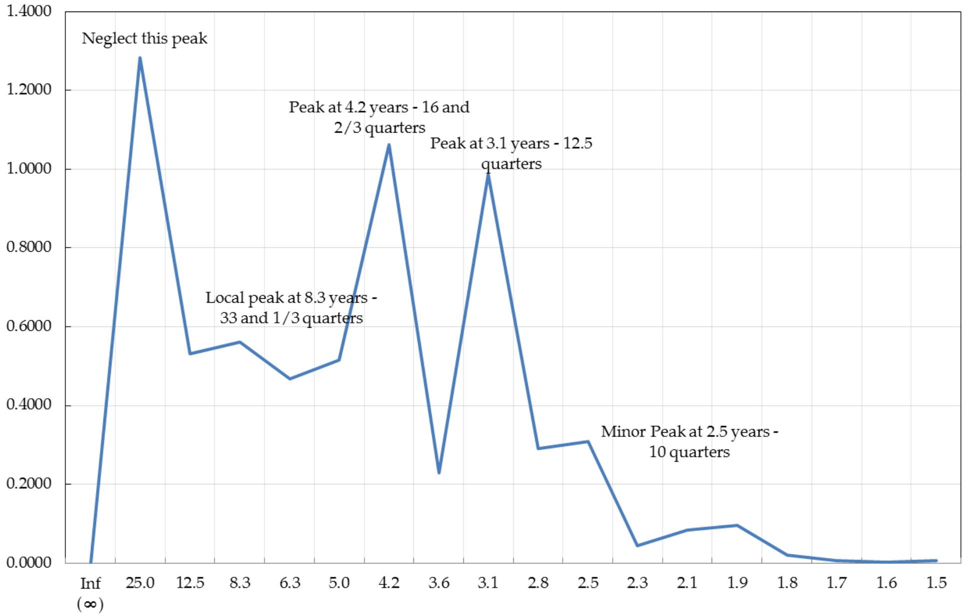
Spectral Density - FR-IPI-DHP



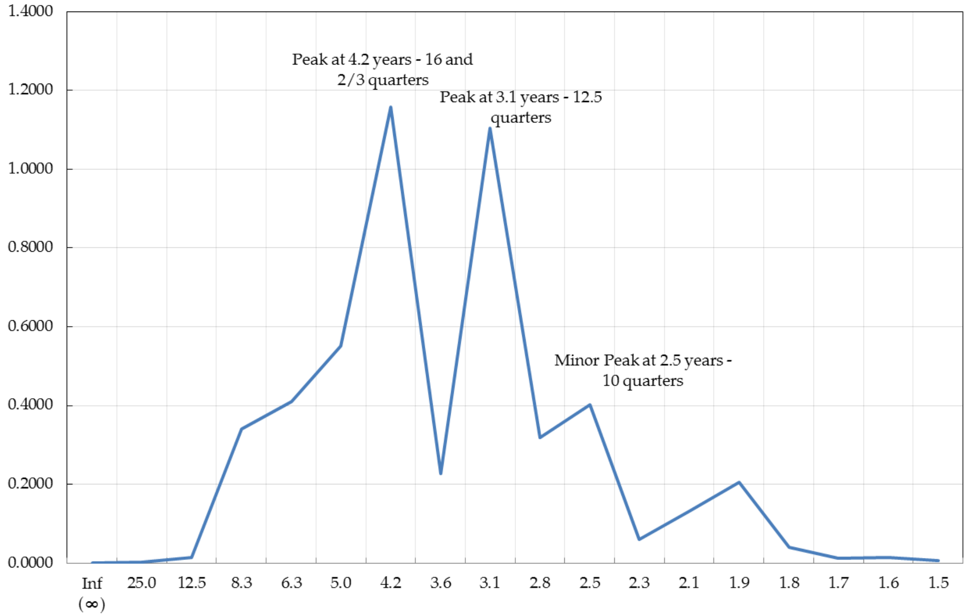
Spectral Density - FR-IPI-CFA

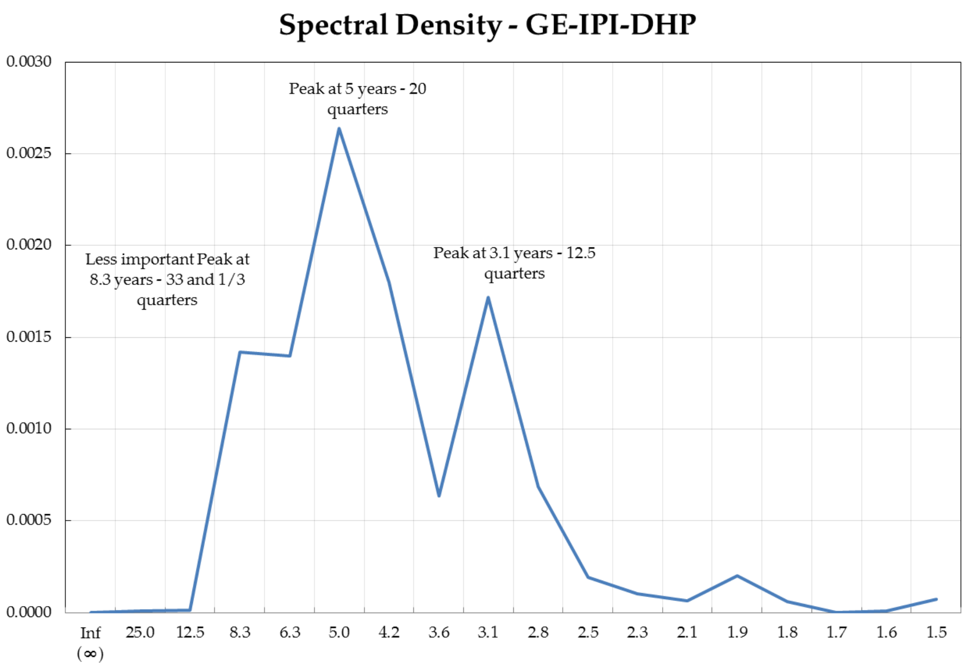


Spectral Density - BCI-FR-HP

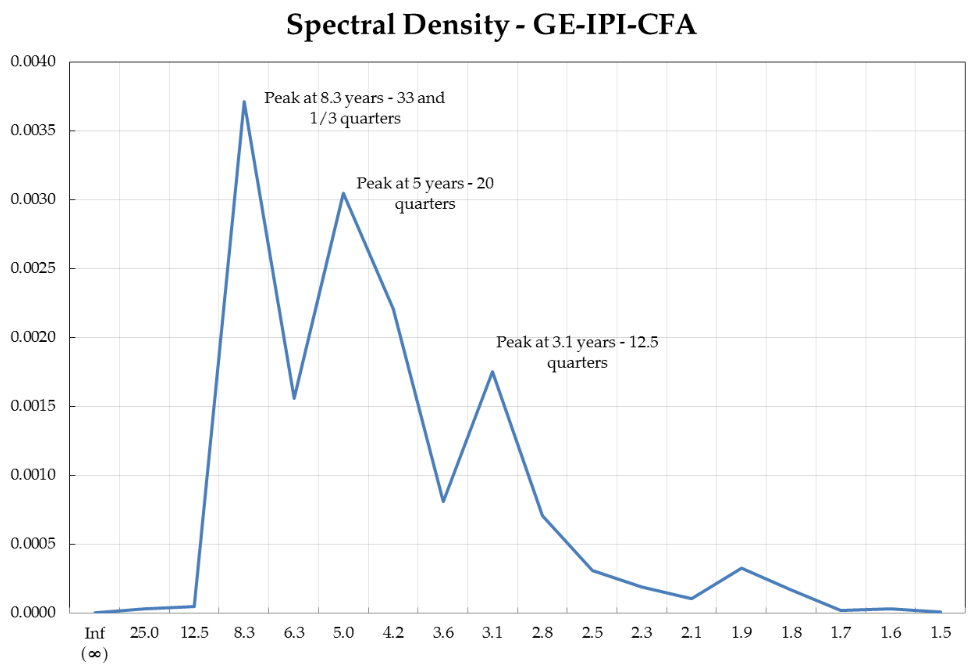


Spectral Density - BCI-FR-CFA

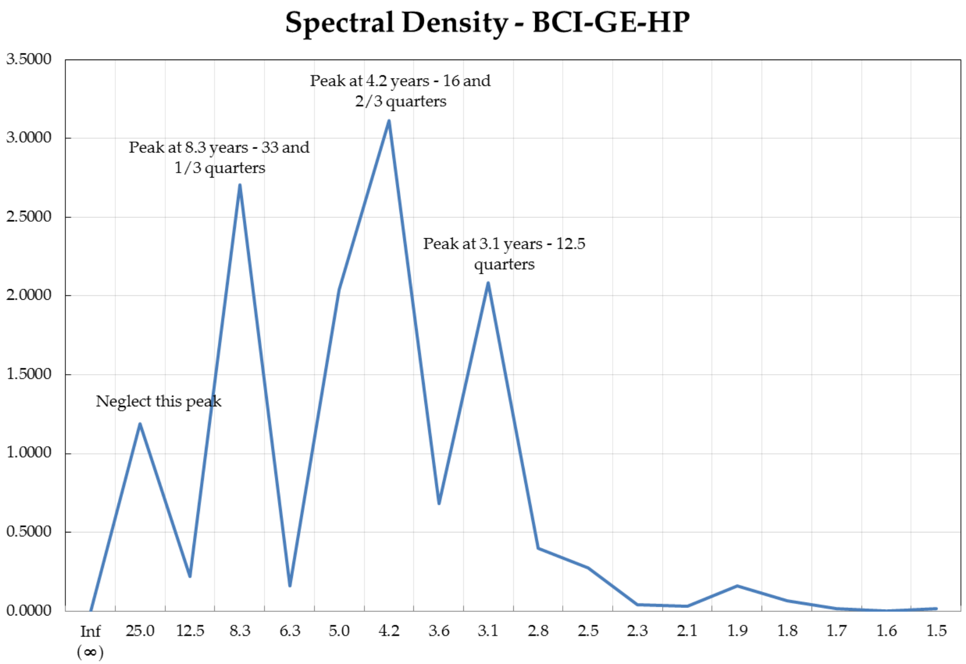




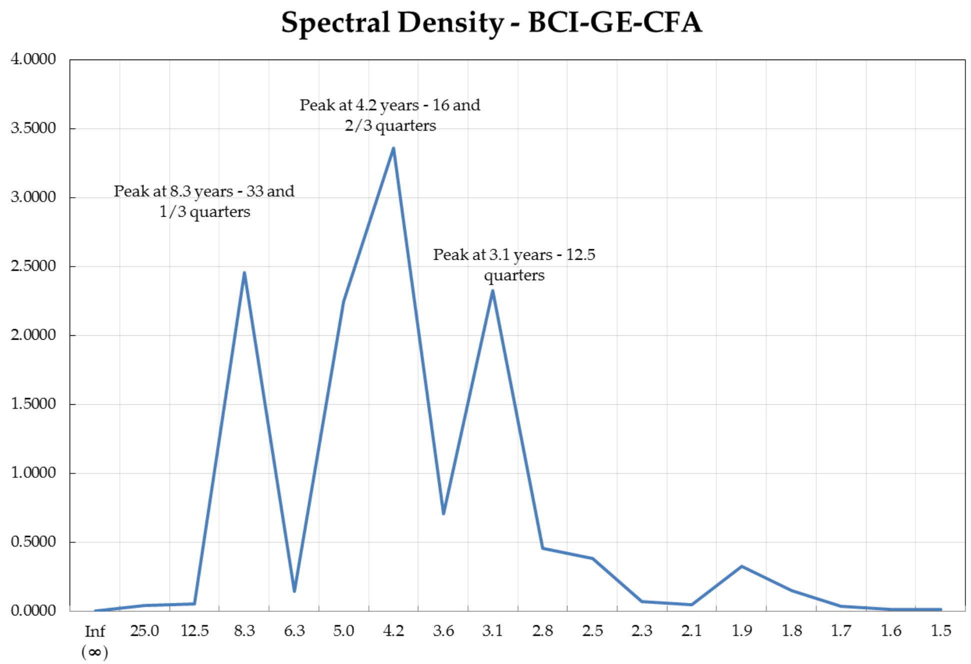
Spectral Density - GE-IPI-DHP
Peak at 5 years - 20 quarters
Less important Peak at 8.3 years - 33 and 1/3 quarters
Peak at 3.1 years - 12.5 quarters
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
Inf (∞)
25.0
12.5
8.3
6.3
5.0
4.2
3.6
3.1
2.8
2.5
2.3
2.1
1.9
1.8
1.7
1.6
1.5



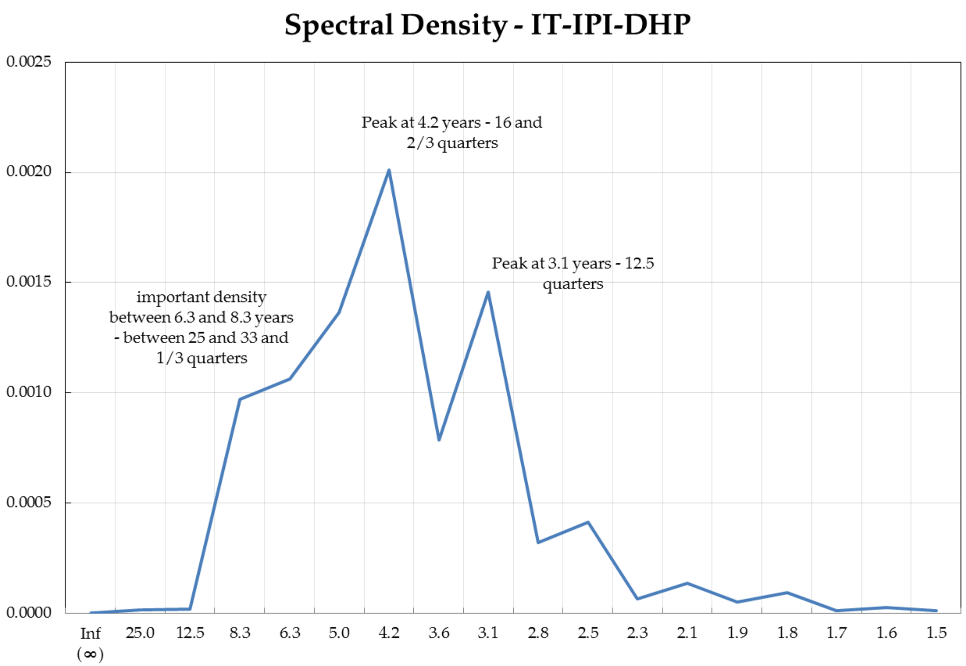
Spectral Density - GE-IPI-CFA
Peak at 8.3 years - 33 and 1/3 quarters
Peak at 5 years - 20 quarters
Peak at 3.1 years - 12.5 quarters
0.0040
0.0035
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
Inf (∞)
25.0
12.5
8.3
6.3
5.0
4.2
3.6
3.1
2.8
2.5
2.3
2.1
1.9
1.8
1.7
1.6
1.5

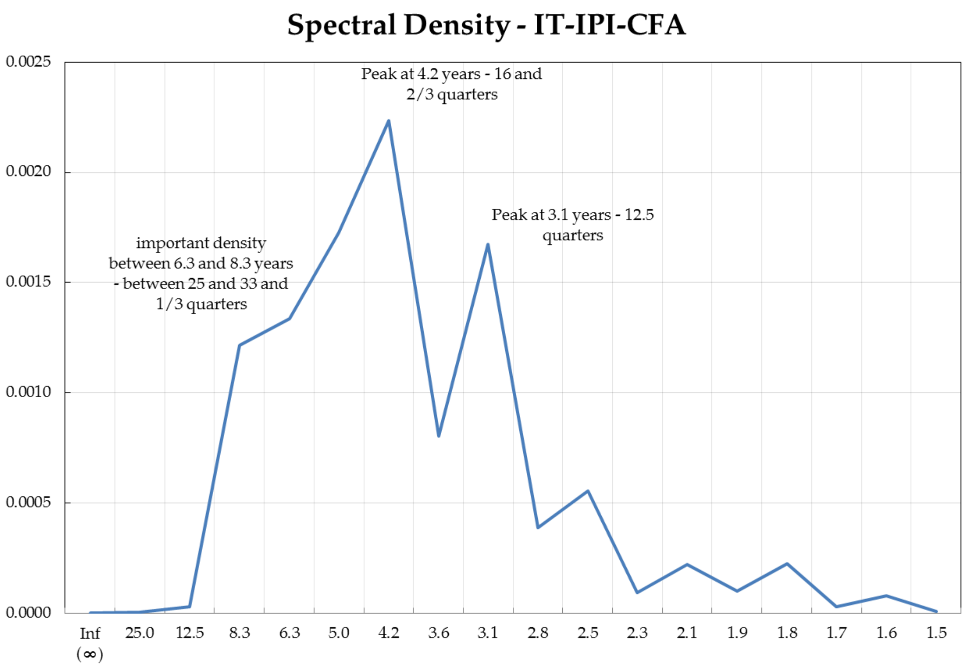


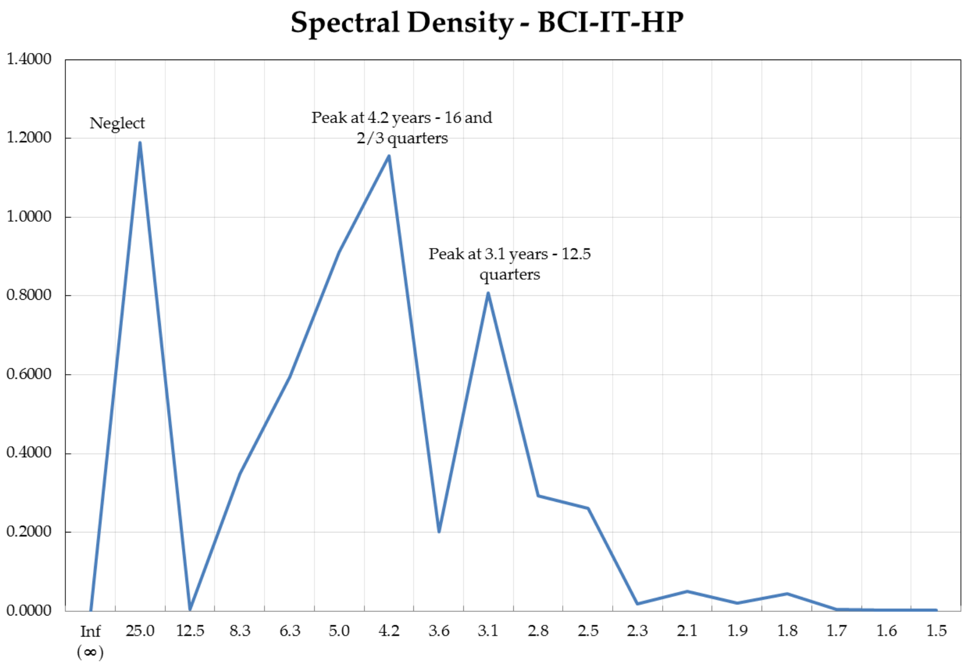
Spectral Density - BCI-GE-HP
Peak at 4.2 years - 16 and 2/3 quarters
Peak at 8.3 years - 33 and 1/3 quarters
Peak at 3.1 years - 12.5 quarters
Neglect this peak
3.5000
3.0000
2.5000
2.0000
1.5000
1.0000
0.5000
0.0000
Inf (∞)
25.0
12.5
8.3
6.3
5.0
4.2
3.6
3.1
2.8
2.5
2.3
2.1
1.9
1.8
1.7
1.6
1.5

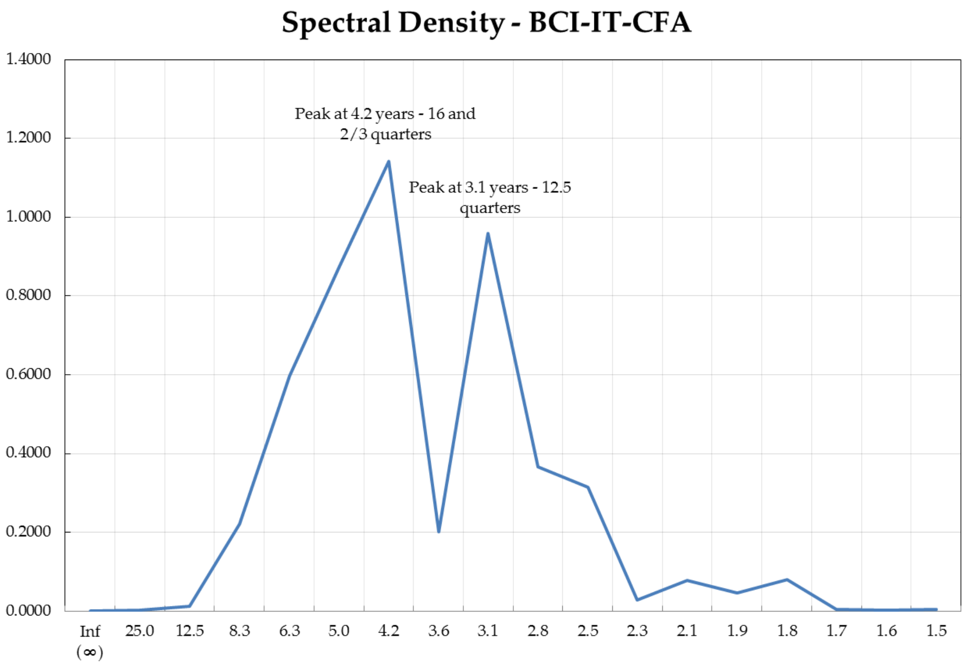


Spectral Density - BCI-GE-CFA
Peak at 4.2 years - 16 and 2/3 quarters
Peak at 8.3 years - 33 and 1/3 quarters
Peak at 3.1 years - 12.5 quarters
4.0000
3.5000
3.0000
2.5000
2.0000
1.5000
1.0000
0.5000
0.0000
Inf (∞)
25.0
12.5
8.3
6.3
5.0
4.2
3.6
3.1
2.8
2.5
2.3
2.1
1.9
1.8
1.7
1.6
1.5

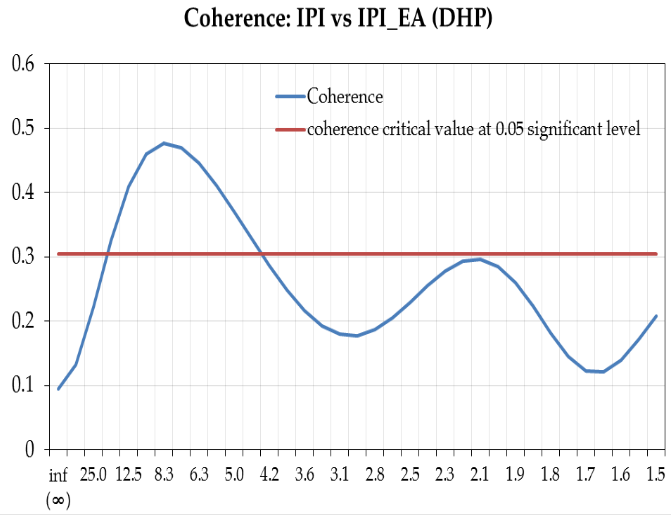

Spectral Density - IT-IPI-DHP
Peak at 4.2 years - 16 and 2/3 quarters
Peak at 3.1 years - 12.5 quarters
important density between 6.3 and 8.3 years - between 25 and 33 and 1/3 quarters
0.0025 0.0020 0.0015 0.0010 0.0005 0.0000
Inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

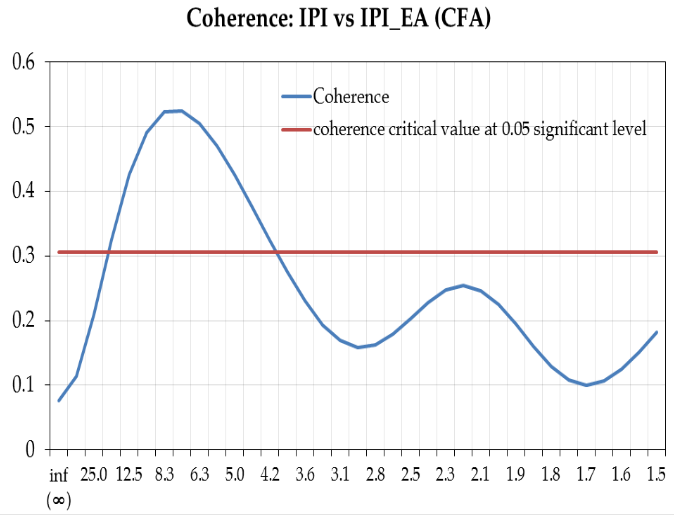

Spectral Density - IT-IPI-CFA
Peak at 4.2 years - 16 and 2/3 quarters
Peak at 3.1 years - 12.5 quarters
important density between 6.3 and 8.3 years - between 25 and 33 and 1/3 quarters
0.0025 0.0020 0.0015 0.0010 0.0005 0.0000
Inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

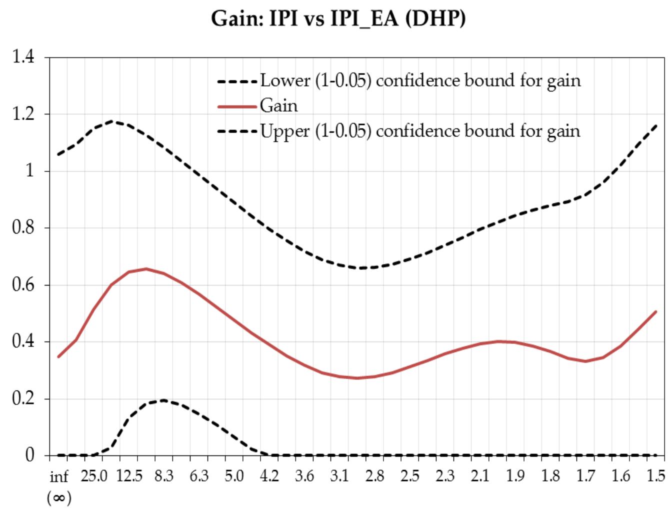

Spectral Density - BCI-IT-HP
Neglect
Peak at 4.2 years - 16 and 2/3 quarters
Peak at 3.1 years - 12.5 quarters
1.4000 1.2000 1.0000 0.8000 0.6000 0.4000 0.2000 0.0000
Inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

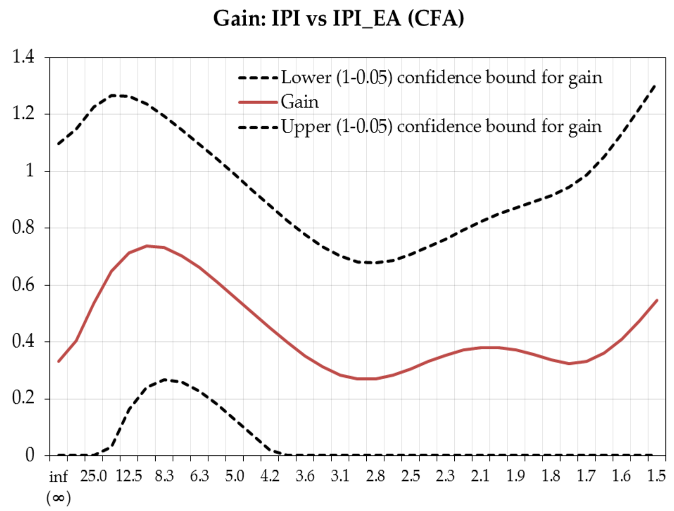

Spectral Density - BCI-IT-CFA
Peak at 4.2 years - 16 and 2/3 quarters
Peak at 3.1 years - 12.5 quarters
1.4000 1.2000 1.0000 0.8000 0.6000 0.4000 0.2000 0.0000
Inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

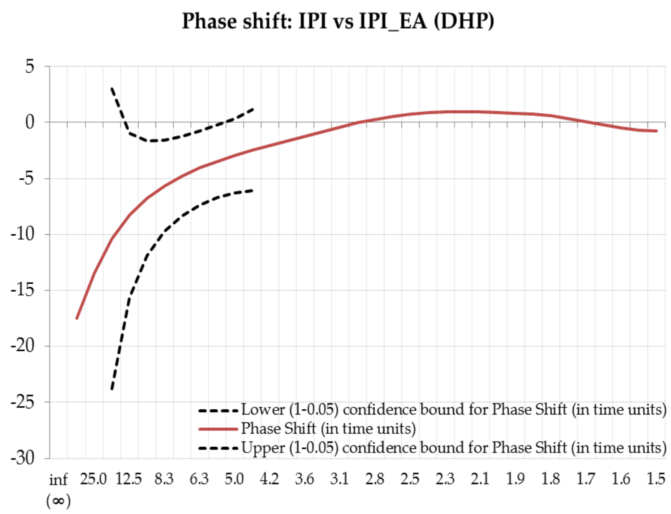
Appendix 5. Cross-spectral analysis results

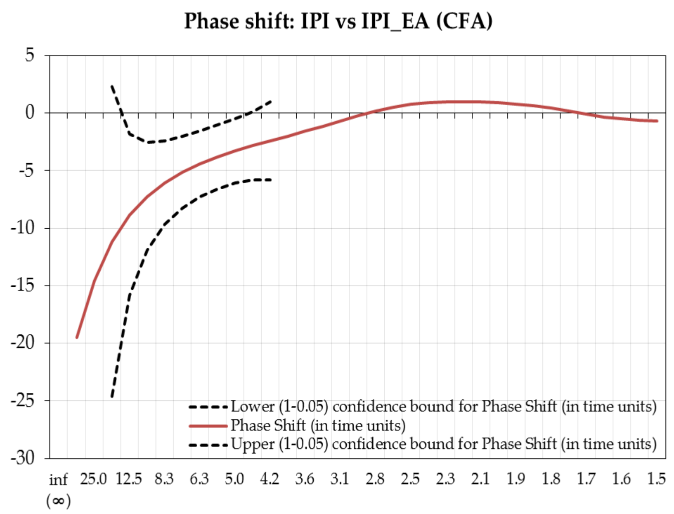




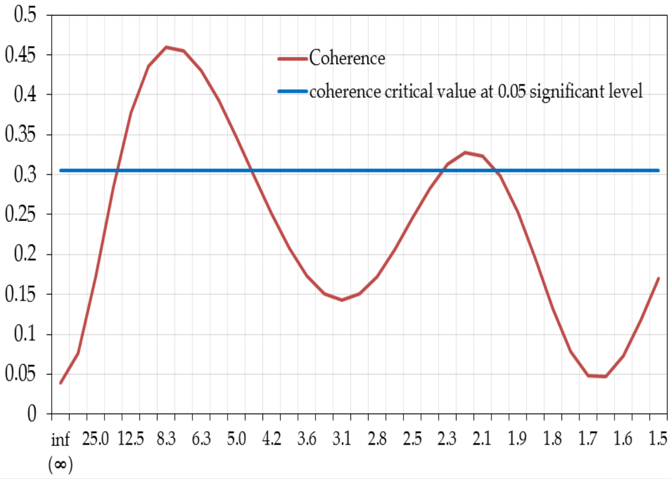




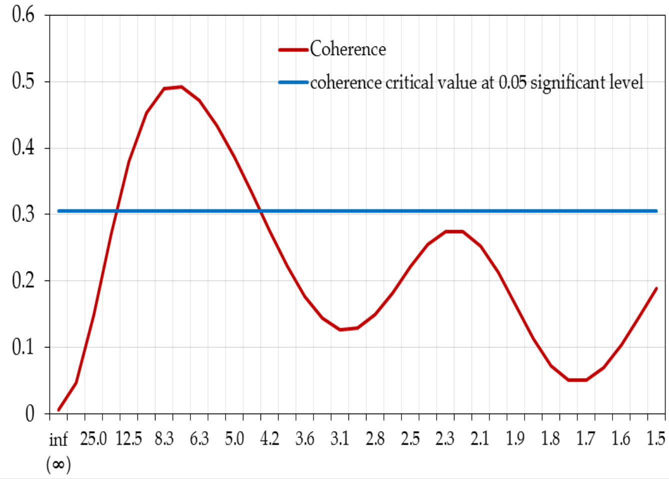




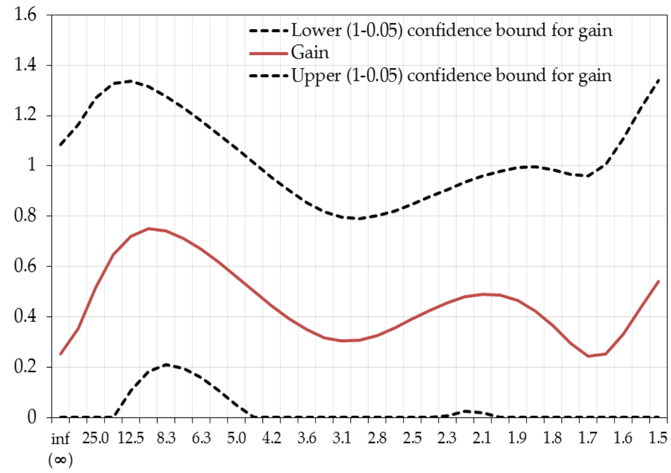
Coherence: IPI vs IPI_FR (DHP)



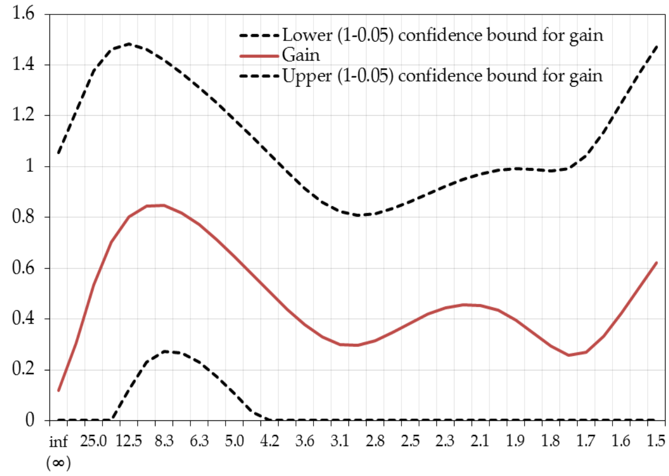
Coherence: IPI vs IPI_FR (CFA)



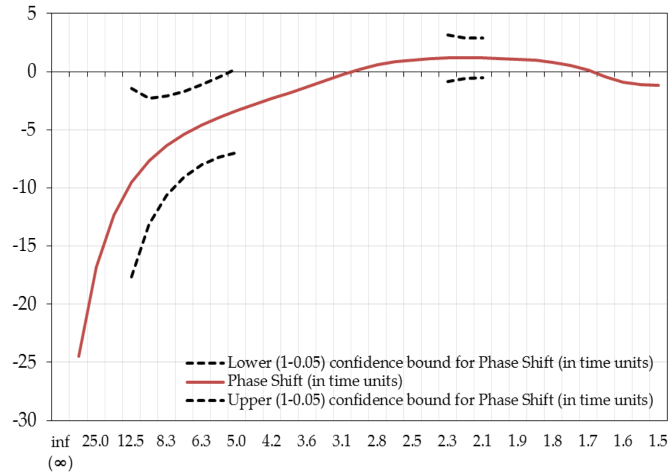
Gain: IPI vs IPI_FR (DHP)



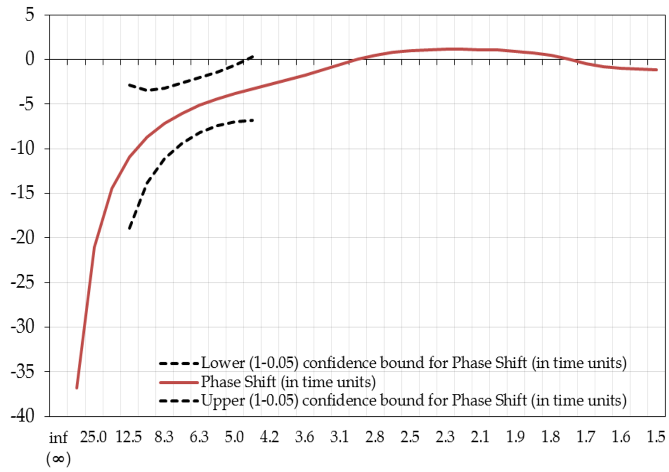
Gain: IPI vs IPI_FR (CFA)

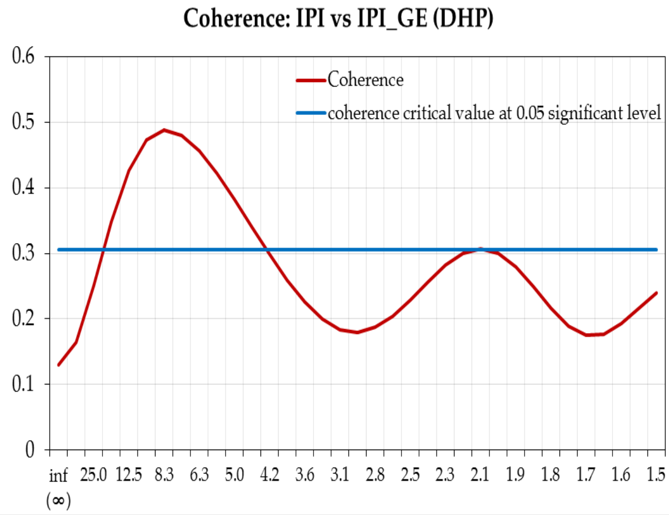


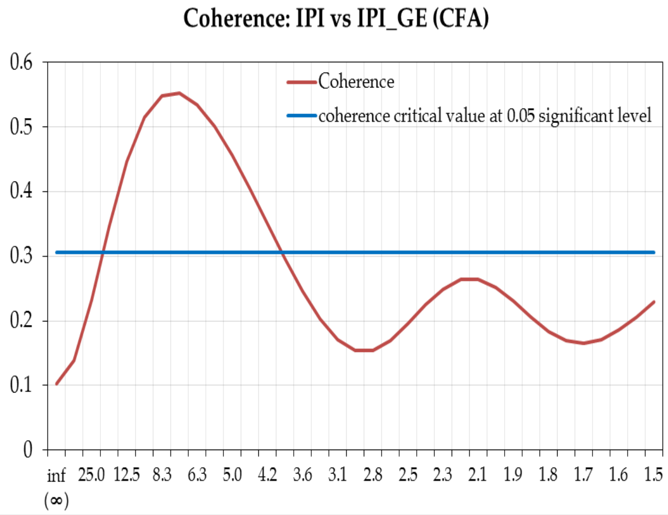
Phase shift: IPI vs IPI_FR (DHP)

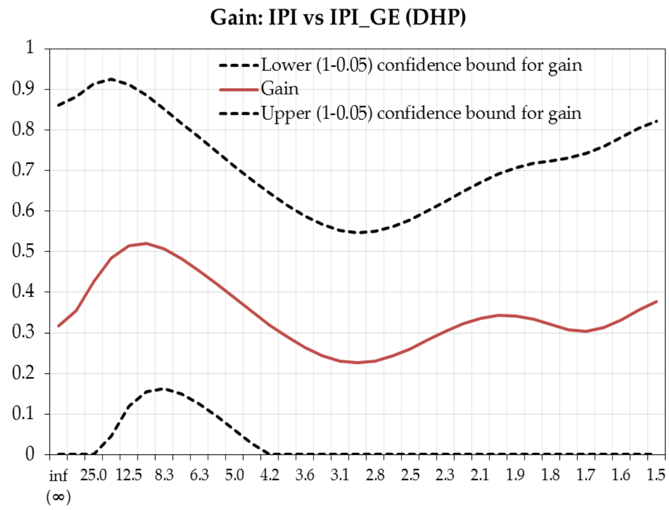


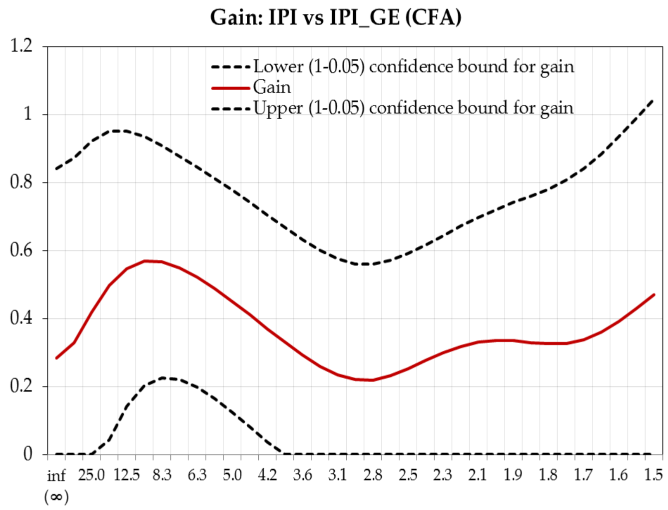
Phase shift: IPI vs IPI_FR (CFA)

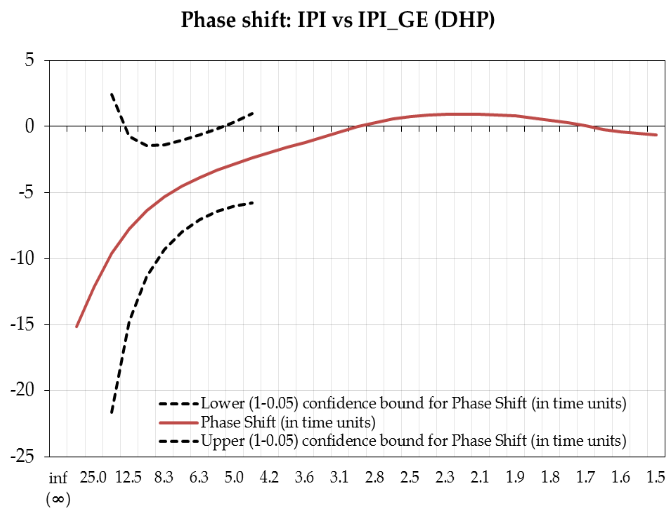


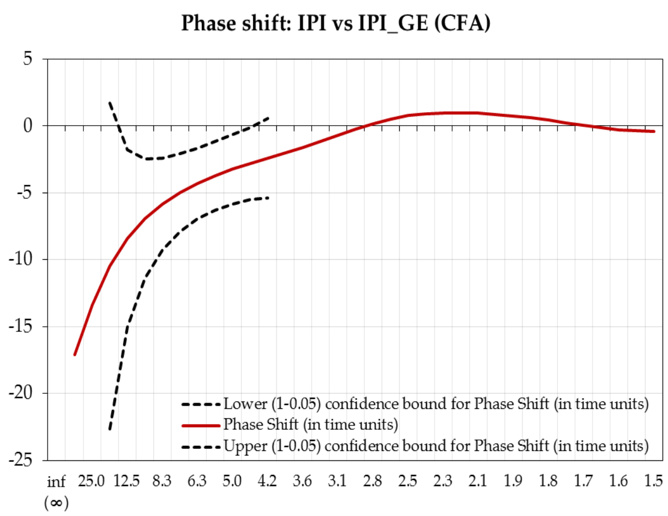

Coherence: IPI vs IPI_GE (DHP)
Coherence
coherence critical value at 0.05 significant level
0.6
0.5
0.4
0.3
0.2
0.1
0
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5


Coherence: IPI vs IPI_GE (CFA)
Coherence
coherence critical value at 0.05 significant level
0.6
0.5
0.4
0.3
0.2
0.1
0
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

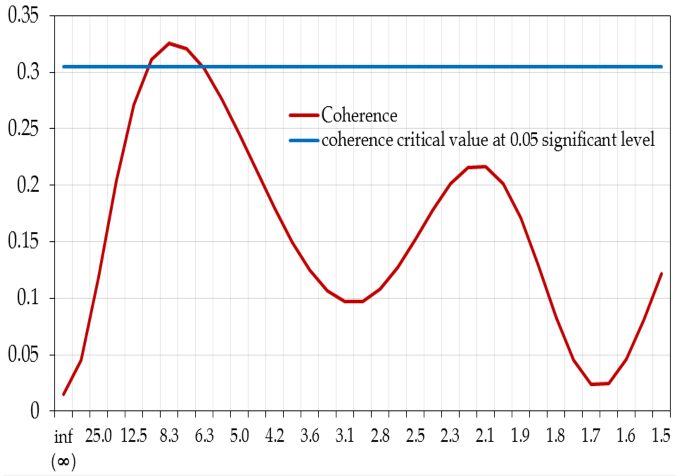

Gain: IPI vs IPI_GE (DHP)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5


Gain: IPI vs IPI_GE (CFA)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain
1.2
1
0.8
0.6
0.4
0.2
0
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

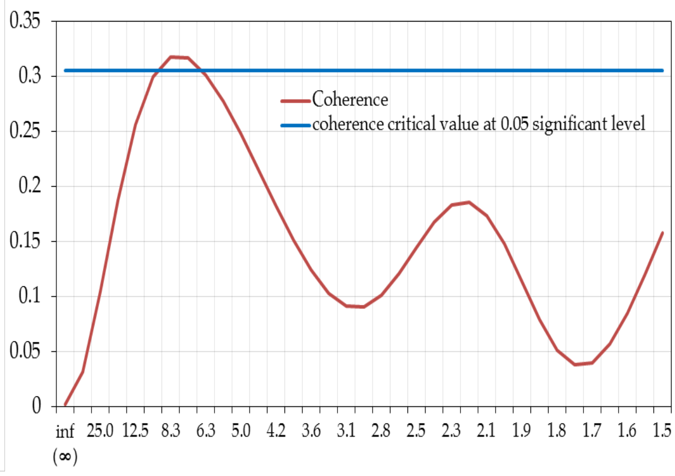

Phase shift: IPI vs IPI_GE (DHP)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)
5
0
-5
-10
-15
-20
-25
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5


Phase shift: IPI vs IPI_GE (CFA)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)
5
0
-5
-10
-15
-20
-25
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

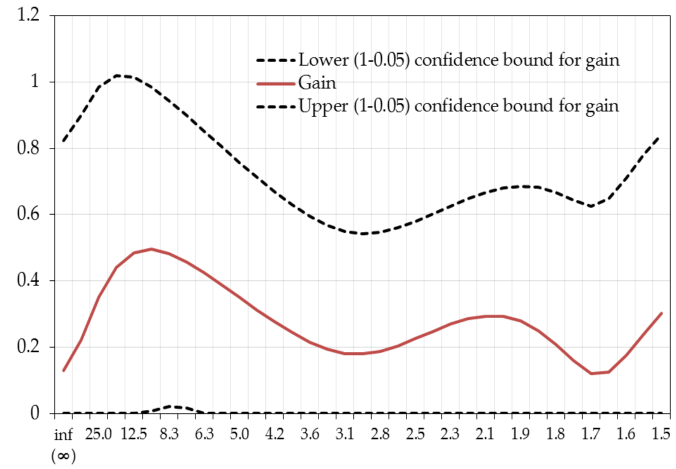
Coherence: IPI vs IPI_IT (DHP)



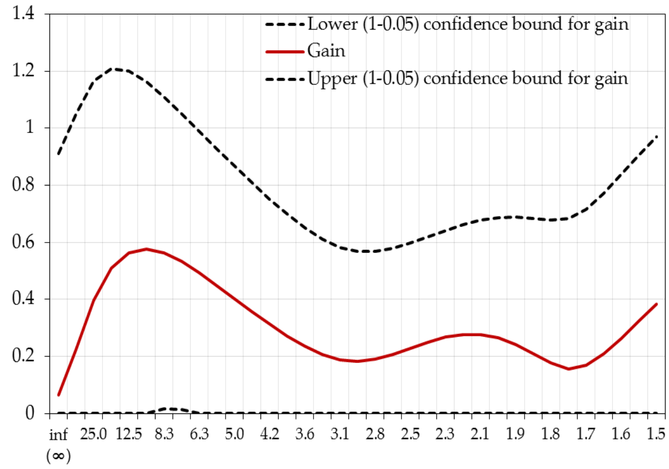
Coherence: IPI vs IPI_IT (CFA)



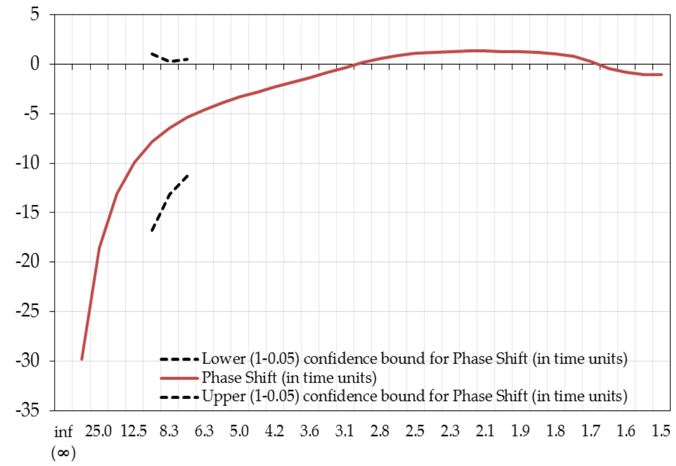
Gain: IPI vs IPI_IT (DHP)



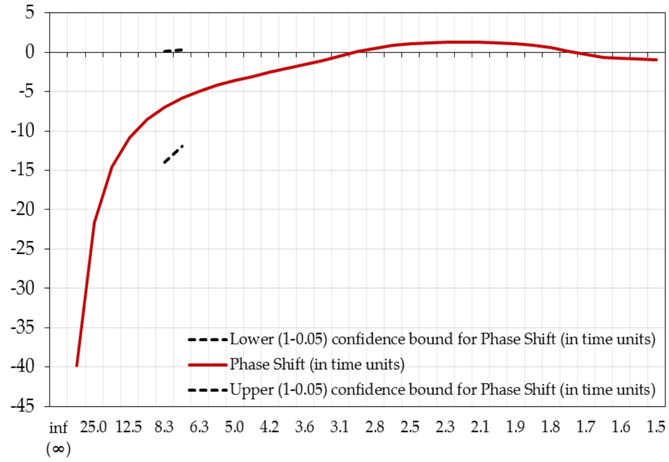
Gain: IPI vs IPI_IT (CFA)



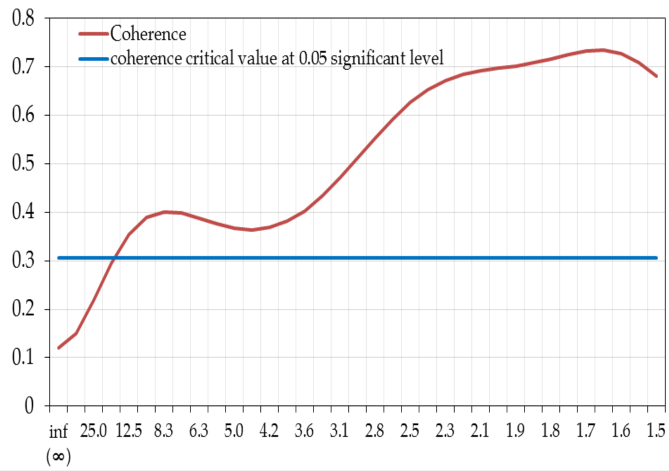
Phase shift: IPI vs IPI_IT (DHP)



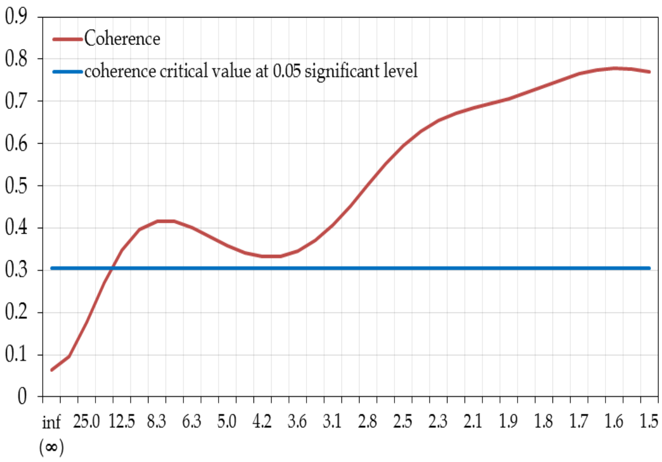
Phase shift: IPI vs IPI_IT (CFA)



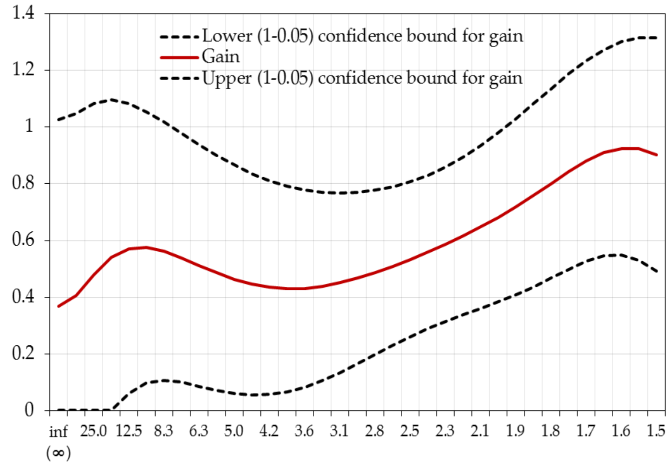
Coherence: IPIM vs IPI_EA (DHP)



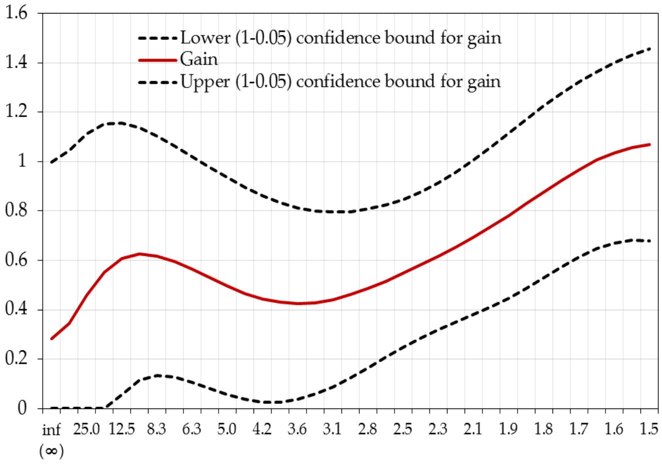
Coherence: IPIM vs IPI_EA (CFA)



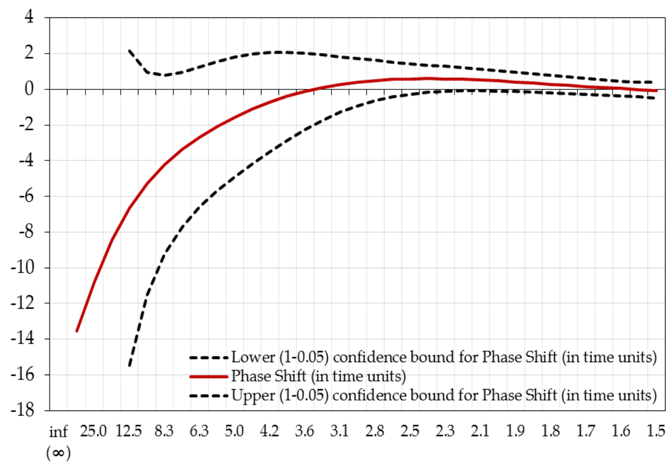
Gain: IPIM vs IPI_EA (DHP)



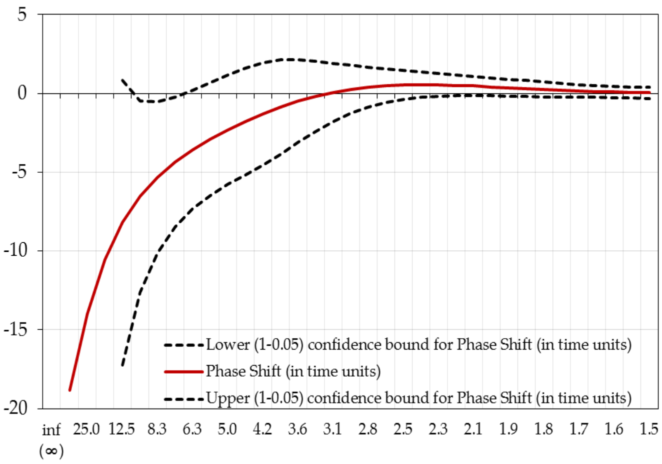
Gain: IPIM vs IPI_EA (CFA)



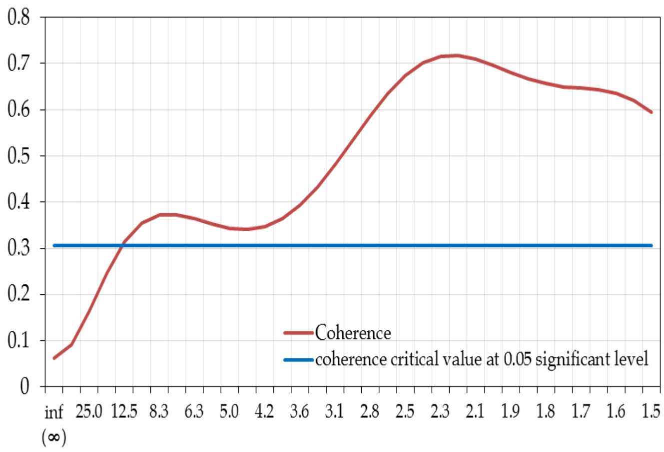
Phase shift: IPIM vs IPI_EA (DHP)



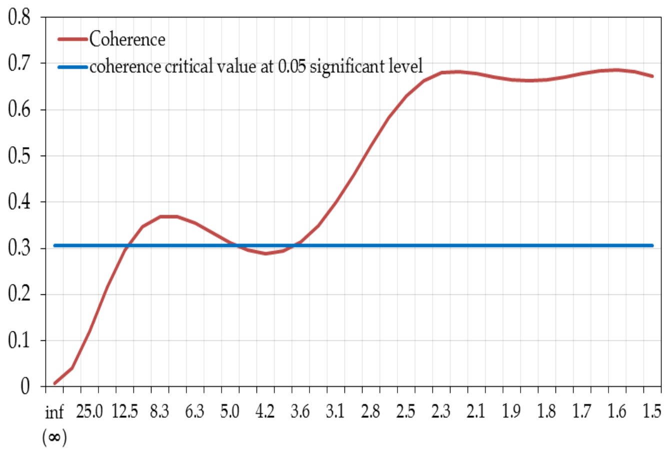
Phase shift: IPIM vs IPI_EA (CFA)



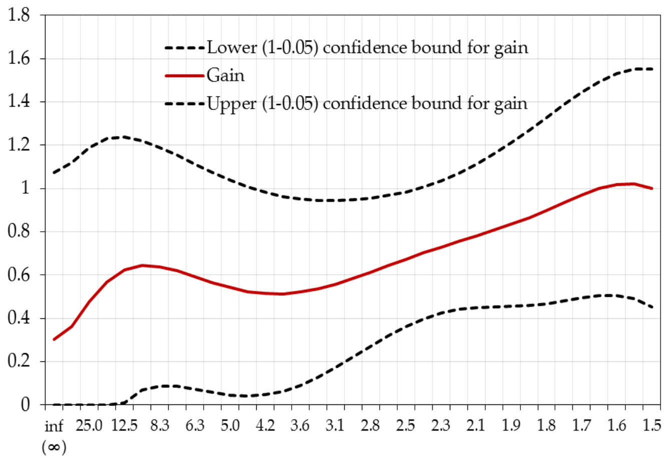
Coherence: IPIM vs IPI_FR (DHP)



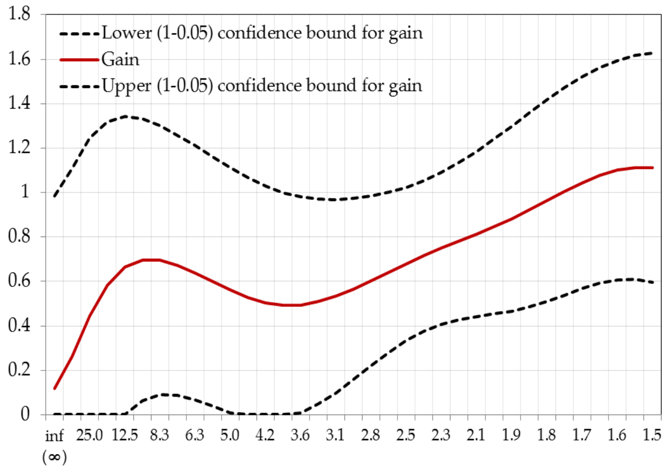
Coherence: IPIM vs IPI_FR (CFA)



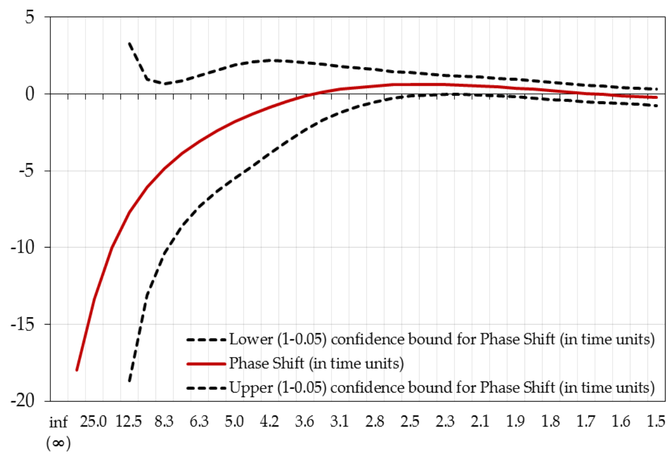
Gain: IPIM vs IPI_FR (DHP)



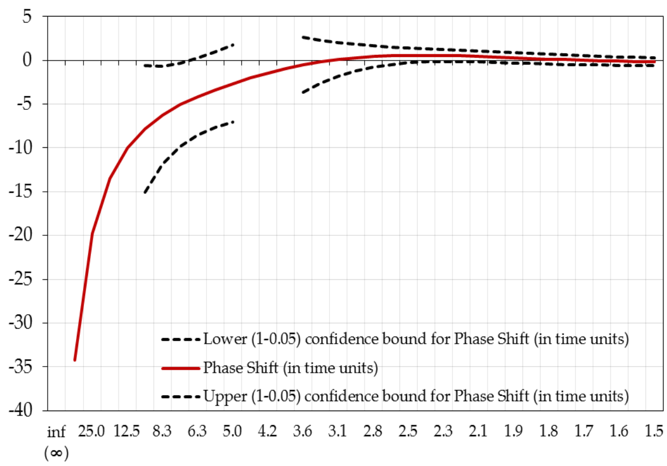
Gain: IPIM vs IPI_FR (CFA)



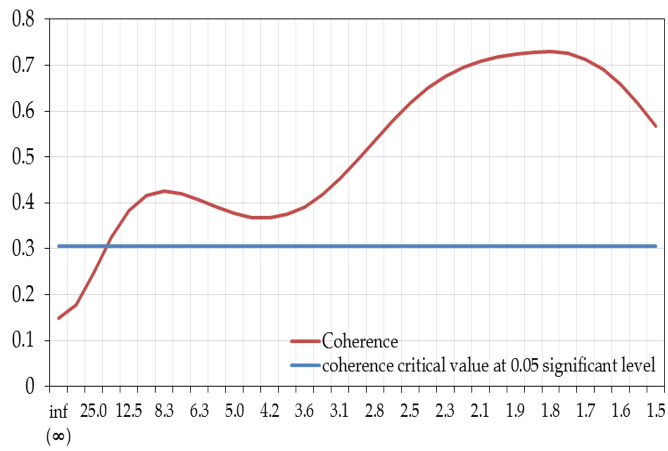
Phase shift: IPIM vs IPI_FR (DHP)



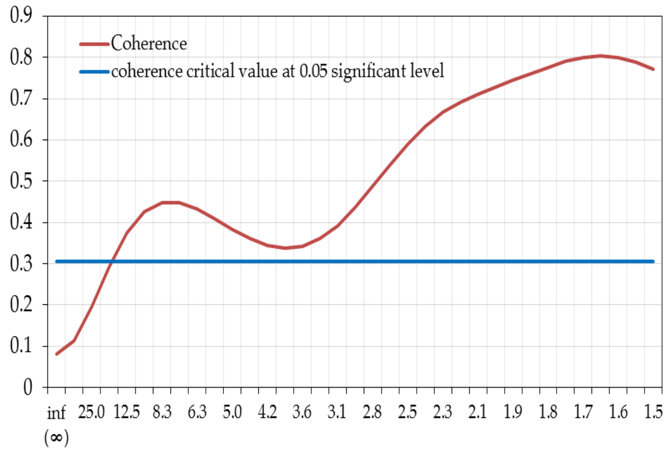
Phase shift: IPIM vs IPI_FR (CFA)



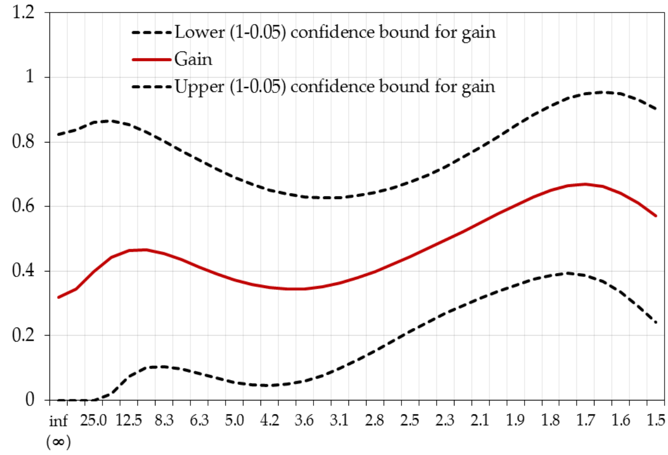
Coherence: IPIM vs IPI_GE (DHP)



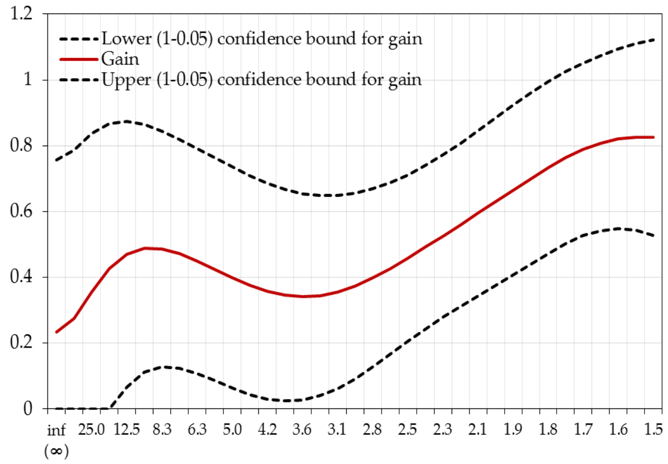
Coherence: IPIM vs IPI_GE (CFA)



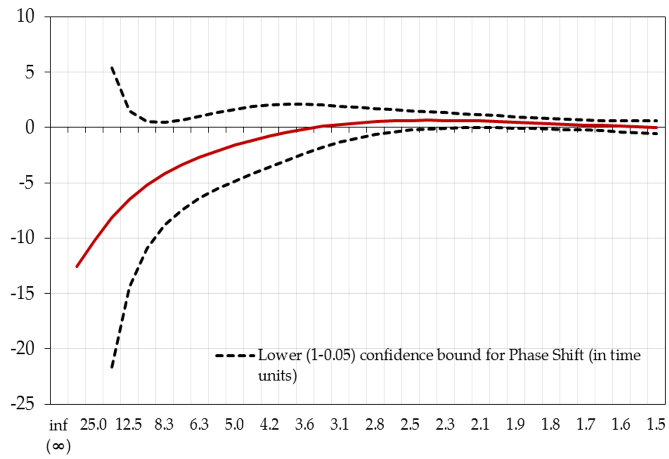
Gain: IPIM vs IPI_GE (DHP)



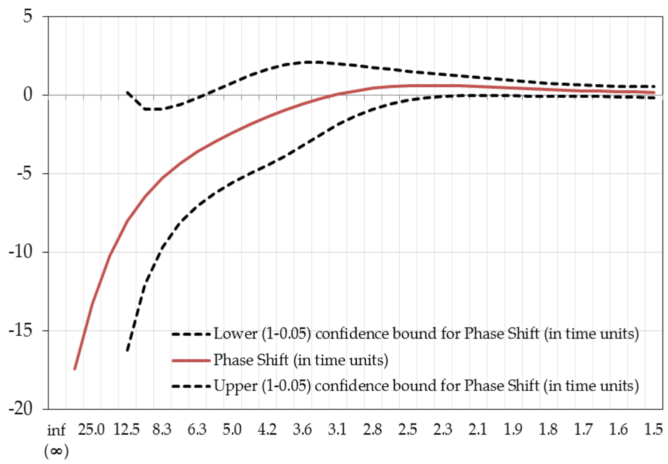
Gain: IPIM vs IPI_GE (CFA)



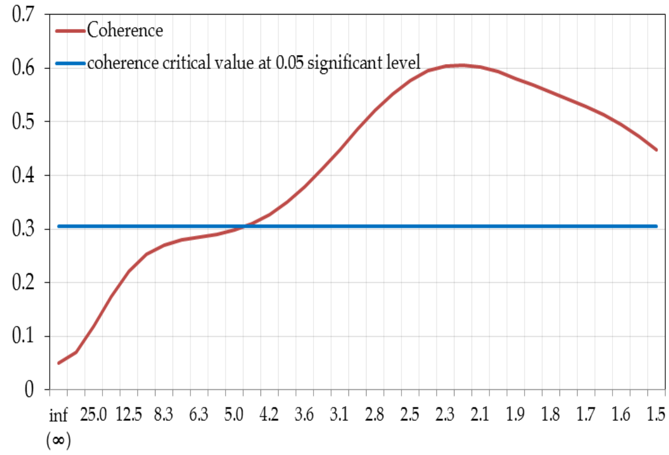
Phase shift: IPIM vs IPI_GE (DHP)



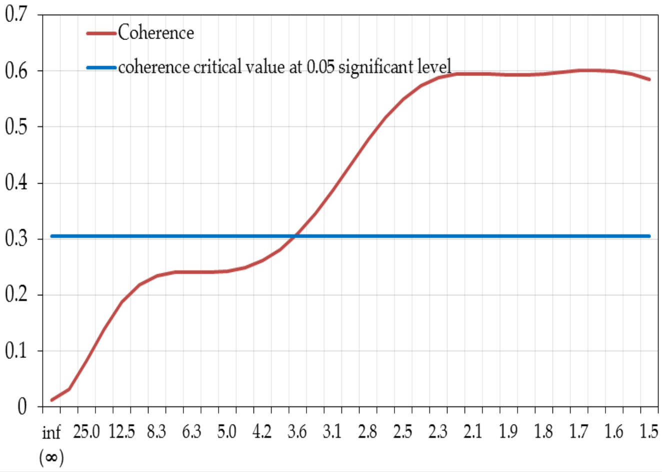
Phase shift: IPIM vs IPI_GE (CFA)



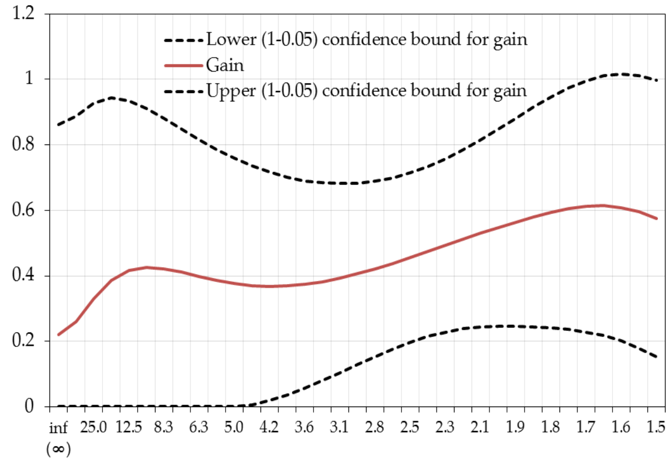
Coherence: IPIM vs IPI_IT (DHP)



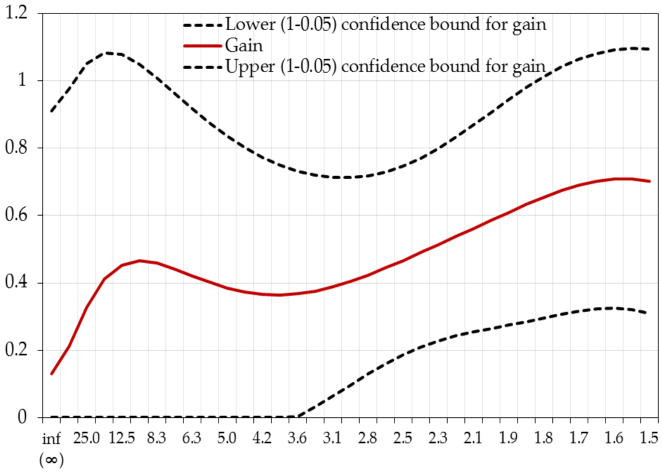
Coherence: IPIM vs IPI_IT (CFA)



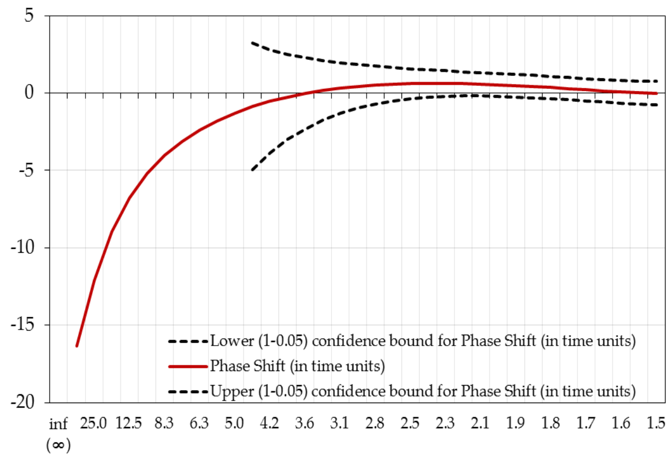
Gain: IPIM vs IPI_IT (DHP)



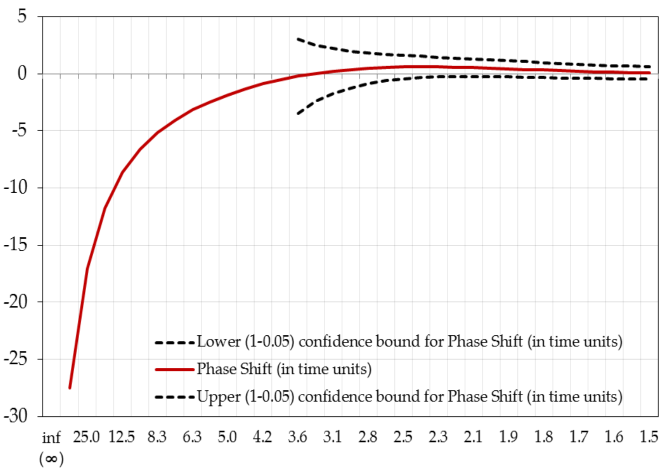
Gain: IPIM vs IPI_IT (CFA)

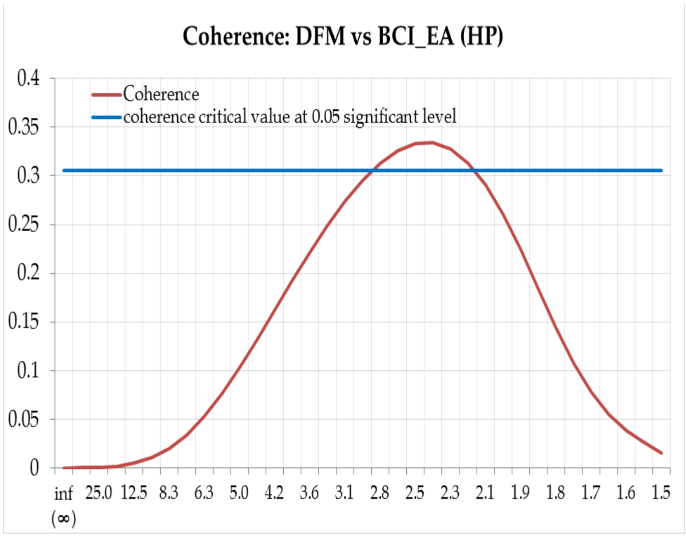


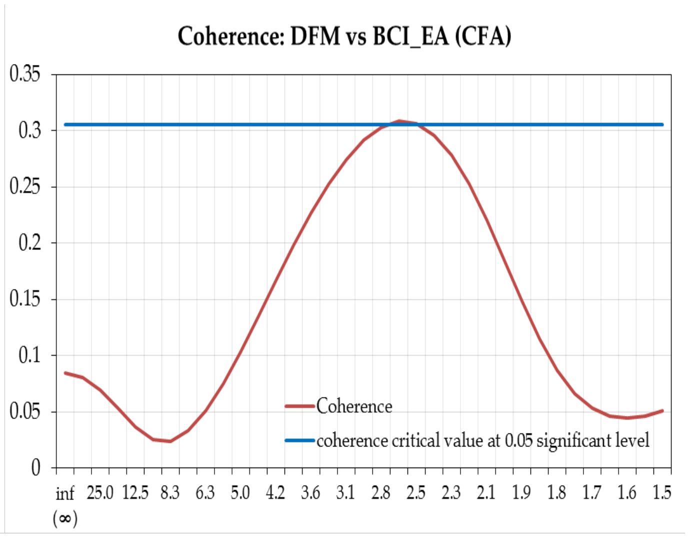
Phase shift: IPIM vs IPI_IT (DHP)

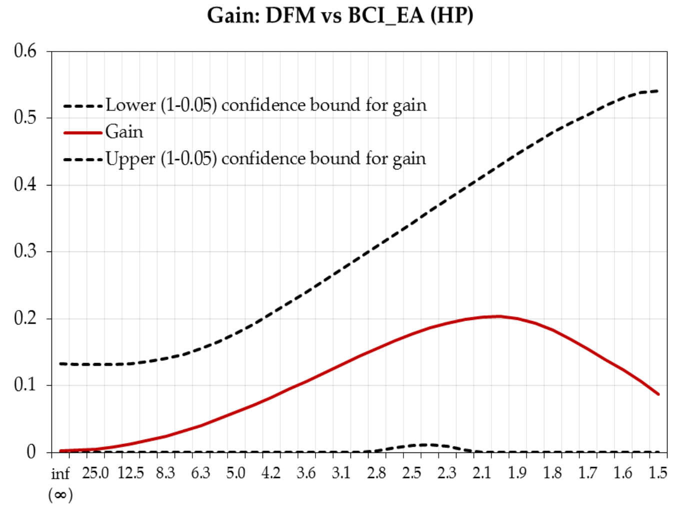


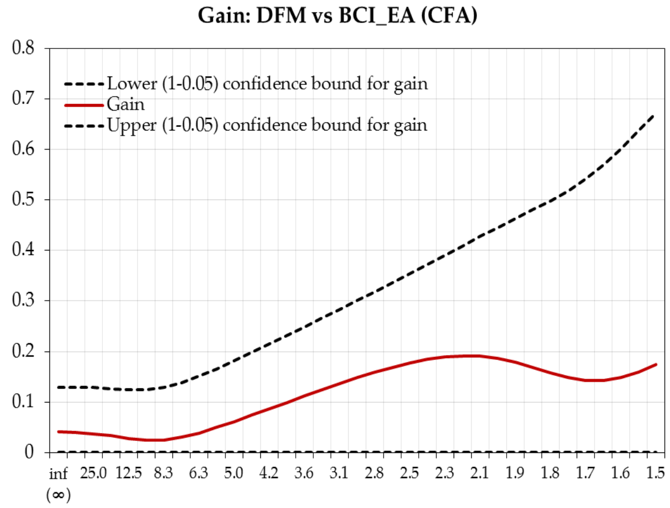
Phase shift: IPIM vs IPI_IT (CFA)

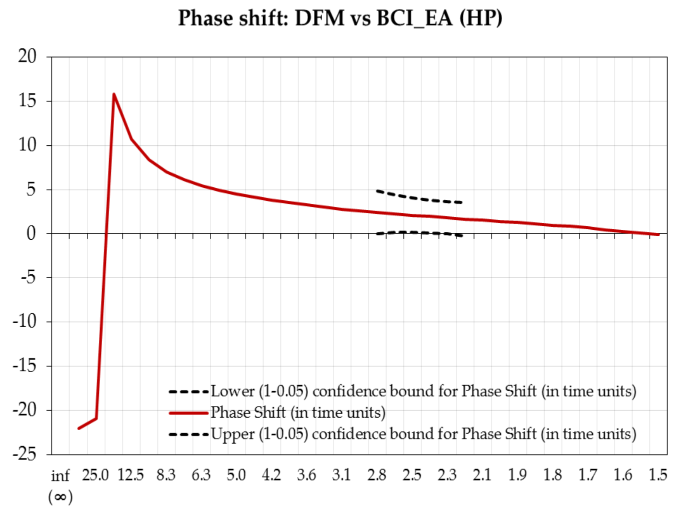


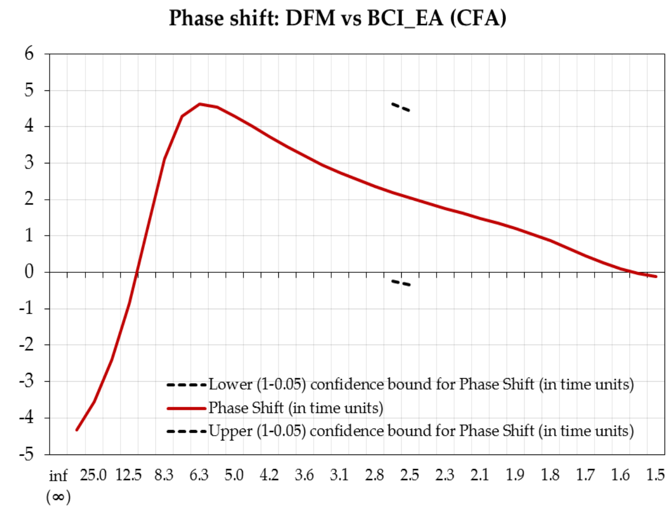

Coherence: DFM vs BCI_EA (HP)
Coherence
coherence critical value at 0.05 significant level
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5


Coherence: DFM vs BCI_EA (CFA)
Coherence
coherence critical value at 0.05 significant level
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

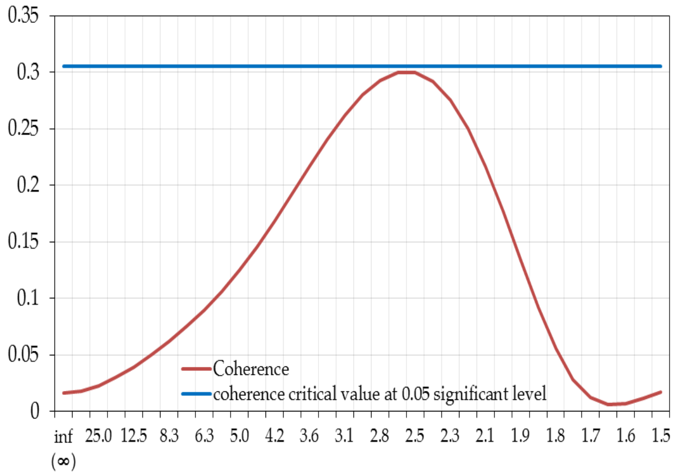

Gain: DFM vs BCI_EA (HP)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5


Gain: DFM vs BCI_EA (CFA)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

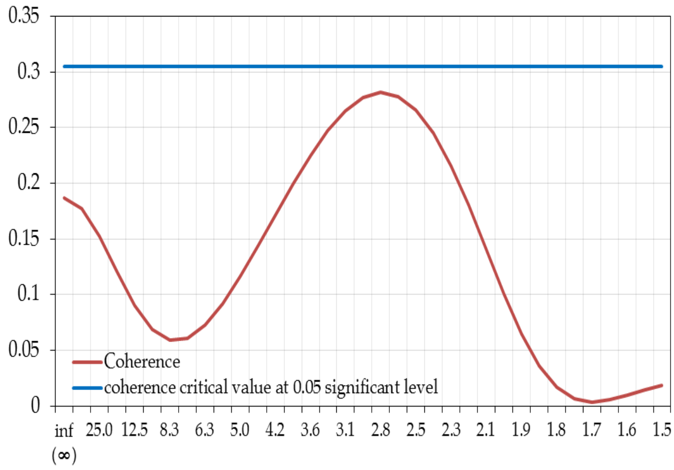

Phase shift: DFM vs BCI_EA (HP)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5


Phase shift: DFM vs BCI_EA (CFA)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)
inf (∞) 25.0 12.5 8.3 6.3 5.0 4.2 3.6 3.1 2.8 2.5 2.3 2.1 1.9 1.8 1.7 1.6 1.5

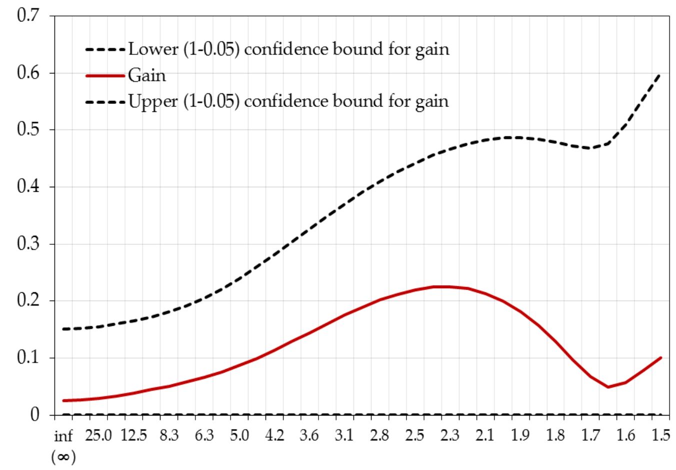
Coherence: DFM vs BCI_FR (HP)



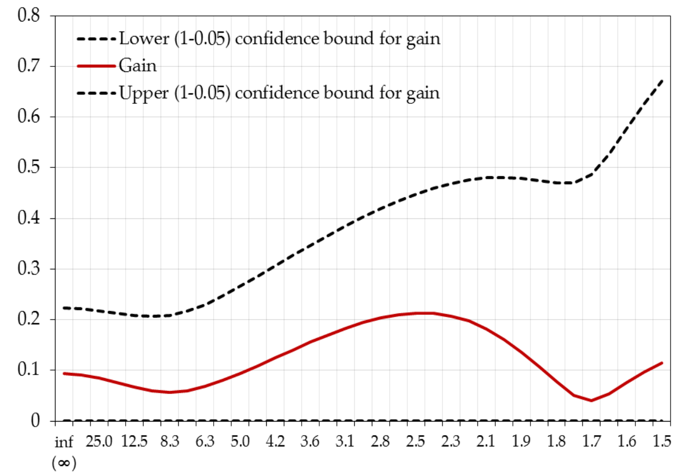
Coherence: DFM vs BCI_FR (CFA)



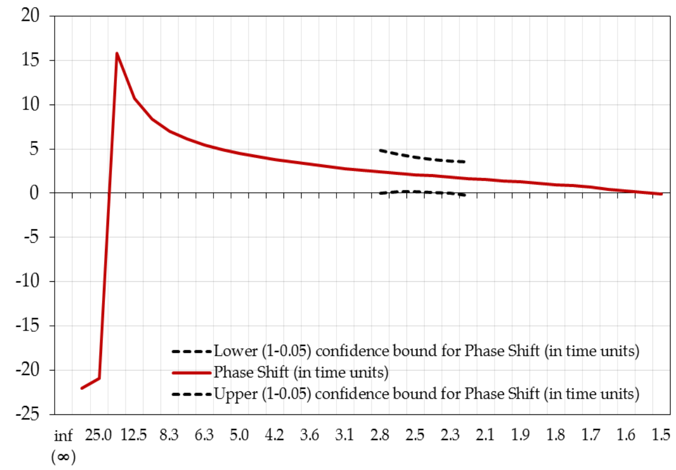
Gain: DFM vs BCI_FR (HP)



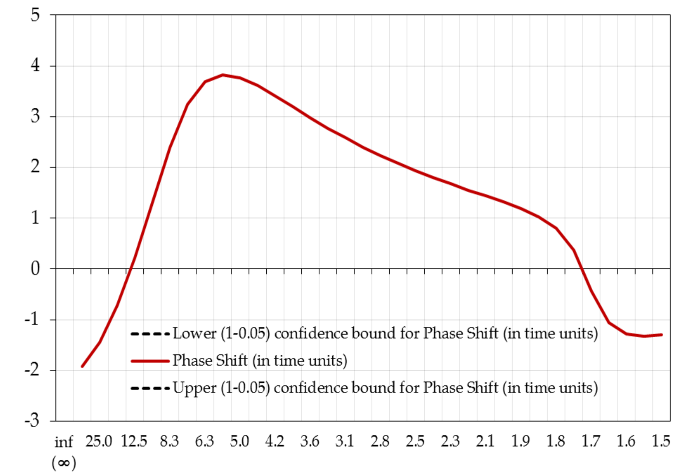
Gain: DFM vs BCI_FR (CFA)



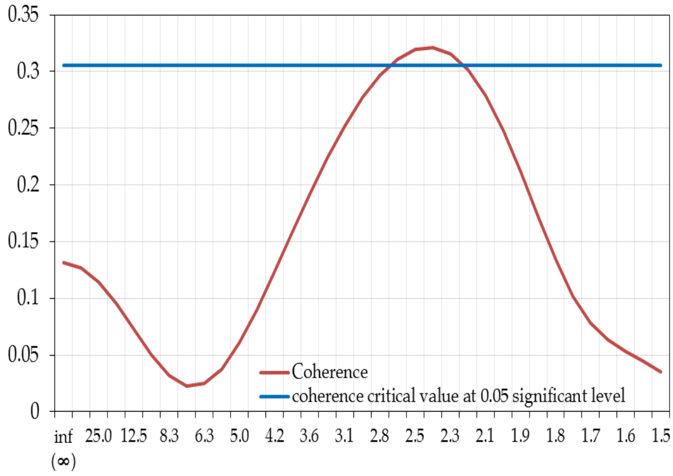
Phase shift: DFM vs BCI_EA (HP)



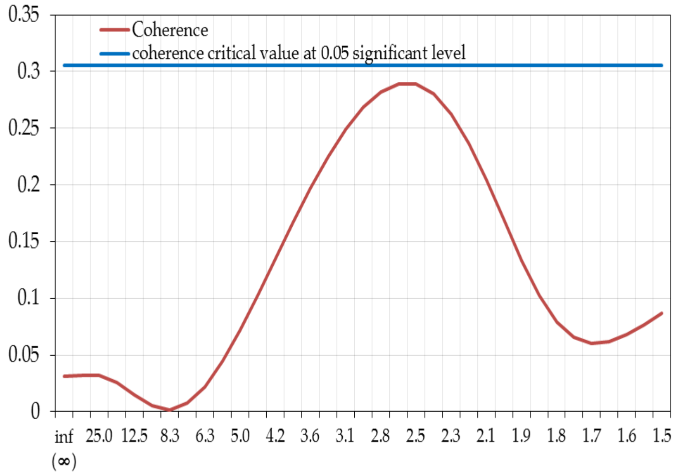
Phase shift: DFM vs BCI_FR (CFA)



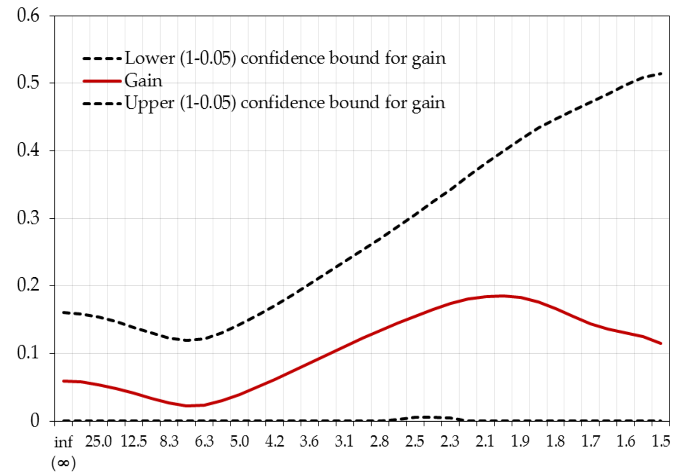
Coherence: DFM vs BCI_GE (HP)



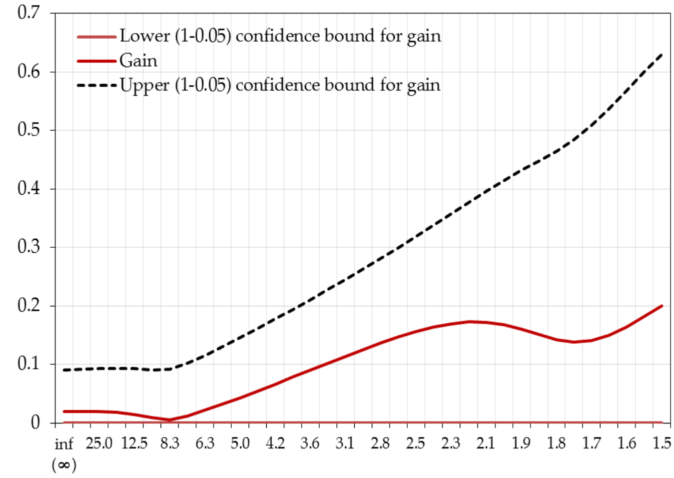
Coherence: DFM vs BCI_GE (CFA)



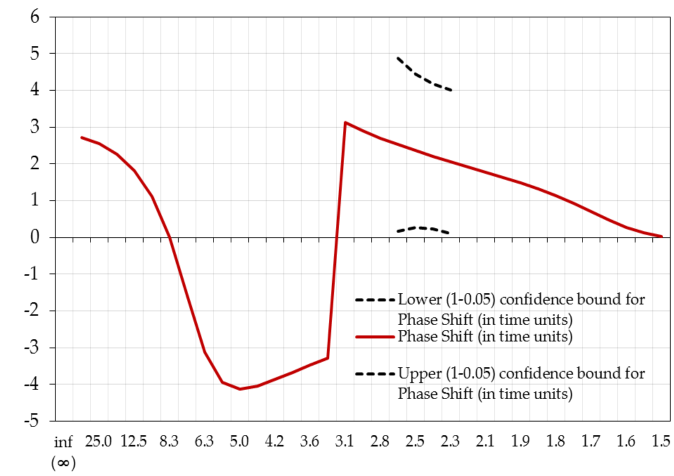
Gain: DFM vs BCI_GE (HP)



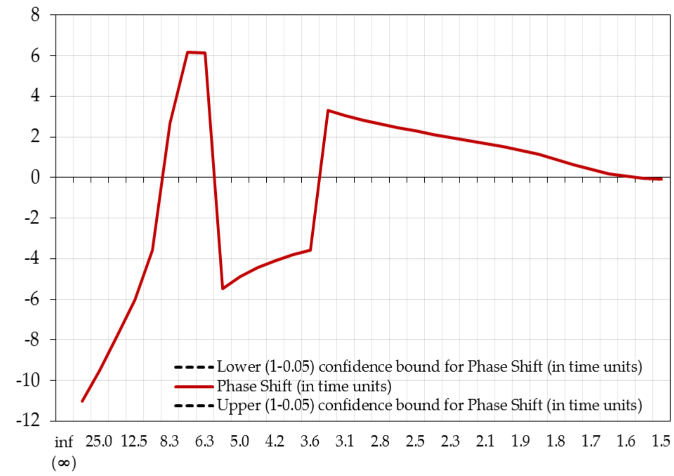
Gain: DFM vs BCI_GE (CFA)

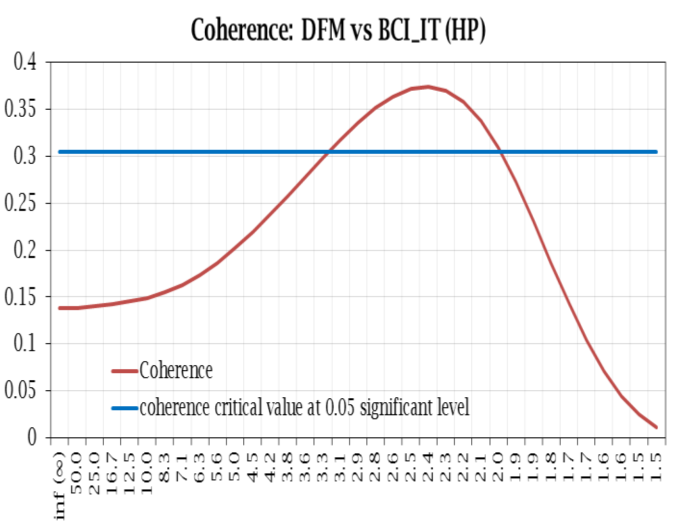


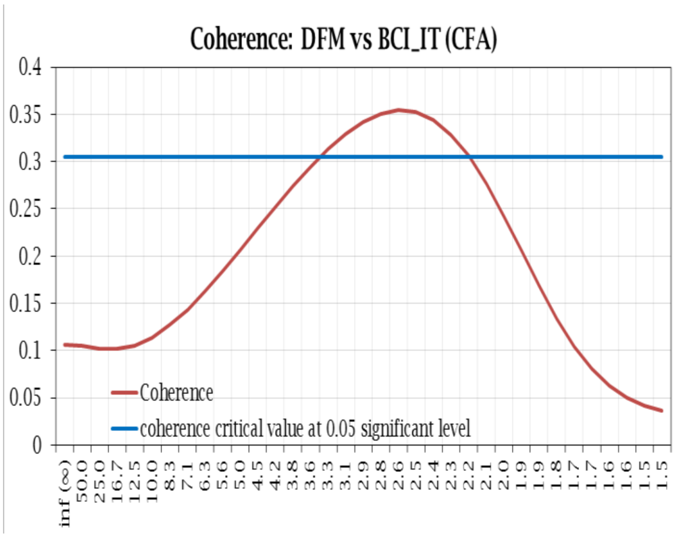
Phase shift: DFM vs BCI_GE (HP)



Phase shift: DFM vs BCI_GE (CFA)




Coherence: DFM vs BCI_IT (HP)
Coherence
coherence critical value at 0.05 significant level


Coherence: DFM vs BCI_IT (CFA)
Coherence
coherence critical value at 0.05 significant level

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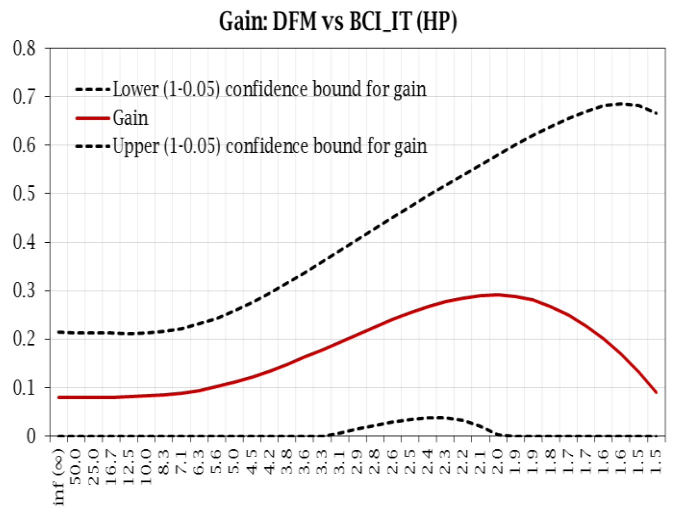
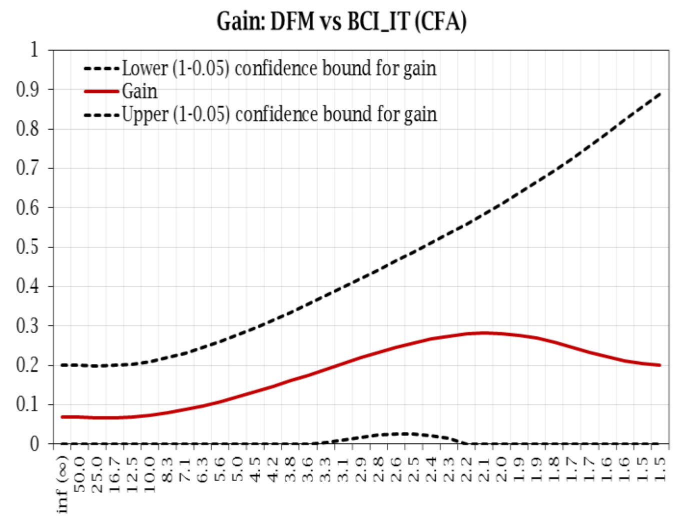
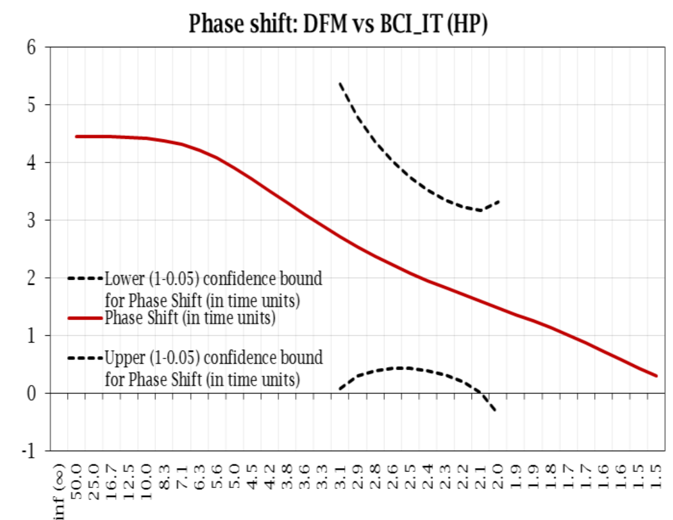
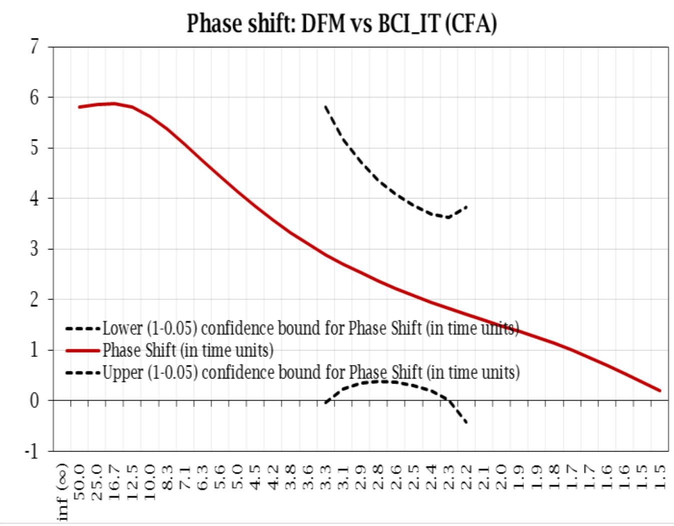
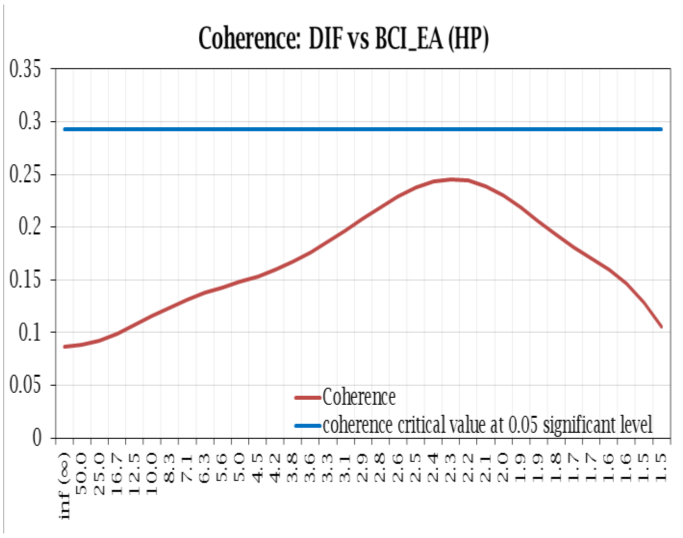
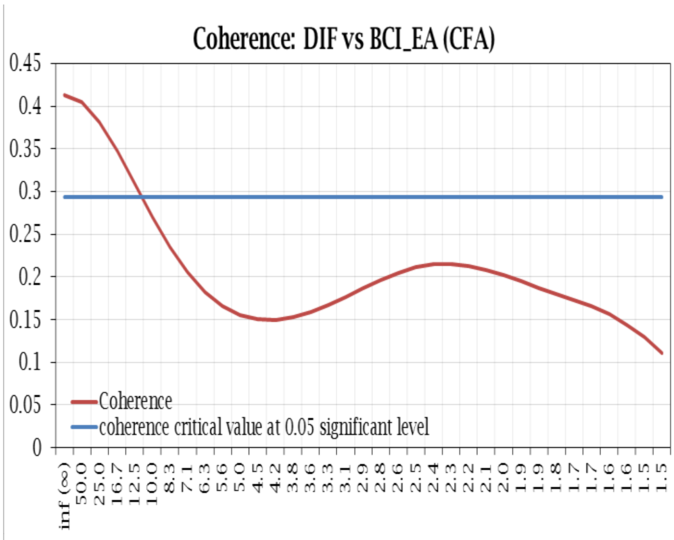

Gain: DFM vs BCI_IT (HP)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain

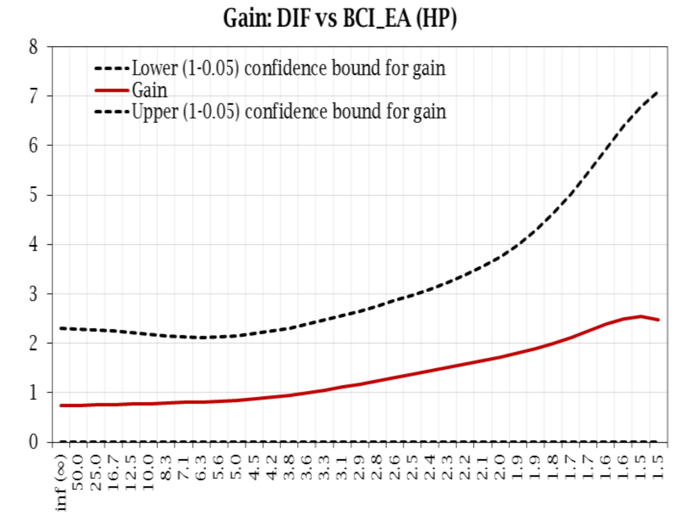

Gain: DFM vs BCI_IT (CFA)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain

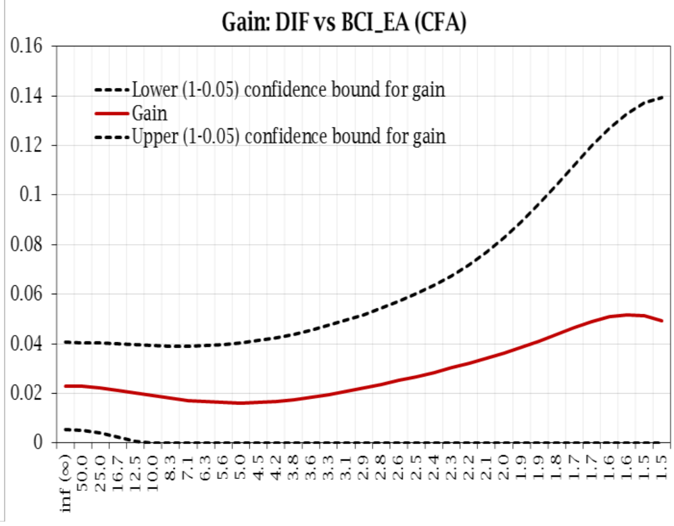

Phase shift: DFM vs BCI_IT (HP)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)

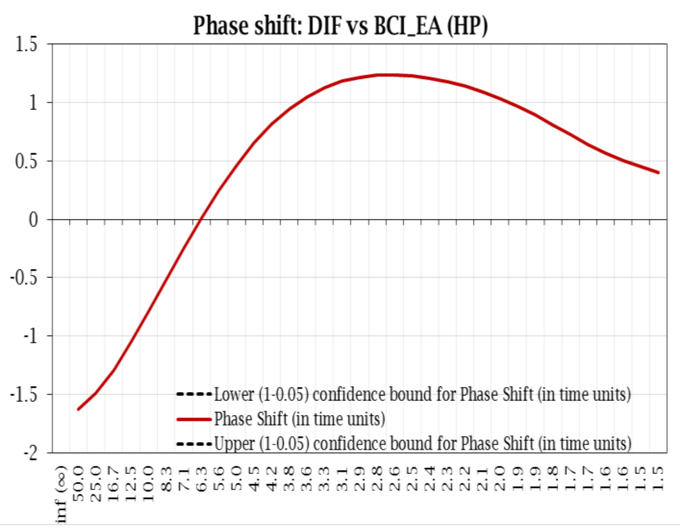

Phase shift: DFM vs BCI_IT (CFA)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)

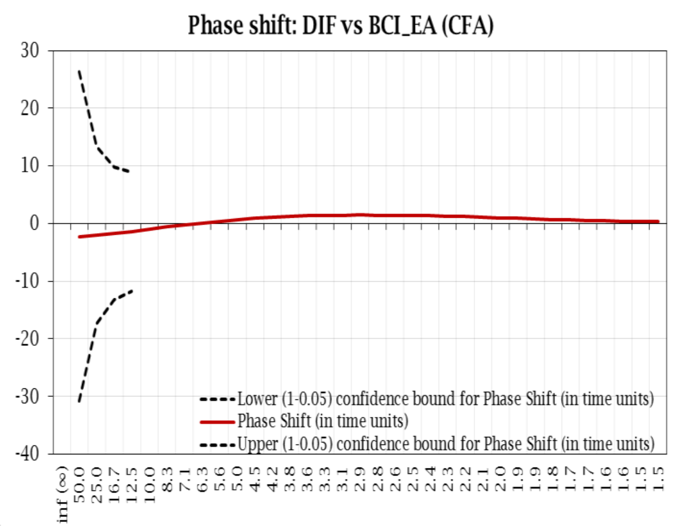

Coherence: DIF vs BCI_EA (HP)
Coherence
coherence critical value at 0.05 significant level

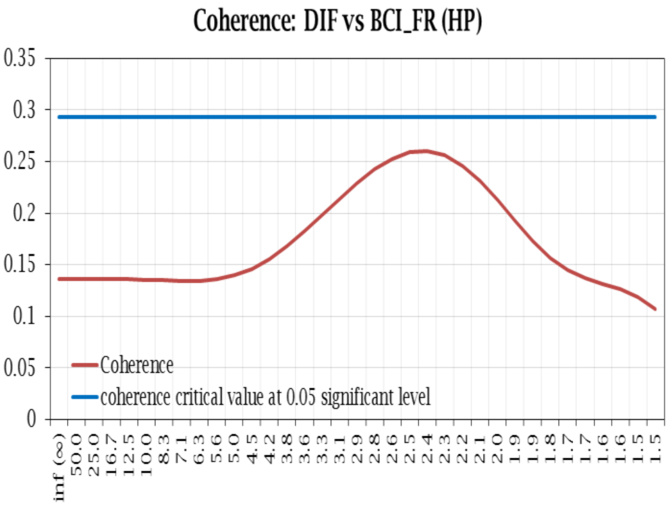

Coherence: DIF vs BCI_EA (CFA)
Coherence
coherence critical value at 0.05 significant level

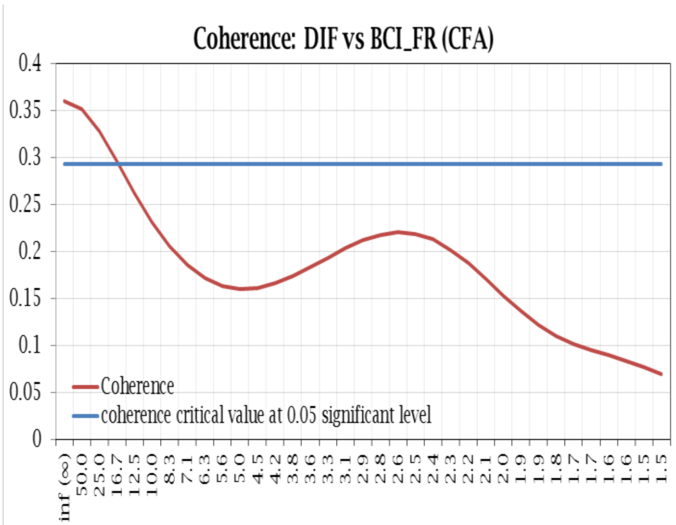

Gain: DIF vs BCI_EA (HP)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain

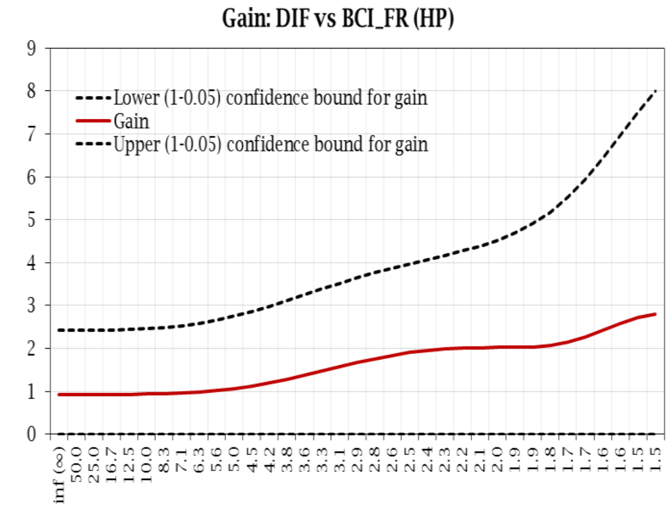

Gain: DIF vs BCI_EA (CFA)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain

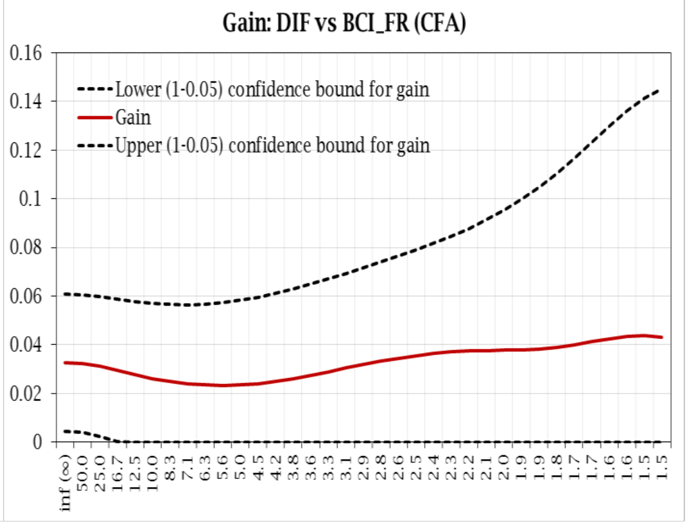

Phase shift: DIF vs BCI_EA (HP)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)

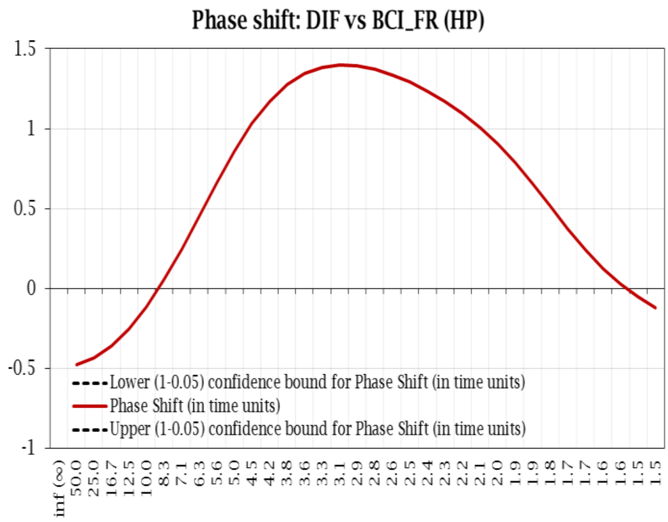

Phase shift: DIF vs BCI_EA (CFA)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)

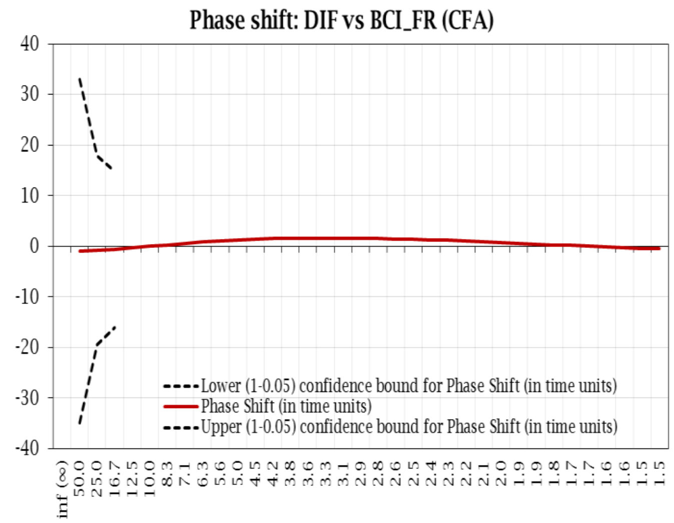

Coherence: DIF vs BCI_FR (HP)
Coherence
coherence critical value at 0.05 significant level

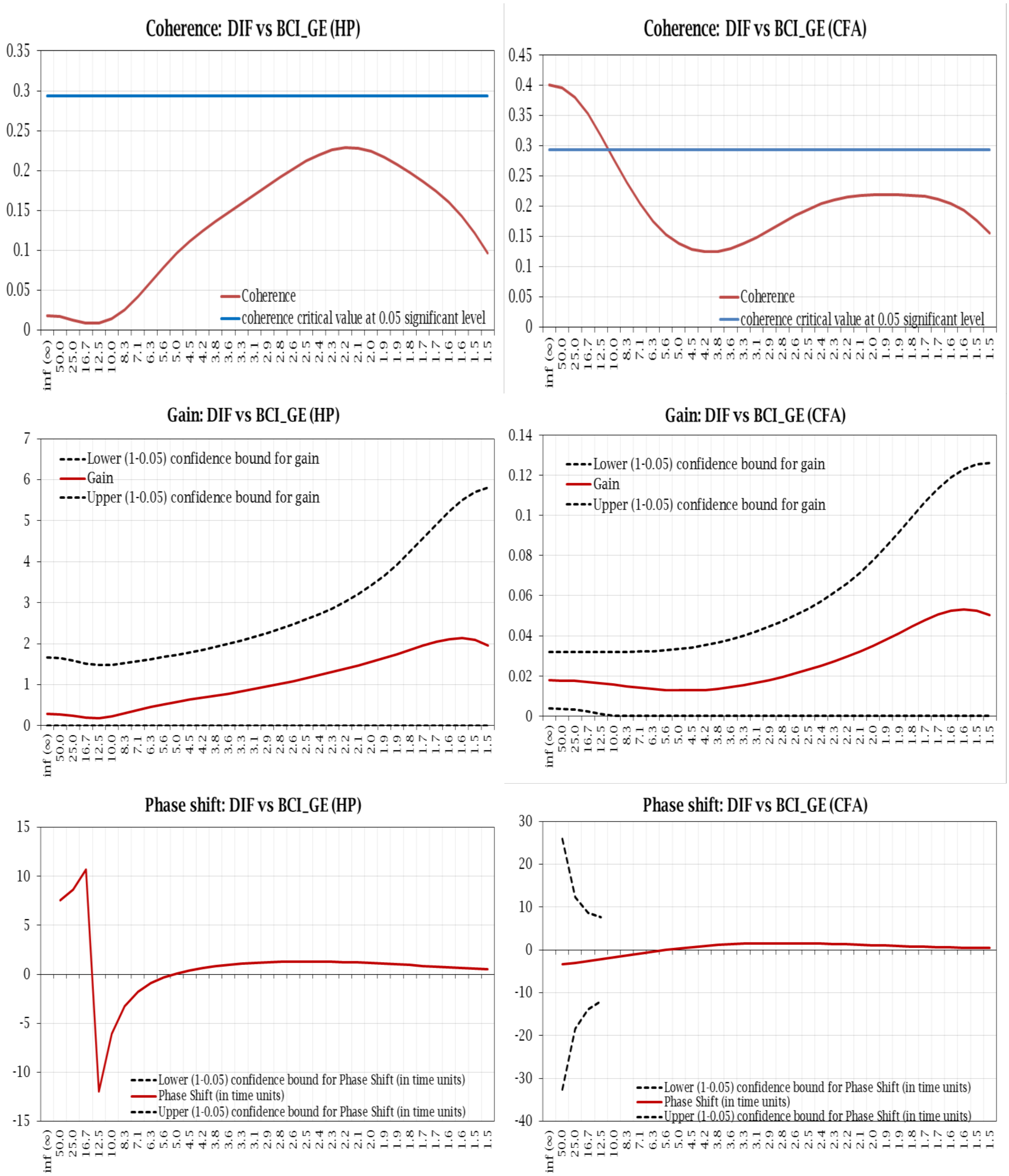

Coherence: DIF vs BCI_FR (CFA)
Coherence
coherence critical value at 0.05 significant level


Gain: DIF vs BCI_FR (HP)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain

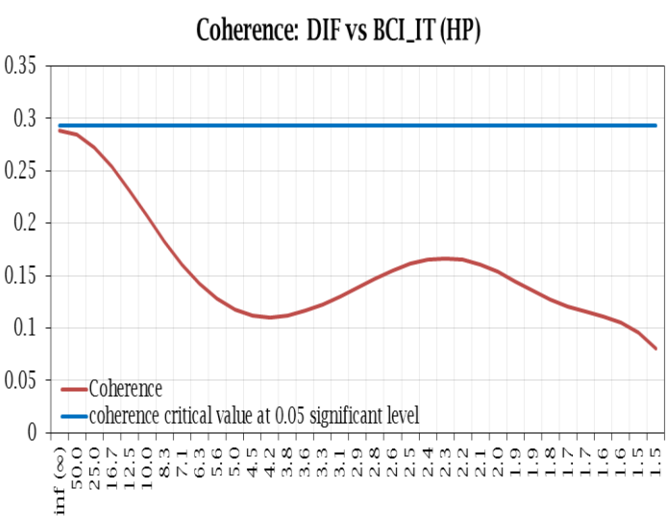

Gain: DIF vs BCI_FR (CFA)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain

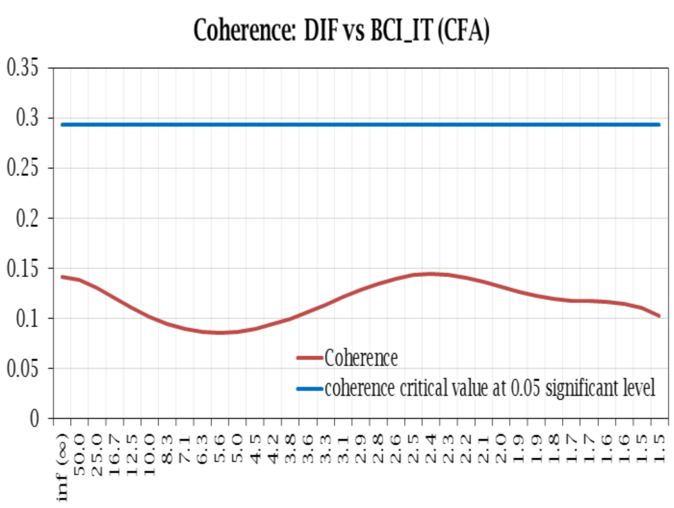

Phase shift: DIF vs BCI_FR (HP)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)

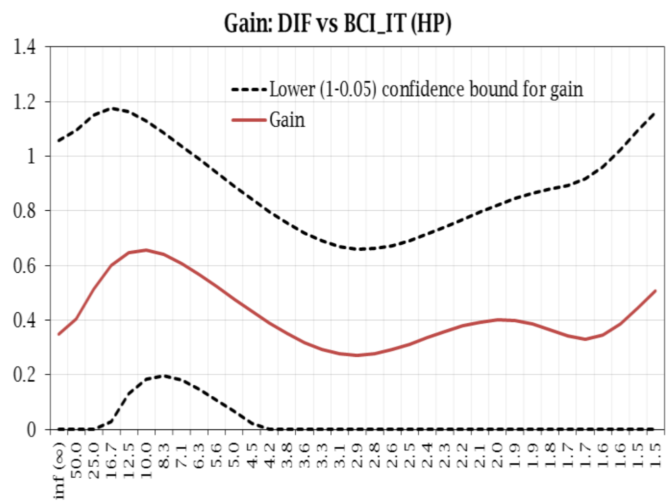

Phase shift: DIF vs BCI_FR (CFA)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)

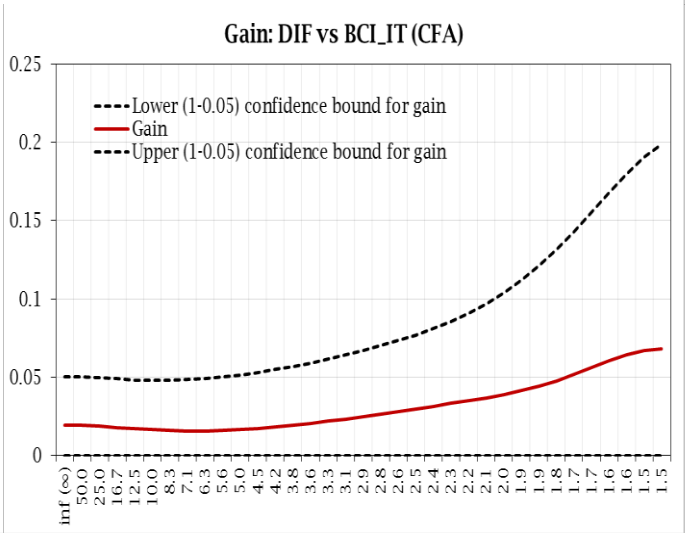


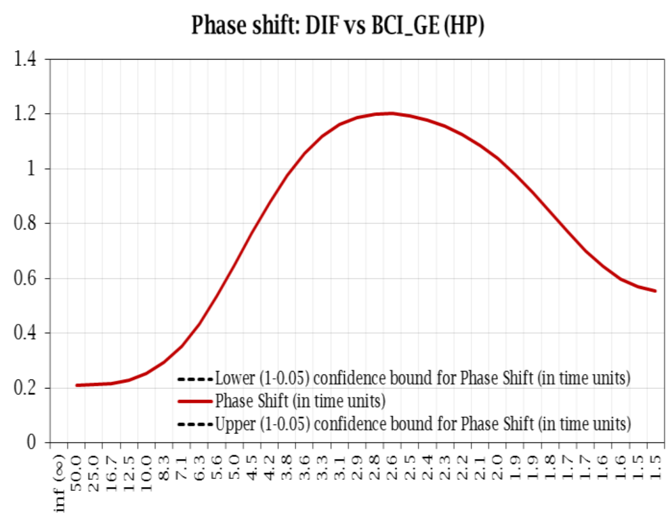
Coherence: DIF vs BCI_GE (HP)
Coherence
coherence critical value at 0.05 significant level
Coherence: DIF vs BCI_GE (CFA)
Coherence
coherence critical value at 0.05 significant level
Gain: DIF vs BCI_GE (HP)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain
Gain: DIF vs BCI_GE (CFA)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain
Phase shift: DIF vs BCI_GE (HP)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)
Phase shift: DIF vs BCI_GE (CFA)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)
inf (∞) 50.0 25.0 16.7 12.5 10.0 8.3 7.1 6.3 5.6 5.0 4.5 4.2 3.8 3.6 3.3 3.1 2.9 2.8 2.6 2.5 2.4 2.3 2.2 2.1 2.0 1.9 1.9 1.8 1.7 1.7 1.6 1.6 1.5 1.5

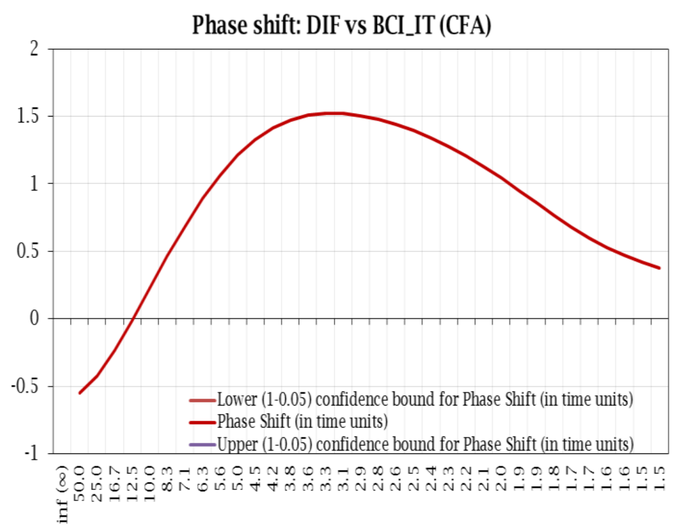

Coherence: DIF vs BCI_IT (HP)
Coherence
coherence critical value at 0.05 significant level


Coherence: DIF vs BCI_IT (CFA)
Coherence
coherence critical value at 0.05 significant level


Gain: DIF vs BCI_IT (HP)
Lower (1-0.05) confidence bound for gain
Gain


Gain: DIF vs BCI_IT (CFA)
Lower (1-0.05) confidence bound for gain
Gain
Upper (1-0.05) confidence bound for gain


Phase shift: DIF vs BCI_GE (HP)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)


Phase shift: DIF vs BCI_IT (CFA)
Lower (1-0.05) confidence bound for Phase Shift (in time units)
Phase Shift (in time units)
Upper (1-0.05) confidence bound for Phase Shift (in time units)