THE GRADUATE INSTITUTE GENEVA

INSTITUT DE HAUTES ÉTUDES INTERNATIONALES ET DU DÉVELOPPEMENT
GRADUATE INSTITUTE OF INTERNATIONAL AND DEVELOPMENT STUDIES

Graduate Institute of International and Development Studies

International Economics Department

Working Paper Series

Working Paper No. HEIDWP14-2021

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July 2021

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Unconventional Credit Policy in an Economy under Zero Lower Bound*

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June 30, 2021

Abstract

In this paper we develop a simple two-period model that reconciles credit demand and supply frictions. In this stylized but realistic model credit and deposit markets are interlinked and credit demand and credit supply frictions amplify each other in such a way that produces in equilibrium very low levels of credit and stronger reductions of the real and nominal interest, so an economy is much closer to the ZLB. However, an unconventional credit policy, that consists on central bank loans to firms that are guaranteed by the government, can undo partially the effects of the credit frictions and prevents the economy from reaching the ZLB. Since central bank loans are not subject to the moral hazard problem between bankers and depositors and are government-guaranteed, credit market interventions rise aggregate credit supply and positively affect the aggregate credit demand, respectively. However, once the economy is at the ZLB the effect of a credit policy is reduced due to a relatively stronger inflation reduction, which in turn reduces entrepreneurs' incentives to demand bank loans.

Keywords: Unconventional credit policy, asymmetric information, moral hazard, zero Lower bound.

JEL Classification: G21, G28, E44, E5.

1 Introduction

The Covid-19 global shock has confronted policy makers with the limits of standard policy tools to stimulate the economy. One important constraint faced by several central

*The authors are very thankful to Céline Poilly for the academic supervision of this paper. The authors also want to thank Professor Cédric Tille and to participants of seminar at the Graduate Institute of International and Development Studies (IHEID) for helpful comments. This research took place through the Coaching Program of the Bilateral Assistance and Capacity Building for Central Banks (BCC), financed by SECO and implemented by the Graduate Institute in Geneva. The views expressed in this paper are solely those of the authors and do not necessarily reflect those of the Central Reserve Bank of Peru. All errors are our own.

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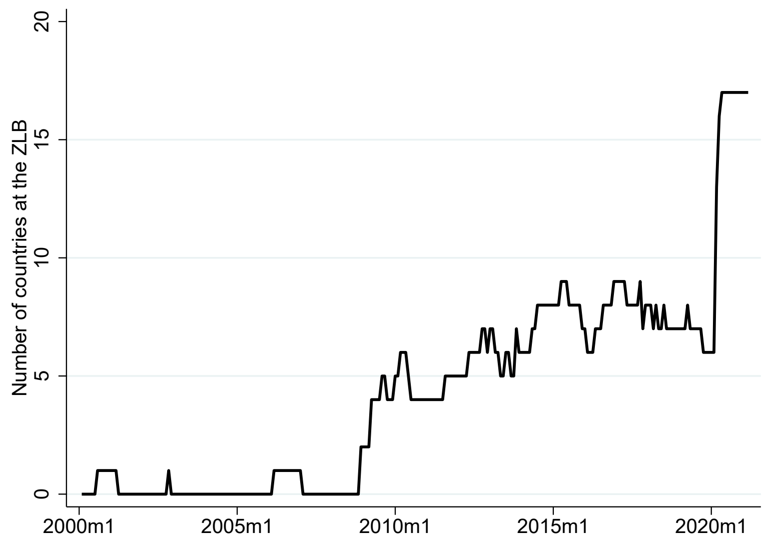
banks is the zero lower bound (ZLB) in the monetary policy rate. Figure 1 shows that during the Covid-19 pandemic the number of ZLB countries more than duplicated, from around 7 to 17. Other countries are also close to the ZLB. However, monetary policy has not stopped and now involves in several countries unconventional measures, such as Quantitative Easing (QE), to reduce the cost of financing and limit the kinds of moral hazard issues that could freeze the credit market. In addition, some central banks adopted other unconventional credit policies to avoid a deep recession, such as additional liquidity facilities or government-guaranteed corporate lending at an interest rate equal to the policy rate. These unconventional credit policies have grown in importance around the world. From the 113 economies that adopted debt finance policies, 41 countries have used these unconventional credit policies to reduce the cost of credit.[1] Governments promptly adopted these unconventional credit policies due to the firms' liquidity shortage shock.

Figure 1. *Number of countries at the ZLB*



Note: Source: IMF, BIS. Own computations. Monthly data: 2000m1-2021m3.

This paper aims to uncover the implications of the unconventional credit policy under a zero lower bound environment in a simple two-period framework with credit supply and credit demand frictions.[2] In particular, our main goal is to study if the ZLB improves or deteriorates the effectiveness of the unconventional credit policy intervention. In the same line, we also assess if the unconventional credit policy intervention allows central banks to exit faster from the ZLB. In that sense, we need to understand what are the effects of this unconventional credit program and the mechanisms that are working on.

[1]Information on policies implemented around to world to face the Covid-19 shock is compiled by the World Bank and reported in the "Map of SME-Support Measures in Response to COVID-19".

[2]In our view, these credit policies are named unconventional, and classified as different to conventional credit policies studied in Cúrdia and Woodford (2011) and Gertler and Karadi (2011), for two reasons: 1) the required return of loans originated by the credit policy is not the market lending rate but the monetary policy rate; 2) loans are originated by a government-guaranteed credit policy. This unconventional credit policy in presence of the ZLB opens the door to policy considerations regarding the role of central bank intermediation for accessing credit in a zero-cost economy.

The simplicity of the two-period model allows us to better understand the intuition of mechanisms and results.

In this two-period framework the inefficiencies created by the credit supply and credit demand frictions give rooms to the implementation of an unconventional credit policy. This is, unconventional credit policy plays a role due to credit frictions that hamper savings flows in financing investment opportunities and prevent banks from adequately monitoring projects. The model includes households, banks, entrepreneurs and firms. Households make bank deposits and banks give loans to entrepreneurs, who in turn create capital. Intermediate goods firms demand capital to produce. Price stickiness is introduced by assuming that a fraction of them cannot update prices and allows to model conventional central bank monetary interventions. And final goods firms demand intermediate goods to produce final goods. In addition, the monetary policy rate is subject to a ZLB constraint. We assume that the monetary authority successfully reaches its target inflation unless the economy has reached the ZLB.

The novelty of our framework lies in the modeling of frictions on both the credit demand and supply sides (as in Pozo and Rojas 2020). Credit demand frictions are modeled à la Bernanke, Gertler and Gilchrist (1999). This arises from an asymmetric information problem between entrepreneurs and banks. Ex-ante identical entrepreneurs face an idiosyncratic shock, which is not observable by banks, and for which a risk premium is charged. Entrepreneurs might prefer to hold enough equity as collateral to ensure a not very high risk premium. Credit supply frictions are modeled à la Gertler and Karadi (2011). This is, there is a moral hazard problem between banks and depositors. An endogenous leverage constraint arises in order to ensure that banks do not divert banks' assets and hence can operate. As a result, firms' and banks' equity is crucial to determine aggregate credit demand and supply, respectively.

The unconventional credit policy consists of central bank liquidity injection to banks so these latter have the commitment to use these resources to issue loans (named, indirect central bank loans) that are guaranteed by the government. The liquidity is provided in auctions where the winners offer the lower (non-default) lending interest rate. Since central bank has better enforcement power over banks than depositors and since indirect CB loans are government-guaranteed, the (non-default) lending interest rate is the risk-free interest rate. The goal of this policy is to lessen the impact of a negative shock on the economy.

As expected the credit supply and demand frictions yield to an inefficiently low capital allocation and credit level. In particular, the credit supply frictions deteriorate the credit supply and the credit demand frictions reduce entrepreneurs' incentives to demand loans. In addition, we observe that the interaction of these two frictions produces a stronger reduction of the real interest rate and hence of the nominal interest rate. This is because the small credit demand pushes the return of bank loans down, which in turn further deteriorates banks' profits and hence capacity to demand households' deposits. This pushes the real interest rate down. As a result, the interaction of the credit supply and demand frictions takes us closer to the ZLB.

We find that the unconventional credit policy has a positive impact on capital and credit. More importantly, our model suggests that this policy intervention can take the economy out of the ZLB. Since the policy intervention is funded with lump-sum taxes on households, with this policy the government is moving households' wealth across time and hence in order to smooth consumption households reduce their supply of deposits. This raises the real interest rate and the nominal interest rate as well. As a result, a strong enough policy intervention might take the economy out of the ZLB. In the same line, the unconventional credit policy can reduce the likelihood of reaching the ZLB. Finally, according to the model when the ZLB binds (even after the policy intervention) the effectiveness of the credit policy in increasing capital and hence total credit is reduced.

The remainder of this chapter is partitioned as follows. Section 2 presents the literature review. In section 3 we develop the simple two-period model. Section 4 studies how the credit and the deposit market are interlinked and how both credit frictions interact. In section 5 we study the implications of the unconventional credit policy. In section 6 we study the consequences of the zero lower bound. Finally, section 7 concludes.

2 Literature Review

This paper is related to the literature of demand side credit frictions and supply side credit frictions. In the case of demand side credit frictions this paper is related to Kiyotaki and Moore (1997) and, Bernanke, Gertler and Gilchrist (1999), or henceforth BGG 1999. In particular, we follow BGG 1999. It features an asymmetric information problem with costly state verification between entrepreneurs and financial intermediaries. The authors study the implications of the monetary policy in an economy with financial frictions. In the case of supply side credit frictions, this paper is related to the literature that incorporates financial intermediaries in DGSE models and develops a moral hazard problem between banks and depositors (see, e.g., Gertler and Kiyotaki, 2011; Gertler and Karadi, 2011; Gertler and Kiyotaki, 2015; Gertler, Kiyotaki and Queralto, 2012). The moral hazard problem consists in the fact that bankers can divert a fraction of bank assets and hence depositors might want bankers to put some of their money (as equity) to fund bank assets to the point that the bank charter value is higher than the value of diverting bank assets. This results in an endogenous leverage constraint. Our contribution to this literature is to model both supply and demand credit frictions in order to provide more realism and more importantly in order to assess their interaction and how they might affect an economy that faces a lower limit on the nominal interest rate.

Our paper is also related to the literature on modeling the interaction of both demand and supply credit constraints aimed to analyze the dynamics of credit markets in allocating resources, as in Elenev, Landvoigt, and Van Nieuwerburgh (2017); Justiniano, Primiceri and Tambalotti (2019); Segura and Villacorta (2020) and Pozo and Rojas (2020). Our contribution to this literature is studying the interaction of the supply and demand frictions in a simple two-period framework and its importance in a context of a ZLB constraint.

The credit policy developed in this paper is also related to the previous literature, as in Cúrdia and Woodford (2011); Gertler and Karadi (2011); Gertler and Kiyotaki (2011) and Gertler, Kiyotaki and Queralto (2012). The key differences with this previous literature are the following: (i) we assume for simplicity that central bank intermediation does not involve any efficiency cost; and (ii) the lending interest rate on central bank loans to firms is the risk-free interest rate since we assume that central bank loans are insured by the government. Indeed, in this paper, we call the credit policy "unconventional" because the government guarantees. In addition, our contribution to this literature is to assess the implications of the credit policy in a ZLB environment.

Our work is also related to the literature on the ZLB: Krugman (1998); Eggertsson and Krugman (2012); Eggertsson and Woodford (2003); Eggertsson and Egiev (2019); and Eggertsson and Singh (2019). This literature focus on the troubles generated by the zero lower bound in order to implement conventional monetary policy and the risks of observing a deflationary spiral. We technically depart from this literature, since in the two-period framework that we develop inflation moves above its target when the ZLB binds. However, we claim this does not affect at least qualitatively our main results. Our contribution to the literature involves discussing the role of credit market interventions by a central bank under the ZLB constraint.

This work is also related to the literature that studies the effects of the credit policy under zero lower bound. Schenkelberg and Watzkab (2013) find that a quantitative easing shock leads to a significant increase in output and price level for the post-1995 Japanese data. Similarly, Wu and Xia (2016) find evidence that the unconventional monetary policy implemented by the Fed has succeed in lowering unemployment. In addition, in a panel VAR for eight advanced countries for the 2008 global financial crisis, Gambacorta et al. (2014) find that the increase of central banks' balance sheet at the ZLB temporarily increases economic activity. Gertler and Kiyotaki (2011) in its DSGE model with financial intermediaries and with only credit supply frictions discuss the implications of the ZLB and the effects of a credit policy under a ZLB. They find that the credit policy diminishes the negative effects of the ZLB after a capital quality shock. We complement this literature by focusing on the effects of the ZLB on the effectiveness of the credit policy and by clearly illustrating the mechanisms behind our results using a simple two-period model.

Our work is also part of the current Covid-19 literature on policy interventions through credit markets as in Segura and Villacorta (2020); Céspedes, Chang and Velasco (2020); and Drechsel and Kalemli-Ozcan (2020). Segura and Villacorta (2020) study optimal government support in a lockdown in a framework with firm-bank linkages. Without government intervention, output losses are amplified. In a minimalist framework Céspedes, Chang and Velasco (2020) also models a lockdown shock and amplification effects. They find that unconventional policies are more effective than conventional ones. And Drechsel and Kalemli-Ozcan (2020) recommend direct cash transfers to support small and medium-sized enterprises, implemented via a negative tax. We seek to contribute with an additional dimension to this literature regarding the interaction of monetary policy constraints and credit policy, the former being the new element in the analysis.

3 A two-period model

In this paper we present a stylized model that incorporates together the basic features of the New Keynesian model and demand and supply credit frictions. It is a two-period model where the only factor of production is capital. In this economy, capital allows the economy to transfer goods across periods, and thus current and future conditions are interlinked[3].

For simplicity, we assume no aggregate uncertainty, but only idiosyncratic risk faced by entrepreneurs investing in capital services. In this economy, we have 5 types of agents: households, entrepreneurs, banks, intermediate and final goods firms. Households own banks and all businesses.

The timeline and the role of each agent in the economy is as follows. At time $t = 1$, households are endowed with y_1 units of goods and decide how much to allocate to consumption, c_1, and savings via bank deposits, D_2. In addition, given that households own entrepreneurs' business and banks, they make exogenous transfers in a fixed amount to entrepreneurs, $N_{1,e}$, and to bankers, $N_{1,b}$. This assumption captures the idea that initial equity is needed by entrepreneurs and banks to operate. Bankers, endowed with $N_{1,b}$ goods, demand households' deposits D_2 and lend to entrepreneurs $B_2 = N_{1,b} + D_2$. Hence, banks screen entrepreneur's projects and intermediate funds. At period $t = 2$, banks pay the gross interest rate on deposits funds, R_2, and charge the gross interest on loans, R_2^l.

Entrepreneurs, endowed with $N_{1,e}$ goods, at period $t = 1$ take also a loan from a bank, B_2, and invest it into a risky project to produce capital, $K_2 = N_{1,e} + B_2$, to earn $\omega_2 R_2^k$ per unit of capital at period $t = 2$, where ω_2 is the idiosyncratic shock that has a lognormal distribution, and R_2^k is the gross rate of return on capital. At period $t = 2$, a competitive monopolistic firm buys capital, pays a given price R_2^k, and produces intermediate goods with a decreasing return to scale technology. This intermediate goods firm sets prices under pricing frictions: we assume that a share of firms can not set prices optimally but at a prior exogenous level. Perfect competitive final goods firms demand intermediate goods to produce final goods firms with a Dixit-Stiglitz technology. At the end of period $t = 2$, households consume all earnings from deposits and profits made by bankers, entrepreneurs and intermediate goods and final goods firms.

3.1 Households

Households make deposits, D_2, decide consumption allocation, $\{c_1, c_2\}$, and take dividends and profits as lump-sum transfers. A representative household solves the following problem:

$$\max_{c_1, c_2, D_2} u(c_1) + \beta u(c_2)$$

[3]This feature makes our model different from a standard New Keynesian (NK) model, as in Galí (2015), where labor is the only input for production and transfers of goods across periods is not possible. In the standard NK model, the labor market equilibrium depends on current economic conditions, and the real interest rate clears the goods market, so there are not incentives for households to store goods across periods via savings. These assumptions conveniently simplify the analysis.

subject the budget constraints at period 1 and 2, respectively,

$$\begin{aligned} c_1 + D_2 &= y_1 - N_{1,b} - N_{1,e} \\ c_2 &= R_2 D_2 + \pi_2^T \end{aligned} \tag{3.1.1}$$

where $u(c)$ represents a standard utility function of consumption, with $u'(c) > 0$ and $u''(c) < 0$, β is the discount factor, y_1 is the exogenously given endowment, $-N_{1,b}$, $-N_{1,e}$ are fixed amount of dividends at period 1 and $\pi_2^T = \pi_2^e + \pi_2^b + \pi_2^f$ dividends at period 2 received from entrepreneurs, banks and intermediate firms. The economic justification of having $N_{1,b} + N_{1,e}$ as outflows at period 1 is that households own banks and entrepreneurs' business. Thus, they do not only receive dividends at period 2, but also are responsible for equity (capital) injections $N_{1,b}$ and $N_{1,e}$ to both banks and business, respectively. $R_2 = (1 + i_1)/(P_2/P_1)$ is the gross real interest rate, where i is the nominal interest rate and P_1 and P_2 are nominal prices.

The optimal consumption allocation across the two periods is given by the first order condition with respect to deposits, D_2, or the Euler equation,

$$u'(c_1) = \beta R_2 u'(c_2), \tag{3.1.2}$$

which establishes an equilibrium condition between the intertemporal marginal rate of substitution in consumption and the real interest rate. The real interest rate affects the allocation of consumption across periods, as it affects the relative valuation of consumption between periods. Other things equal, a rise in the interest rate stimulates a household to save more via bank deposits by discouraging consumption at period 1, i.e., incentivizing consumption at period 2.

We can further gain more insights by assuming an isoelastic utility function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where σ is the inverse of the intertemporal elasticity of substitution. As a result, equation (3.1.2) becomes $c_2 = c_1 \left(\beta R_2\right)^{1/\sigma}$, which after some algebra by substituting into (3.1.1) yields the households' supply curve of deposits,

$$D_2 = \frac{(y_1 - N_{1,b} - N_{1,e}) \left(\beta R_2\right)^{1/\sigma} - \pi_2^T}{\left(\beta R_2\right)^{1/\sigma} + R_2}, \tag{3.1.3}$$

which is a different way to write the Euler equation. The equation of supply of deposits (3.1.3) shows that due to intertemporal smoothness, other things being equal, a higher level of endowment at period 1, y_1, increases supply of deposits, D_2, but higher profits at period 2, π_2^T, reduce supply deposits, D_2.

3.2 Banks: Demand of deposits and credit supply frictions

Banks capture deposits from households, D_2, that together with their initial exogenous equity (cash) $N_{1,b}$, are used the fund the loans issued to firms, B_2, i.e., bank balance sheet is,

$$B_2 = D_2 + N_{1,b}. \tag{3.2.4}$$

The process of demand of deposits is not frictionless, and there is a moral hazard problem between banks and depositors. It creates a credit supply friction that prevents a free flow from deposits to loans. Since banks are identical in what follows we discuss the problem of the representative bank.

A bank receives a gross return R_2^l per unit of loans and pays R_2 per unit of deposits. We assume that there is no aggregate uncertainty and banks can perfectly diversify their loans across entrepreneurs, that face idiosyncratic risk, and as a result the lending rate R_2^l is agreed and known at $t = 1$.

We introduce a moral hazard problem, as in Gertler and Kiyotaki (2011), to motivate a limit on banks' ability to expand their assets indefinitely by borrowing additional funds from households. At period 2, a banker can choose to intermediate loans or to divert some fraction λ of available funds and transfer them back to the household of which she is a member. The cost to a banker that diverts is that the depositors can force the bank into bankruptcy and recover only the remaining fraction $1 - \lambda$ of all available funds. As a result, to ensure the existence of bank loans, the following incentive constraint (IC) must be satisfied,

$$V_1 \geq \lambda B_2 R_2^l. \tag{3.2.5}$$

where V_1 is the value of future bank profits,

$$V_1 = R_2^l B_2 - R_2 D_2. \tag{3.2.6}$$

Equation (3.2.5) says that the charter value of the bank, the benefits of continuing operating, should be greater than the benefits of diverting bank assets. Hence, banks optimally choose the size of deposits D_2 in order to maximize (3.2.6) subject to the incentive constraint (3.2.5), where $B_2 = D_2 + N_{1,b}$. Notice that the only difference with the frictionless case is the presence of this incentive constraint. The first order condition with respect to D_2 leads to:

$$R_2^l - R_2 = \frac{\nu\lambda}{(1+\nu)}, \tag{3.2.7}$$

where $\nu \geq 0$ is the Lagrange multiplier associate with the incentive constraint. We calibrate our model so that (3.2.5) binds, so $\nu > 0$ always[4].

According to equation (3.2.7), a positive spread arises between the funding costs of banks and the lending interest rate, which is zero without credit supply frictions, $R_2^l = R_2$. This positive spread captures the idea that banks need to generate enough profits, so V_1 is high enough that banks do not divert and prefer intermediate loans. This positive spread that bankers earn can be called, *credit spread* or *credit risk premium*.

From the binding incentive constraint (3.2.5) we obtain the demand curve of deposits

[4]See Appendix A for a proof.

D_2,

$$D_2 = N_{b,1}\left[\frac{(1-\lambda)R_2^l}{R_2-(1-\lambda)R_2^l}\right]. \tag{3.2.8}$$

Equation (3.2.8) not only represents the demand curve of deposits but also determines the supply curve of loans, since $B_2 = D_2 + N_{1,b}$ and bank's equity, $N_{1,b}$ is exogenous. In fact, it can be expressed simply as:

$$B_2 = N_{b,1}\frac{R_2}{R_2-(1-\lambda)R_2^l}. \tag{3.2.9}$$

Due to the moral hazard problem (financial friction) bank's capacity to fund supply of loans with deposits is constrained to a proportion of its equity level. This means that the higher the bank net worth, the higher the bank capacity to demand deposits and issue loans. When banks put more of their skin, they have less incentives to perform this costly divertion, which in turn allows banks to capture more deposits and hence to issue more loans. Equation (3.2.9) also shows, other things equal, that the higher the lending rate R_2^l the higher bank loans supply, as a bank's incentives are lower.

Thus, equations (3.2.8) and (3.2.9) show how closely related bank's capacity to demand deposits and hence to supply credit are. In particular, ceteris paribus, the higher the deposit rate R_2, the lower bank's future profits and hence the higher the bank's incentives to divert which in turn leads to a tighter incentive constraint and less capacity to capture deposits. This describes the negative slope of the deposit demand curve by banks. Similarly, ceteris paribus the higher the lending rate R_2^l, the higher bank future profits and hence the lower the bank incentives to divert which in turn leads to a looser incentive constraint and more capacity to capture deposits and hence to issue loans. This describes the positive slope of the credit supply curve by banks.

Shifts on the deposits demand and credit supply curves: According to equation (3.2.8) the higher the bank ability to divert, i.e., the higher the λ, the lower the bank capacity to supply loans per unit of equity. The intuition is that a higher λ increases the ability of a bank to divert, so the depositors will require banks to holds more equity per unit of loans in order to diminish their incentives to divert via the IC; i.e., for a given level of bank equity, a higher λ decreases deposits demand and credit supply. Notices also that, ceteris paribus, a higher bank lending rate of funding R_2^l increases deposits demand and a higher banks' cost of funding R_2 decreases credit supply. The intuition is that a higher R_2^l (higher R_2) increases (decreases) bank profits, which in turn looser (tighten) the IC, and increases deposits demand (decreases credit supply). As a result, there is a movement to the right (left) of the deposits demand curve (credit supply curve).

3.3 Entrepreneurs: Lending diversification and credit demand frictions

In order to study credit demand frictions and demand for capital, we assume lending is also frictionless. We adopt the same modeling device as in Bernanke, Gertler and Gilchrist (1999): a Costly State Verification (CSV) problem. An entrepreneur has asymmetric information of the state of her firm and banks need to pay a monitoring cost to observe the entrepreneur's realized return.

At the end of period $t = 1$, entrepreneurs start a firm, indexed by $j \in [0,\ 1]$, that transforms, under a linear technology, funding, composed by an initial equity or net worth, N_2^j, and bank's loans, B_2^j, into capital, K_2^j. i.e entrepreneur j balance sheet is,

$$K_2 = B_2 + N_{1,e} \tag{3.3.10}$$

where we forget index j as entrepreneurs are ex-ante identical[5], so we solve the problem as of a representative entrepreneur.

Entrepreneurs are risk-neutral and ex-ante identical and face an idiosyncratic shock. Specifically, the ex-post gross return of each unit of capital is $\omega_2 R_2^k$, where ω_2 is a random idiosyncratic disturbance to entrepreneur j and R_2^k is the aggregate return of capital. We set that the random variable ω_2 is i.i.d. across entrepreneurs follows a c.d.f $F(\omega)$ equal to the lognormal distribution with $\mathbb{E}_1\{\omega_2\} = 1$.

The asymmetric information problem consists that banks cannot observe ω_2, the idiosyncratic shock faced by each entrepreneur. However, banks can pay a monitoring cost to observe the realized gross value of entrepreneurs' payoffs. The monitoring cost is equal to a fixed proportion $\mu > 0$ of entrepreneurs realized return: $\mu\omega_2 R_2^k K_2$.

Entrepreneur chooses K_2, and hence the level of borrowing B_2 prior to the realization of ω_2 taken as given the aggregate return of capital R_2^k. The optimal lending contract offered by a bank is given by the gross non-default bank loan rate Z_2 and a threshold value of the idiosyncratic shock, ω_2, defined as,

$$\bar{\omega}_2 R_2^k K_2 = Z_2 B_2. \tag{3.3.11}$$

For values of the idiosyncratic shock higher than the threshold value, $\omega_2 \geq \bar{\omega}_2$, an entrepreneur fully repays bank loan, otherwise, it defaults. Banks are repaid in full if entrepreneurs do not default, so banks do not have any incentive to pay a monitoring cost to verify the entrepreneurs' performance. However, when an entrepreneur defaults, banks do have incentives to pay the monitoring cost to observe the realized payoffs. They pay the auditing cost, seize the entrepreneur's project and obtain $(1 - \mu)\omega_2 R_2^k K_2$. i.e a defaulting entrepreneur receives nothing. Notice that $F(\bar{\omega}_2)$ is the probability of default. i.e It is the probability that entrepreneur j defaults at $t = 2$ (or the fraction of entrepreneurs that defaults at $t = 2$). Then, a higher default probability of entrepreneurs raises the agency cost of monitoring projects, but it also increases the repayment value, $Z_2 B_2$.

On one hand, the lending contract must reflect the opportunity cost of funding loans. Recall that the required return of bank loans, compatible with a no diverting deposits equilibrium, is R_2^l. Banks can perfectly diversify the idiosyncratic risk involved in lending across entrepreneurs, and as a result an optimal lending contract requires banks receiving a certain gross return of R_2^l per unit of bank loans. In other words, banks hold a perfectly safe portfolio (it perfectly diversifies the idiosyncratic risk involved in lending) that ensures

[5] Also, as we will see later, given that entrepreneurs use a constant return technology, even ex-post there is a factor of proportionality between the demand for capital and net worth that is independent of entrepreneurs' specific factors. Thus, aggregation is easier. Bernanke, Gertler and Gilchrist (1999) provide a more detailed discussion.

a certain return R_2^l for their loans. Hence, the bank's loan contract ($\bar{\omega}_2$, Z_2) must satisfy:

$$[1-F(\bar{\omega}_2)]Z_2B_2 + (1-\mu)\int_0^{\bar{\omega}_2} \omega R_2^k K_2 dF(\omega) = R_2^l B_2. \tag{3.3.12}$$

The left-hand side of equation (3.3.12) is the expected return on the loan to the entrepreneurs and the right-hand side is the opportunity cost of lending. By definition, in equilibrium since a positive number of entrepreneurs default (i.e., since $\bar{\omega}_2 > 0$), the (non-default) bank lending rate, Z_2, is higher than R_2^l. Intuitively, the positive spread charged to the required return of bank loans aims to compensate bank revenues for that fraction of entrepreneurs that are not able to fully repay bank loans. As a result, the spread Z_2-R_2^l represents the idiosyncratic risk premium.

On the other hand, the lending contract offered by a bank must maximize the expected profits to the entrepreneur, which may be expressed as:

$$\int_{\bar{\omega}_2}^{\infty} (\omega R_2^k K_2 - Z_2 B_2) dF(\omega). \tag{3.3.13}$$

Entrepreneurs aim to maximize (3.3.13) optimally choosing K_2 and $\bar{\omega}_2$ subject to the constraint implied by the bank loan contract, equation (3.3.12), the repayment value, in equation (3.3.11) and where B_2 is solved in bank balance sheet equation (3.3.10) taking as given R_2^k, R_2 and R_2^l, which are endogenously determined in the general equilibrium. Formally, the solution the this problem yields the credit demand curve of the representative entrepreneurs[6], given by

$$\left[[1-\Gamma(\bar{\omega}_2)] \frac{1-F(\bar{\omega}_2)-\mu\bar{\omega}_2 f(\bar{\omega}_2)}{1-F(\bar{\omega}_2)} + (\Gamma(\bar{\omega}_2)-\mu G(\bar{\omega}_2)) \right] R_2^k - R_2^l = 0\,, \tag{3.3.14}$$

and equation (3.3.12), that determines the portfolio allocation from banks,

$$B_2 = N_2 \left[\frac{\left(\Gamma(\bar{\omega}_2)-\mu G(\bar{\omega}_2)\right)\frac{R_2^k}{R_2^l}}{1-\left(\Gamma(\bar{\omega}_2)-\mu G(\bar{\omega}_2)\right)\frac{R_2^k}{R_2^l}} \right] \tag{3.3.15}$$

where, $\Gamma(\bar{\omega}_2) = \int_0^{\bar{\omega}_2} \omega dF(\omega) + (1-F(\bar{\omega}_2))\bar{\omega}_2$ is the gross share of entrepreneurs' profits going to pay banks and $\mu G(\bar{\omega}_2) = \int_0^{\bar{\omega}_2} \omega dF(\omega)$ are expected monitoring costs.

By construction $F(\bar{\omega}_2)$ is positive[7], and therefore the term that multiplies R_2^k in equation (3.3.14) is lower than one, which implies a (positive) spread between the marginal productivity of capital and the lending rate (bank loans return), i.e.,

$$R_2^k - R_2^l > 0.$$

This premium is also called external finance premium. The asymmetric information prob-

[6]See Appendix B

[7]We assume $ln(\omega) \sim \mathcal{N}\left(-0.5\sigma_\omega^2, \sigma_\omega^2\right)$ so we have $\mathbb{E}(\omega) = 1$ and then $\Gamma(\bar{\omega}) = \Phi(z-\sigma_\omega) + \bar{\omega}[1-\Phi(z)]$, $G(\bar{\omega}) = \Phi(z-\sigma_\omega)$, $\partial\Gamma(\bar{\omega})/\partial\bar{\omega} = 1-\Phi(z)$ and $\partial G(\bar{\omega})/\partial\bar{\omega} = \bar{\omega}\Phi'(z)$, where $\Phi(.)$ and $\Phi'(.)$ are the c.d.f. and the p.d.f., respectively, of the standard normal and z is related to $\bar{\omega}$ through $z = (ln(\bar{\omega}) + 0.5\sigma_\omega^2)/\sigma_\omega$.

lem distorts entrepreneur's incentives to demand capital. Put it differently, the costly information problem between entrepreneurs and banks reduces net marginal benefits of capital, which produces a shift to the left of the credit demand curve (i.e., smaller demand of credit). If we assume that there is not any asymmetric information problem, then $\mu = 0$, and hence equilibrium condition becomes, $R_2^k - R_2^l = 0$, which is the typical equilibrium condition, where the expected marginal productivity of capital equates the expected marginal cost of capital.

In this environment, net worth position is a key determinant for the cost of external finance. From equation (3.3.15), a higher equity amplifies exogenous changes in the economy. First, there is a direct effect of higher net worth on lending. Secondly, From equation (3.3.11), ceteris paribus, the higher the entrepreneur equity, the lower the likelihood that it defaults (i.e., the lower $\bar{\omega}_2$) and hence the smaller the distortions (i.e., the smaller $R_2^k - R_2^l$) and increases demand for lending. i.e a higher net worth mitigates the agency problems and reduces external finance premium faced by entrepreneurs in equilibrium.

As a result, this new curve of demand for lending (and hence supply of capital) not only shifts to the left but is also steeper than the frictionless curve. This is because a higher credit (and hence supply of capital) increases entrepreneurs' default probability which in turn increases monitoring costs and reduces the effective return of capital and hence entrepreneurs are willing to borrow at a lower lending interest rate, R_2^l. As a result, we should observe a stronger reduction of the lending rate after a higher demand of credit.

To gain more intuition of how the frictions affect the decision process of entrepreneurs, we insert equation (3.3.12) into equation (3.3.13),

$$[1 - \mu G(\bar{\omega}_2)] \, R_2^k K_2 - R_2^l B_2, \tag{3.3.16}$$

where $\mu G(\bar{\omega}_2) R_2^k K_2$ represents the cost of entrepreneur defaulting. As a result, if the monitoring cost is $\mu = 0$ or equivalently if the asymmetric information is overcome costlessly, we are back to a model without frictions on the credit demand. One can see that the interaction of entrepreneur default probability (captured by $\bar{\omega}_2$) and the monitoring cost (μ) leads to a reduction of the net marginal benefit of demanding a unit of bank loans from entrepreneur perspective. This is, ceteris paribus a higher default probability or a higher μ reduces demand of credit. And hence in that sense this equation shows how the distortions in the market affect entrepreneur decisions on their demand of credit (B_2) or equivalently their purchases of capital (K_2). In other words, the asymmetric information problem reduces entrepreneur capacity to demand loans and hence to invest. As stated before, a higher entrepreneur's equity reduces the likelihood of reduction of the net marginal effects of issue loans from bankers' perspective.

Notice that from the credit demand equations (3.3.14) and (3.3.15), the risk premium and the amount of borrowing relative to equity of the entrepreneur does not depend on idiosyncratic characteristics of financial position of the entrepreneur. These features of the model makes aggregation straightforward. And in fact, the aggregate credit demand curve is identical to the credit demand curve of the representative entrepreneur, (3.3.14), where the aggregate loan contract (which is also the aggregate bank's balance sheet .) is thus given by equation (3.3.15).

3.4 Sticky prices: Final goods firms and intermediate goods firms

Next, we add sticky prices to the model as is tradition in a standard NK models[8]. Final goods are produced competitively by final firms that transform substitute intermediate domestic goods, $Y_{i,2}$, into a homogeneous good, Y_2, using the following constant elasticity of substitution (CES) production function,

$$Y_2 = \left[\int_0^1 Y_{i,2}^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}} ,$$

where θ is the elasticity of substitution between different intermediate inputs, with $\theta > 1$ to ensure input substitutability. As the final good is produced competitively, the demand schedule for a domestic intermediate i is:

$$Y_{i,2} = \left(\frac{P_{i,2}}{P_2} \right)^{-\theta} Y_2, \tag{3.4.17}$$

and the aggregate price index, P_2, is

$$P_2 = \left[\int_0^1 \; P_{i,2}^{1-\theta} di \right]^{\frac{1}{1-\theta}} , \tag{3.4.18}$$

where $P_{i,2}$ is the price of an domestic intermediate good i.

Firms: Intermediate good producers

Differentiated intermediate goods are produced by monopolistic competitive firms indexed by $i \in [0, 1]$. These firms set price, $P_{i,2}$, and produce $Y_{i,2}$ using a decreasing return to scale technology:

$$Y_{i,2} = a \left(K_{i,2} \right)^{\alpha} , \quad \text{with } \alpha < 1, \tag{3.4.19}$$

where $K_{i,2}$ is capital and a is technical innovation constant or a productivity level. The problem of a firm i is to choose $\{K_{i,2} \, , P_{i,2}\}$[9] such as to maximize profits, subject to a demand function, its production technology and the total cost function,

$$\max_{\{K_{i,2} \, , P_{i,2}\}} \left[\left(\frac{P_{i,2}}{P_2} \right) Y_{i,2} - \mathcal{C}(Y_{i,2}) \right] , \tag{3.4.20}$$

subject to demand curve, (3.4.17), and the production function, (3.4.19), and where $\mathcal{C}(Y_{i,2}) = R_2^k K_{i,2}$ is the total cost function.

To model price rigidities we assume a fraction γ of firms have sticky prices and their prices are set to a predetermined value equal to the aggregate price in period $t = 1$, i.e

[8] See Appendix C for a more detailed derivation of this section.

[9] In this problem choosing $K_{i,2}$ is identical as choosing $Y_{i,2}$, as there is a one-to-one relationship between these two given the production function. In fact, as derived in the Appendix C, this problem can be transformed in such a way that there is just one decision variable, $Y_{i,2}$.

$P_{i,2} = P_1$. A fraction $1-\gamma$ of firms can update prices. In other words, a fraction γ does not internalize the decisions of $Y_{i,2}$ on prices $P_{i,2}$, the inverse demand curve, while a fraction $1-\gamma$ does.

A firm i that has the opportunity to change prices, solves the profits maximization problem, (3.4.20). The solution of this problem yields the optimal pricing, in which we undo price distortions[10]:

$$\frac{P_{i,2}}{P_2} = \mathfrak{c}_{i,2} \, ,$$

where $\mathfrak{c}_{i,2} = \frac{1}{\alpha a^{1/\alpha}} R_2^k \left(Y_{i,2}\right)^{\frac{1-\alpha}{\alpha}}$ is the marginal cost, which with $\alpha < 1$ is itself endogenous. In fact after inserting back (3.4.17) and solving for $\frac{P_{i,2}}{P_2}$ we get that firms optimally choose the same price $\frac{P^o_{i,2}}{P_2}$:

$$\frac{P_{i,2}}{P_2} = \frac{P_2^o}{P_2} = \left(\frac{1}{\alpha a^{1/\alpha}} R_2^k \left(Y_2\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{\alpha}{\alpha+\theta(1-\alpha)}} \, , \tag{3.4.21}$$

Notice that optimal pricing equation (3.4.21) imposes a relationship between prices and real variables in the economy in equilibrium. From this equation its clear that increases in the real return of capital impact positively the marginal cost and with it the real prices that firms set.

In equilibrium, equation (3.4.21) and the consistent aggregate price index, P_2 (3.4.18),

$$P_2 = \left[(1-\gamma)\left(P_2^o\right)^{1-\theta} + \gamma \left(P_1\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \, , \tag{3.4.22}$$

determine the aggregate supply curve of economy or the Phillips curve [11]. In the aggregate price index we set $P_{i,2} = P_2^o$ for all firms of measure $1-\gamma$ that can adjust their prices and $P_{i,2} = P_1$ for all firms that can not set prices[12].

3.5 Market Clearing

In equilibrium, market clearing in the capital market requires:

$$K_2 = \int_0^1 K_{i,2} di = \left(\frac{Y_2}{a}\right)^{1/\alpha} \Delta$$

[10]Note that with capital subsidy to the firm,

$$\frac{P_{i,2}}{P_2} = (1-\tau)\mathcal{M}\mathfrak{c}_{i,2}$$

. As shown in Gali (2012) one can eliminate the markup distortion on prices by considering a capital subsidy for the firm: $\tau = \frac{1}{\theta}$. Where $\mathcal{M} = \frac{\theta}{\theta-1}$ denotes the constant markup of the monopolistic firm. See more details in the Appendix C.

[11]See Appendix D for log linear representation of the Phillips curve.

[12]For simplicity, we assume that P_1/P_2 is high enough so that the equilibrium is governed by the demand curve, i.e., at P_1/P_2 there is a excess of supply of intermediate goods.

Thus, solving the GDP at $t = 2$ we find:

$$Y_2 = \Delta^{-1} a K_2^{\alpha},$$

where, $\Delta = \int_0^1 \left(\frac{P_{i,2}}{P_2}\right)^{-\theta/\alpha} di$ is the price dispersion. The equilibria in the deposit market and credit market requires

$$B_2 = D_2 + N_{1,b},$$
$$K_2 = B_2 + N_{1,e}.$$

The market clearings in final goods market are,

$$c_1 = y_1 - K_2 \tag{3.5.23}$$
$$c_2 = Y_2 - \mu G(\bar{\omega}_2) R_2^k K_2 = \Delta^{-1} a K_2^{\alpha} - \mu G(\bar{\omega}_2) R_2^k K_2, \tag{3.5.24}$$

In equilibrium, the aggregate demand curve of the economy is given by Euler equation, (3.1.2) which is also the supply curve of deposits in the economy,

$$R_2 = \frac{1}{\beta} \left(\frac{\Delta^{-1} a (D_2 + N_{1,b} + N_{1,e})^{\alpha} - \mu G(\bar{\omega}_2) R_2^k (D_2 + N_{1,b} + N_{1,e})}{y_1 - (D_2 + N_{1,b} + N_{1,e})} \right)^{\sigma} \tag{3.5.25}$$

where GDP, Y_2, reflects price dispersion. This supply curve is the aggregate version of equation (3.1.3), and it shows a positive relationship between R_2 and D_2.

Appendix E shows that the demand curve for capital, that relates the return on capital with the marginal productivity of capital, and which represent the supply side of the economy is given by

$$R_2^k = \mathcal{W}\left(\frac{P_1}{P_2}\right) \alpha a K_2^{\alpha - 1}, \tag{3.5.26}$$

where $\mathcal{W}\left(\frac{P_1}{P_2}\right) = \left(\frac{1-\gamma\left(\frac{P_1}{P_2}\right)^{1-\theta}}{(1-\gamma)}\right)^{\frac{\alpha+\theta(1-\alpha)}{(1-\theta)\alpha}} \Delta^{\frac{1-\alpha}{\alpha}}$ is a wedge between the return on capital and marginal productivity of capital. This wedge only reflect nominal rigidities, and it is a function of gross inflation or the ratio P_2/P_1. If $\gamma = 0$ (i.e., flexible prices), $\mathcal{W} = 1$.

3.6 Conventional Monetary policy: Taylor rule

We assume that the central bank sets the nominal interest rate i_1 via a Taylor rule, where the short-term nominal rate is related to inflation:

$$i_1 = max(i_{min}, R_2^* (1 + \pi_2)^{\phi_\pi} - 1), \tag{3.6.27}$$

where i_{min} is the lower bound for the nominal interest rate, R_2^* is the natural real interest rate or the real interest rate under flexible prices, and $\pi_2 = (P_2/P_1 - 1)$, with the Fisher equation being :

$$R_2 = \frac{1 + i_1}{1 + \pi_2}. \tag{3.6.28}$$

Furthermore, in this simple model we assume that the central bank follows an optimal monetary policy rule or equivalently follows an Inflation Targeting at a targeted inflation of zero, i.e., $\pi_2 = 0$[13]. Therefore, if there is not an ZLB equilibrium, inflation is zero and the nominal interest rate is set as follows:

$$i_1 = R_2^* - 1,$$

and the central bank is able to replicate the flexible price equilibrium. In particular, with $\pi_2 = 0$, via the Fisher equation:

$$R_2 = 1 + i_1 = R_2^*.$$

However, if the economy is at a ZLB equilibrium where $R_2^* (1 + \pi_2)^{\phi_\pi} - 1 < i_{min}$, the central bank cannot implement a flexible price equilibrium, and $i_1 = i_{min} = 0$, $R_2 \neq R_2^*$, $\pi_2 \neq 0$.

3.7 Equilibrium

In this economy the equilibrium is the set of 9 endogenous variables that solve the system of 9 equations for a given set of as a function of the exogenous variables y_1, a, $N_{1,e}$, $N_{1,b}$. In particular, $\{D_2\,, R_2\}$ is the solution to the deposit market equilibrium, (3.5.25), (3.2.8), $\{B_2\,, R_2^l, \bar{\omega}_2\}$ is the solution to the credit market equilibrium, (3.2.4), (3.3.14), (3.3.15); $\{K_2\,, R_2^k\}$ the capital market equilibrium, (3.5.26), (3.3.10); and $\{i_1\,, \pi_2 = P_2/P_1 - 1\}$ satisfy the MP rule,(3.6.27), and the Fisher identify, (3.6.28).

3.8 Calibration used in numerical examples

In all our next numerical examples we turn off the effect of the price dispersion variable, i.e., $\Delta = 1$. Price dispersion has second order effects, and for simplicity, we ignore them to abstract in our analysis from second order effects of price distortions.

For illustrative purposes, we set $\beta = 0.99$, $\sigma = 2$, $a = 5$, $\alpha = 0.33$, $\theta = 4.1$, $\phi_\pi = 1.25$, $\gamma = 0.7$. In addition, we set y_1, σ_ω, λ, μ, $N_{1,b}$ and $N_{1,e}$ so that without price rigidities we have a bank leverage ratio ($B_2/N_{1,b}$) of 4, an entrepreneur leverage ratio ($K_2/N_{1,e}$) of 4, an annualized entrepreneur default probability of 15%, an annualized spread $R_2^k - R_2^l$ of 5%, an annualized spread $R_2^l - R_2$ of 5% and an annualized net real interest rate of 5%. Clearly, in the baseline the ZLB constraint does not bind. Hence, in section 6 when assessing the implications of the ZLB, we recalibrate the model so that the ZLB binds.

[13]See Appendix F

4 Analysis of Deposit and Credit Market

In this section we study the how credit and deposit markets are interlinked and the interaction of the credit demand and credit supply frictions. In equilibrium Figure 2 depicts 4 deposit and credit markets equilibria: an equilibrium in an economy without credit frictions, the equilibrium with credit demand and credit supply frictions, an equilibrium with only credit demand frictions and equilibrium with only credit supply frictions.[14]

As it is shown by point A in figure (2), without credit frictions the equilibrium in the deposit market is given by the intersection of the Euler equation (supply curve) and a perfectly elastic demand curve, i.e a horizontal line at $R_2 = R_2^l$ (demand curve). And the equilibrium in the credit market is given by the intersection of the capital marginal productivity (demand curve), R_2^k, and the horizontal line at $R_2^l = R_2$ (a perfectly elastic supply curve). As a result, in equilibrium $R_2 = R_2^l = R_2^k$, and there are not risk premia.

For convenience, figure (2) also shows the equilibrium when there is either only credit supply frictions, point B1, or only credit demand frictions, point B2. Relative to the frictionless benchmark, both types of frictions reduce the amount of credit in equilibrium but due to different reasons. In point B1, credit supply frictions prevent the flow of deposits to loans. Bank's demand for deposits is constrained by their net worth. So, relative to the frictionless benchmark credit supply frictions reduce banks' deposit demand, and as a consequence there is a decrease in the deposit interest rate in equilibrium. Given that lower deposits intermediation limits lending, the new credit market equilibrium is one with lower credit and higher lending rate.

In the second case, in point B2, the credit demand friction lowers the marginal value of capital and distorts the credit demand. Since entrepreneurs are constrained by its net worth, the demand for credit falls. Relative to the frictionless benchmark, the new credit market equilibrium is one with lower credit and lower lending rates. Consequently, it drives a reduction in the demand for deposits and a fall in the deposit interest rate. Also notice the credit demand frictions create losses (due to the monitoring cost) which in turn reduces future households' consumption increasing households' incentive to supply deposits in order to smooth consumption, this is capture by the shift to the left of the credit supply curve. However, this positive effect on deposits does no dominate.

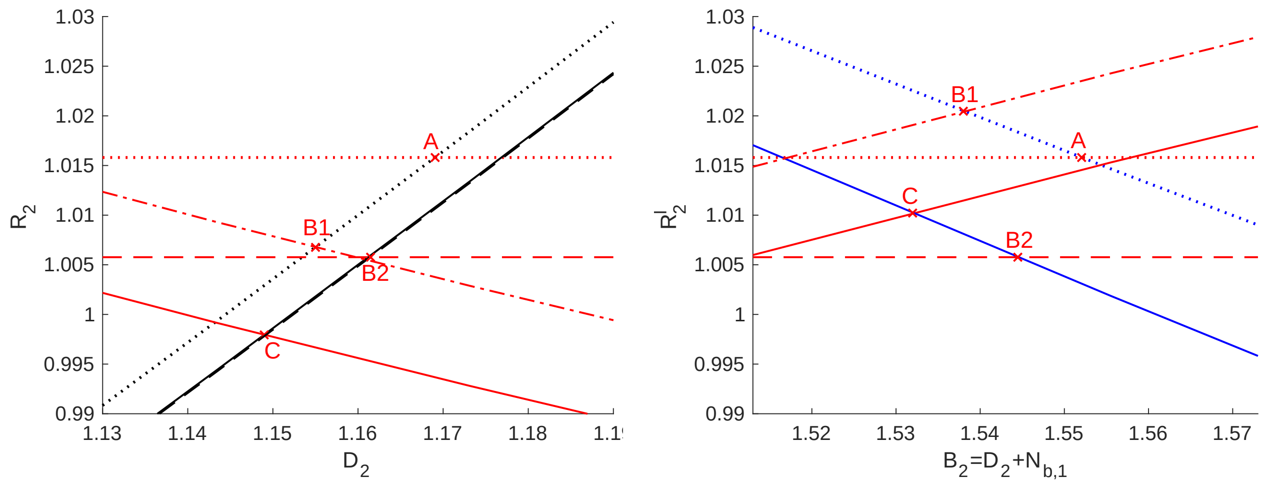
Point C in figure (2) shows the equilibrium when both credit frictions are in place. Supply and demand frictions amplify each other and the credit reduction in equilibrium is stronger. As the flow of deposits to loans is now distorted by a restriction on the deposit demand (an hence loan supply) and lower credit demand, lending and deposit intermediation is much lower. One clear result is that deposit interest rates fall to clear the deposit market. This is, we observe a stronger reduction of the deposit interest rate when both frictions are in place. Starting from an economy with credit supply frictions, when adding credit demand frictions, there is a smaller demand of credit which in turn reduces the lending interest rates and reduces banks' capacity to demand deposits, shifting to the left the deposit demand curve and reducing even more the deposit interest rate. Therefore, the presence of credit frictions takes the economy closer to the ZLB equilibrium.

[14] In any scenario for comparison reasons we set R_2^* as R_2 as in the baseline calibration, so in equilibrium $P_2/P_1 = 1$, i.e., the central bank MP can replicate the flexible price equilibrium.

However, the impact on the lending rate will depend on the relative forces of the credit supply and demand frictions. In the particular case of point C in figure (2) the demand side credit friction is relatively much stronger than the supply side credit friction and hence it is observed a lower lending rate in equilibrium.

Figure 2. *Deposit and Credit Market Equilibrium*

Panel (a): Deposit Market | Panel (b): Credit Market



Note: A: No credit frictions. B1: Only credit supply frictions (moral hazard problem between banks and depositors). B2: Only credit demand frictions (asymmetric information and CSV). C: Both credit frictions.

5 Unconventional Credit Policy

In this section we study the effects of a tool that central bank use during periods of crisis to prevent the amplifying effects of credit frictions during a period of crisis: a unconventional credit policy characterized by central bank (CB) lending facilities to entrepreneurs guaranteed by the government. We follow similar modeling strategy as in Pozo and Rojas (2020).[15] We assume that CB credit facilities are given to entrepreneurs indirectly through banks, which is closer to the empirical evidence. Indirectly we are assuming the CB loans are not subject to the credit supply frictions[16], as the central bank is not constrained by the size of its net worth. Given that CB lending is guaranteed by the government there is no incentive for the central bank to run[17]. This is, CB injects liquidity to banks charging

[15] For simplicity, we assume that the central bank obtains the funds from households through lump-sum taxes at $t = 1$.

[16] We are conscious that this assumption implies that the central bank can act better as a lender than a traditional private bank, or that the central bank can replace banks in lending intermediation. However, we model the unconventional credit policy as one that is only active during periods of crisis/boom. Also, the CB loans are not better than traditional loans when dealing with credit demand frictions. i.e, CB loans can also be defaulted.

[17] We assume that the government pays the monitoring costs of observing the entrepreneur's realized revenues that go to repay CB loans. Since CB loans are guaranteed, banks do not have any incentives to pay the monitoring costs associated to observe realized revenues that go to pay CB loans. Similarly, the central bank does not have any incentive to do so due to the government guarantee. Hence, we believe

some interest rate, and then banks use these funds to issue loans.[18] We refer to these as (indirect) CB loans, B_2^g.

We assume central bank is willing to provide a fraction $\psi_{CB,2}$ of the total external funding (traditional loans + indirect CB loans) of entrepreneurs through banks, i.e.,

$$B_2^g = \psi_{CB,t}(K_2 - N_{1,e}). \tag{5.1}$$

Notice that since entrepreneurs are ex-ante identical, (5.1) holds at the individual entrepreneur level. We assume that entrepreneur does not internalize the effects of her capital and credit decisions on the CB loans injections. Hence, from entrepreneur's perspective the marginal cost of external funding is still given by the required return of bank loans[19].

5.1 CB credit policy and supply side frictions

We assume that the central bank is willing to give funding (or inject liquidity) to banks at the risk-free gross interest rate R_2 with the commitment that (1) banks give at least the same amount of loans (CB loans) to entrepreneurs[20] and (2) charge some agreed (non-default) lending interest rate Z_2^g to entrepreneurs for these indirect CB loans. This is given in three steps. Step 1: CB offers the funds in an auction. Step 2: banks demand these funds and propose Z_2^g. Step 3: CB gives the funding to those banks that offer the lowest Z_2^g in order to benefit the most to entrepreneurs. Since all banks are identical and compete perfectly with other banks, at the end of the day they all offer the same and the smallest feasible lending rate, Z_2^g. We assume that CB can costlessly enforce banks to perform (1) and (2).

First, since these indirect CB loans are guaranteed by the government, if an entrepreneur is not able to fully pay back Z_2^g per unit of CB loans, government transfers enough resources so it ensures the bank receives the agreed return, R_2^l.[21]. Hence, in equilibrium the required return for indirect CB loans from bank perspective, $R_2^{L,g}$, is the same as the (non-default) lending interest rate.

$$Z_2^g - R_2^{L,g} = 0 \tag{5.1.2}$$

In other words, in contrast to the traditional bank loans, banks do not need to add any entrepreneur default risk premium to the (no-default) lending interest rate, Z_2^g [22].

it is a reasonable assumption to say that since the government takes care of her budget, she is the more interested in recover as much as it can from entrepreneur revenues and hence pays the monitoring costs.

[18] Pozo and Rojas (2020) study this policy on a dynamic setup and study both cases, when the CB lending credit is given to firms indirectly through banks and directly. They thoroughly explain under what circumstances both forms are equivalent.

[19] This seems a reasonable assumption in a context of unconventional credit policy, in the sense that entrepreneur cannot predict if central bank will provide lending facilities.

[20] i.e CB funding to banks and CB loans to firms intermediated by banks are both equal to B^g.

[21] We assume government funds their activities with lump-sum taxes to households at $t = 2$.

[22] Notice that we implicitly assume that there is government credibility. In other words, banks ex-ante believe that government will honour the guarantee for the CB loans. And we also assume, for simplicity, that ex-post the government always honor the guarantee.

Second, we assume that there is not a moral hazard problem between banks and CB. In other words, we assume that bankers cannot divert bank assets that are funded by the CB liquidity (indirect CB loans)[23]. As a result, banks do not need to put more equity due to the CB loans or equivalently banks do not need to reduce their traditional bank loans in order to issue indirect CB loans. In addition, we assume that banks do not incur in any administrative cost (or these are negligible) for collecting CB funding and giving these to entrepreneurs as CB loans. Hence, the cost for banks of issuing CB loans is just the interest rate claimed by the central bank, i.e,

$$Z_2^g - R_2 = 0. \tag{5.1.3}$$

In other words, in contrast to the traditional bank loans, there is not a risk premium due to a moral hazard problem between CB and bankers and hence banks do not need to add any spread to the required return for CB loans, $R_2^{L,g} = R_2$.

Finally, jointly equations (5.1.2) and (5.1.3) imply that all banks commit to charge a (no-default) lending rate for the indirect CB loans equals to the risk-free interest rate R_2. As a result all banks equally obtain the funding from CB to issue the indirect CB loans.[24] As a result, CB loans are going to be cheaper than bank loans due to (i) government guarantees and (ii) the fact that indirect CB loans cannot be diverted.

Entrepreneurs are going to demand and deplete first these cheaper CB loans and then bank loans. As a result, we cannot expect a one to one multiplier effect of the credit policy on aggregate lending. With this in mind, we can see how the credit policy will affect aggregate credit supply.

- First, as in Gertler and Kiyotaki (2011) since CB loans cannot be diverted by banks, the CB credit policy is diminishing the impact moral hazard problem between banks and depositors on this economy. In other words, banks required equity per unit of aggregate credit decreases, which allows for a smaller required return of bank loans for a given aggregate credit level. As a result, credit policy increases the aggregate supply of credit.

- Second, since the required return of central bank loans is smaller than those of the traditional bank loans, entrepreneurs now face a limited supply of cheap CB loans in addition to the supply curve of bank loans, equation (3.2.8), which is not affected by the credit policy.

In the margin bank loan supply curve matters since the last external funding comes from bank loans. As a result, we can say that the aggregate credit supply curve changes, but the aggregate supply curve of bank loans that in the margin (together with the aggregate demand curve) determines the equilibrium level of credit is not affected.

[23] We believe this is a realistic assumption, since the central bank might have more monitoring and enforcement power over banks than depositors

[24] Notice that it doesn't make sense that banks propose a (non-default) lending rate below the risk-free interest rate.

In equilibrium, banks issue CB loans by exactly the same amount of funds received from the CB.[25] Hence, banks balance sheet becomes,

$$B_2^g + B_2 = B_2^g + D_2 + N_{1,b}, \tag{5.1.4}$$

where B_2^g is not only the amount of CB loans issued to entrepreneurs but also the amount of funds received from CB to finance these loans. As a result, equation (5.1.4) collapses to the balance sheet (3.2.4) and so traditional bank loans are still funded with both households' deposits and bank's initial equity. Hence, banks profits are not affected given that by definition CB revenues are perfectly cancelled out with their own funding costs. Thus, the maximization problem of banks is not affected. This implies that the demand curve of deposits and the supply curve of traditional bank loans, equations (3.2.8) and (3.2.9), still hold with credit policy intervention.

In addition, the aggregate supply curve of deposits (Euler equation) is indeed affected by the policy intervention,

$$R_2 = \frac{1}{\beta}\left(\frac{\Delta^{-1}a(D_2 + B_2^g + N_{1,b} + N_{1,e})^{\alpha} - \mu G(\bar{\omega}_2)R_2^k(D_2 + B_2^g + N_{1,b} + N_{1,e})}{y_1 - (D_2 + B_2^g + N_{1,b} + N_{1,e})}\right)^{\sigma}.$$

Since in the calibration μ and entrepreneur default probability are very small, a positive credit policy intervention, i.e., $B_2^g > 0$, which transfer households' resources across periods, reduces households' incentives to supply deposits. In other words, $B_2^g > 0$ produces a shift to the left of the deposit supply curve.

And, as it is shown next the maximization problem of entrepreneurs is also affected by the policy intervention.

5.2 CB credit policy and demand side frictions

Recall, we assume that the entrepreneur is not aware of this credit injection rule, equation (5.1), and hence she cannot internalize the effects of their decisions on B_2^g. Entrepreneur balance sheet becomes,

$$K_2 = B_2^g + B_2 + N_{1,e}. \tag{5.2.5}$$

In this paper, we solve the model assuming that bank loans and central bank loans have the same seniority[26]. This is, when entrepreneur defaults at $t = 2$, realized revenues are use to repay CB loans and bank loans proportionally to their values at $t = 2$.

[25] Clearly, banks are not willing to issue central bank loans funded with households deposits and/or bank equity, since the cost of collecting households deposits end up being higher than the risk-free interest rate, due to the moral hazard problem between banks and households, which is the return that they will obtain for issuing central bank loans.

[26] Each time an entrepreneur j defaults, she needs to know the payment order to their creditors (CB and banks). There are three alternative assumptions: (1) Both CB loans and bank loans have the same seniority, (2) bank loans have higher seniority and (3) CB loans have higher seniority. In (1) both loans are paid with the same priority and hence each time entrepreneur defaults she transfers her realized capital payoffs to their creditors proportionally. In (2) if entrepreneur defaults it repays first bank loans, and then she cares on repaying CB loans. In (3) the opposite occurs. Pozo and Rojas (2020) explore in detail the effects of seniority

Banks and government pay monitoring costs to observe entrepreneur' realized return when she defaults. Further, we assume the fixed proportional auditing cost μ is the same for both banks and for central bank. Hence, total monitoring costs must add up $\mu\omega_2 R_2^k K_2$. Thus, the threshold value of the idiosyncratic productivity, $\bar{\omega}_2$, is defined as,

$$\bar{\omega}_2 R_2^k K_2 = Z_2 B_2 + R_2 B_2^g. \tag{5.2.6}$$

Furthermore, if $\omega_2 < \bar{\omega}_2$, government makes sure that CB loans are fully repaid by collecting lump-sum taxes. A defaulting entrepreneur receives nothing.

Combining equations (5.1) and (5.2.6) yields

$$\bar{\omega}_2 R_2^k K_2 = Z_2 B_2 + R_2 B_2^g \Rightarrow \bar{\omega}_2 = \frac{Z_2(1-\psi_{CB,2}) + R_2\psi_{CB,2}}{R_2^k}\frac{\phi_{2,e}-1}{\phi_{2,e}} < \bar{\omega}_2\Big|_{\psi_{CB,2}=0}, \tag{5.2.7}$$

where $\phi_{2,e} = K_2/N_{1,e}$ is the leverage of the representative entrepreneur. From equation (5.2.7), Ceteris paribus, a higher fraction $\psi_{CB,2}$ of cheap loans reduce $\bar{\omega}_2$ and hence entrepreneur default probability, which in turn it results in lower expected monitoring cots. Consequently, it increases the marginal benefit of capital and hence increases demand for bank loans. Furthermore, according to (5.2.7) what drives the smaller $\bar{\omega}_2$ for a given R_2^k is the difference between the cost of CB loans R_2 and the traditional bank loans Z_2, i.e., the difference is that the cost of the CB loans does not include a risk premium due to the moral hazard problem between banks and depositors and an entrepreneur default risk premium.

The bank loan contract ($\bar{\omega}_2$, Z_2), in equation (3.3.12), which satisfies that banks always receive a gross return R_2^l per unit of bank loans, becomes:

$$[1-F(\bar{\omega}_2)]Z_2B_2 + (1-\mu)\int_0^{\bar{\omega}_2} \omega R_2^k K_2 x_2 dF(\omega) = R_2^l B_2, \tag{5.2.8}$$

where $x_2 = Z_2B_2/(Z_2B_2 + R_2B_2^g)$ is the proportion of the realized revenues that goes to repay bank loans when the entrepreneur defaults. For a given K_2 the differences with a bank loan contract without credit policy, equation (3.3.12), are two: i) only a fraction $(1-\psi_{CB,2})$ of external funding comes from bank loans. This is, without credit policy $B_2 = K_2 - N_{1,e}$, while with credit policy $B_2 = (1-\psi_{CB,2})(K_2 - N_{1,e})$, and ii) only a fraction x_2 of $\omega R_2^k K_2$ goes to payback bank loans each time an entrepreneur defaults.

For convenience the bank loan contract is written as,[27]

$$\left(\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)\right) R_2^k K_2 = R_2^l \left(K_2 - B_2^g - N_{1,e}\right) + \Psi(\bar{\omega}_2) R_2 B_2^g, \tag{5.2.9}$$

where Γ and G are already defined in (B.2) and,

$$\Psi(\bar{\omega}_2) = (1-\mu)\frac{1}{\bar{\omega}_2}G(\bar{\omega}_2) + (1-F(\bar{\omega}_2)) < 1.$$

In the left-hand side of equation (5.2.9) we have the resources available to repay fund-

[27]Proof in Appendix G.

ing. From the right-hand side those resources are used to fully cover the required return on the cost of funds, $R_2^l B_2$ and to partially pay the CB loans $\Psi(\bar{\omega}_2) R_2 B_2^g$. So, $\Psi(\bar{\omega}_2) R_2 B_2^g$ is the effective gross return repaid to CB loans by the entrepreneur. It is composed by the amount that non default entrepreneurs transfer to repay central bank loans $(1 - F(\bar{\omega}_2)) R_2 B_2^g$ and the value of seized projects from defaulting entrepreneurs $(1-\mu)\frac{1}{\bar{\omega}_2} G(\bar{\omega}_2) R_2 B_2^g$, net of monitoring costs, to repay central bank loans. Note that each time an entrepreneur defaults, government honours the guarantee and hence transfers resources to ensure CB loans receive the agreed return. This implies that entrepreneurs' transfers are not enough to fully pay CB loans. i.e., $\Psi(\bar{\omega}_2) < 1$, or equivalently the effective cost of CB loans from entrepreneur perspective is smaller than the risk-free interest rate, i.e., $\Psi(\bar{\omega}_2) R_2 < R_2$. This means that government transfers destined to repay CB loans are $(1 - \Psi(\bar{\omega}_2)) R_2 B_2^g$.

With unconventional credit, the entrepreneur aims to maximize their expected profits,

$$\int_{\bar{\omega}_2}^{+\infty} \left(\omega R_2^k K_2 - Z_2 B_2 - R_2 B_2^g\right) dF(\omega),$$

taking as given R_2^k and B_2^g. Using (5.2.6) it yields,

$$[1 - \Gamma(\bar{\omega}_2)]\, R_2^k K_2. \tag{5.2.10}$$

We arrive to an expression identical to the one without credit policy which is independent of the loan seniority assumption. Entrepreneur chooses K_2 and a schedule for $\bar{\omega}_2$ to maximize equation (5.2.10), subject to the state-contingent constraint implied by equation (5.2.9).[28] The aggregate credit demand curve, equation (3.3.14), becomes,[29]

$$\left[[1 - \Gamma(\bar{\omega}_2)] \frac{\Upsilon_2 + 1 - F(\bar{\omega}_2) - \mu \bar{\omega}_2 f(\bar{\omega}_2)}{1 - F(\bar{\omega}_2)} + (\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)) \right] R_2^k - R_2^l = 0\,, \tag{5.2.11}$$

together with (5.2.9) and where,

$$\Upsilon_2 = -\frac{\partial \Psi(\bar{\omega}_2)}{\partial \bar{\omega}_2} \frac{R_2 B_2^g}{R_2^k K_2} > 0. \tag{5.2.12}$$

Since $\Upsilon_2 > 0$, and comparing (5.2.11) with (3.3.14), we observe that credit policy positively affects the net marginal benefit of capital and hence aggregate demand for bank loans. The intuition is that the credit policy is reducing the transfer from entrepreneur to partially repay CB loans (or equivalently is increasing the transfers from government to repay CB loans), which in turn reduces entrepreneur default probability and hence the expected defaulting costs, which in turns raises incentives to demand capital and hence bank loans.

We can say that according to (5.2.7) for a given K_2 credit policy reduces entrepreneur default probability and from (5.2.11) for a given $\bar{\omega}_2$ the net marginal benefit of capital

[28] The first order conditions are found in Appendix G.

[29] This is obtained from aggregating equation (3.3.14) in Appendix G.

increases [30]. Both findings positively affect aggregate credit demand:

- First, since the opportunity cost of the central bank is the risk-free interest rate R_2, banks or the central bank require a lower return per unit of these CB loans than the one required by bank loans, i.e., $R_2 < R^l_2$.[31] This reduces entrepreneur default probability and hence reduces the defaulting costs and pushes up the aggregate demand of capital and hence of demand for credit.

- Second, the guarantee of the government avoids that the (non-default) lending interest rate associated with the CB loans reflects any risk premium. In other words, while the (non-default) lending rate on banks loans is Z_2, the (non-default) lending rate on CB loans is R_2, with $Z_2 > R_2$. Ceteris paribus for given capital and equity, CB loans reduce entrepreneur obligations, default probability and reliance on bank loans. This in turn reduces the defaulting costs and pushes up the aggregate demand of capital and hence of credit.

Hence, the credit policy stimulates the aggregate credit supply and demand. In other words, the credit policy is expected to produce an increase in aggregate credit. Clearly, without frictions on the credit supply side, the credit policy does not increase aggregate credit supply and the first effect on aggregate demand is null.

5.3 The effects of CB credit policy: a simulation exercise

In this subsection, in order to qualitatively assess the effects of the credit policy when the ZLB does not bind, we describe the credit policy intervention as exogenous. In particular, we set $\psi_{CB,2} = 6\%$.[32]

With credit policy intervention, banks substitute expensive traditional loans with cheaper indirect CB loans. But there is not only a substitution effect, since we observe also a higher total level of loans ($B^T_2 = B^g_2 + B_2$). This implies that this credit policy might attenuate a negative impact on the economy. Recall the effects of the credit policy on credit supply and on credit demand. Credit supply: Since CB loans cannot be diverted by banks, there is a higher aggregate supply of credit. Credit demand: Since the

[30] Inserting equation (5.2.9) into equation (5.2.10) yields,

$$[1 - \mu G(\bar{\omega}_2)]\, R^k_2 K_2 - R^l_2 (K_2 - B^g_2 - N_{1,e}) - \Psi(\bar{\omega}_2) R_2 B^g_2. \tag{5.2.13}$$

Equation (5.2.13) says that the marginal cost of capital is not affected directly by the credit policy and so it continues to be R^l_2. This is because entrepreneur is not internalizing the effects of their capital decision on B_2 since they are not aware of the credit policy rule, equation (5.1). Otherwise, they are aware that one unit of external funding is funded with both cheap CB loans and bank loans, reducing the marginal cost of capital from entrepreneur's perspective.

[31] Recall that the return required by bank loans is higher than the risk-free interest rate due to the moral hazard problem between banks and depositors and the asymmetric information problem between banks and firms.

[32] Recall that we assume that even after the credit policy intervention, the central bank can reach the target inflation of zero. This is performed by updating R^*_2 in the Taylor rule so it is equal to the deposit interest rate R_2 without sticky prices.

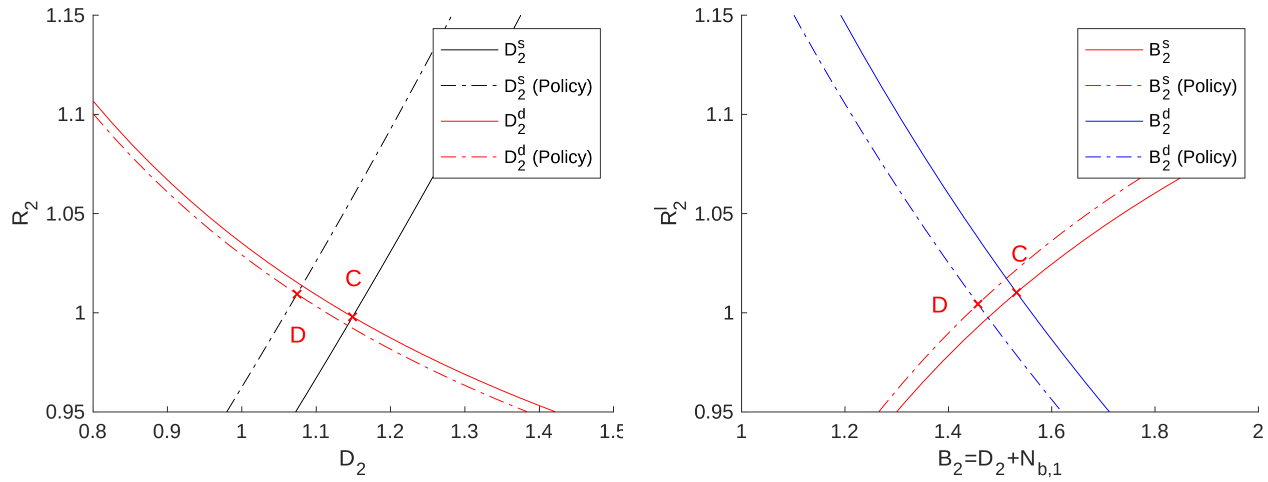
cost of CB loans is the risk-free interest rate and the lending rate does not have any risk premium, entrepreneur default probability decreases, which in turn increases the marginal benefit of capital. This pushes up entrepreneurs' incentives to demand credit.[33]

Figure (3) shows the equilibrium in the deposit and traditional bank loans market with and without the unconventional credit policy intervention. Interestingly, unconventional credit policy intervention raises the deposit interest rate (Panel (a): the deposit market equilibrium moves point C to D). This implies that the credit policy moves the economy away from being closer to the ZLB. This in turn suggests that there is more space for implementing conventional monetary policy. This is because with policy intervention the central bank is collecting lump-sum taxes on households and hence is moving households' wealth across time. As a result, in order to smooth consumption households reduce their supply of deposits. This raises the deposit interest rate and the nominal interest rate as well.

In addition, Panel (b) in figure (3) reports that there is a shift to the left of the credit demand curve due to (i) the higher default probability of entrepreneurs and (ii) the fact that entrepreneurs substitute expensive traditional bank loans with cheap indirect CB loans. This pushes down the required return of traditional bank loans (Panel (b): the traditional bank loans market equilibrium moves point C to D), which leads to a smaller bank capacity to demand deposits moving to the left the deposit demand curve. However, this is not enough to generate a lower deposit rate in equilibrium.

Figure 3. *Deposit and Credit Market*

Panel (a) Deposit Market — Panel (b) Traditional bank loans Market



Note: C: Both credit frictions. D: Equilibrium with unconventional credit policy, $\psi_{CB,2} = 6\%$. D_2^s : deposit supply curve. D_2^d : deposit demand curve. B_2^s : traditional bank loans supply curve. B_2^d : traditional bank loans demand curve.

[33] Although credit policy has a negative effect on entrepreneur default probability pushing up credit demand, the general equilibrium effects of a higher capital on this probability dominates. As a result, we observe that the entrepreneur default probability (or the fraction of defaulting entrepreneurs at $t = 2$) increases.

6 The Impact of the Zero Lower Bound

Here, we study the impact of the ZLB on credit policy effectiveness to diminish the impact of a shock that takes the economy to the ZLB. In particular, we use a productivity level change so that it takes the economy to the ZLB. According to figure 4, a lower productivity level might move the economy to a low enough nominal interest rate so that the ZLB binds.[34] Notice that we assume the central bank successfully implements inflation targeting.[35] We set the credit policy intervention $\psi_{CB,2}$ as a linear and decreasing function of the relative deviation of the productivity level from its baseline, i.e., $\psi_{CB,2} = -3\Delta a$, so it behaves as a "countercyclical" intervention. For the next numerical results, since we assume a ZLB, $i_{min} = 0$.[36] Then, according to our baseline calibration, the distance of the nominal interest rate to is ZLB is 1.23% (5% in annual terms).

When the ZLB binds, the nominal interest rate and the real interest rate (deposit interest rate) stop reducing. This constraint on the nominal interest rate avoids that the central bank can implement a monetary policy (a low enough nominal interest rate) so that inflation yields its target value, which is zero. As a result, inflation moves above its target value.[37] Intuitively, the higher real interest rate increases households' incentives to save[38] and to consume more at $t = 2$, which in turn increases aggregate demand and future inflation. This in turn increases entrepreneurs' incentives to produce and hence to demand credit. Thus, the ZLB produces a positive impact on capital and credit.[39]

According to figure 4, the unconventional credit policy can reduce the likelihood of reaching the ZLB. This is, as commented in the previous section, because the unconventional credit policy reduces the deposit supply of households pushing upwards pressure on the real interest rate. This implies that the credit policy might give more space to implement a stronger conventional monetary policy. In the same line, figure 4 also shows that a strong enough credit policy can take us out of the ZLB environment.

Figure 4 also suggests that in an economy with a ZLB the impact of the credit policy, assuming that the ZLB binds before and after the policy intervention, is weaker. In other words, when the ZLB already binds (even after the policy intervention) the effectiveness

[34]Figure 8 in Appendix I reports the case, when a lower $N_{b,1}$ can take us to the ZLB. In any case, all the conclusions from this section hold and hence the impact of the ZLB on the effectiveness of credit policy to improve capital and credit is qualitatively the same.

[35]In other words, we update R_2^* with the movement of a and with the policy intervention. This means that when the ZLB does not bind inflation is zero. However, when the ZLB binds, the central bank cannot implement inflation targeting and it becomes positive. This assumption is due to the two-period feature of the model; otherwise, the model might suggest the central bank can never reach its target inflation.

[36]It is easy to see that the results qualitatively holds for the case of a different value of i_{min} below the baseline value of the nominal interest rate.

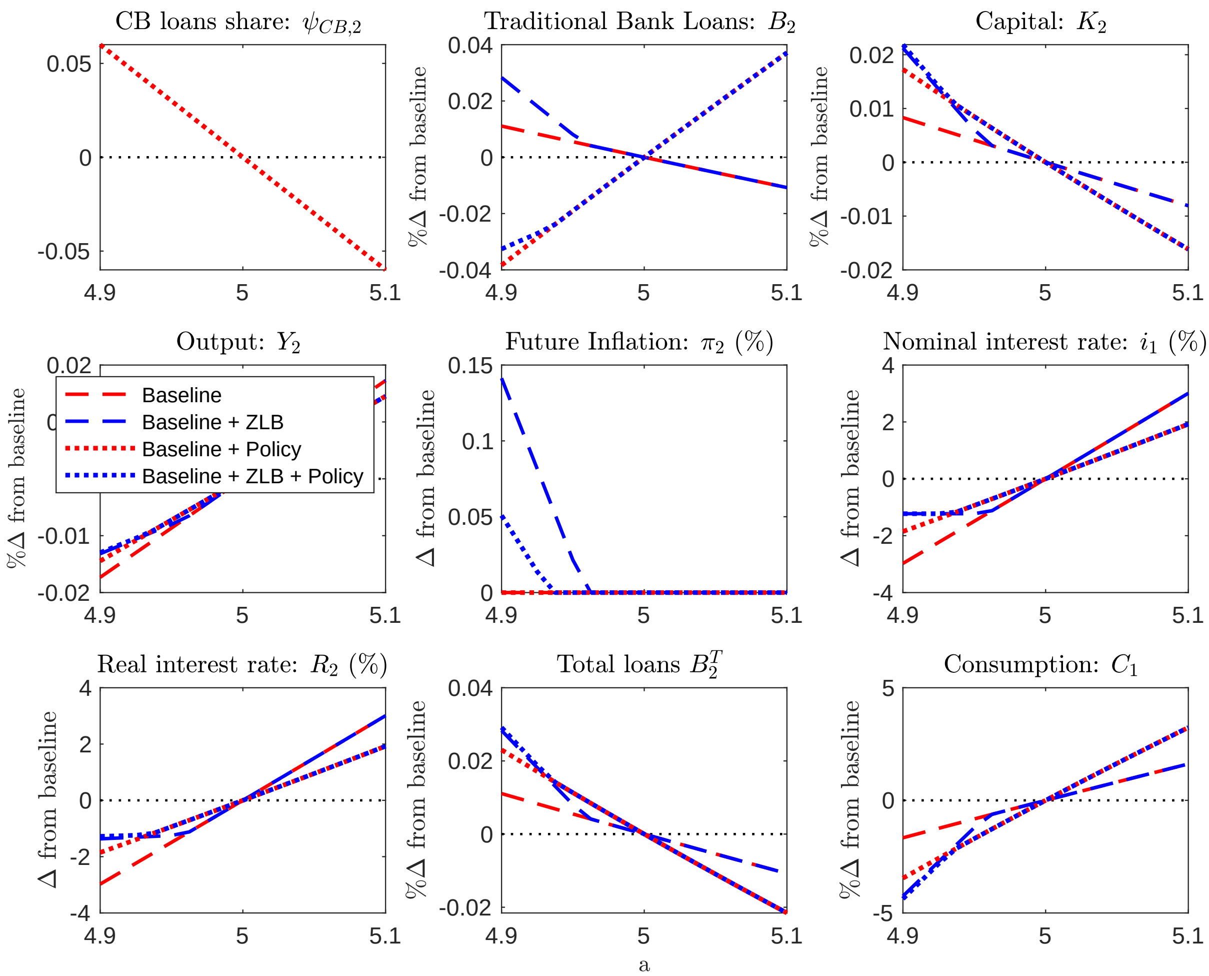
[37]Note that this departs from the literature (based on dynamics models) that suggest that when the zero lower bound binds, the economy falls in a deflation spiral. In other words, in a dynamic NK model it is possible to find a stable solution for inflation. However, in this two-period model this is not the case.

[38]Under the model calibration a lower a produces an increase in credit and capital due to an increase of the deposit supply of households as the wealth effects dominate the substitution effect of a lower real interest rate.

[39]In Appendix H we display the shifts of the supply and demand curves in the deposit and credit market due to the ZLB.

of the credit policy to increases total credit and hence capital is diminished. We compare the impact of the credit policy on capital, credit and output in figure 5 in an economy with and without ZLB. For example, at a productivity level 2% smaller than its baseline value (i.e., at $a = 4.9$, the lowest value in the figure), the policy intervention (i.e., the participation of CB loans to total loans) of 6% produces increments of 1.2% in total loans ($B_2 + B_2^g$) and 0.9% in capital in an economy without a ZLB; while these increments become 0.08% and 0.06% respectively in an economy with an already binding ZLB.

Figure 4. *ZLB and credit policy*

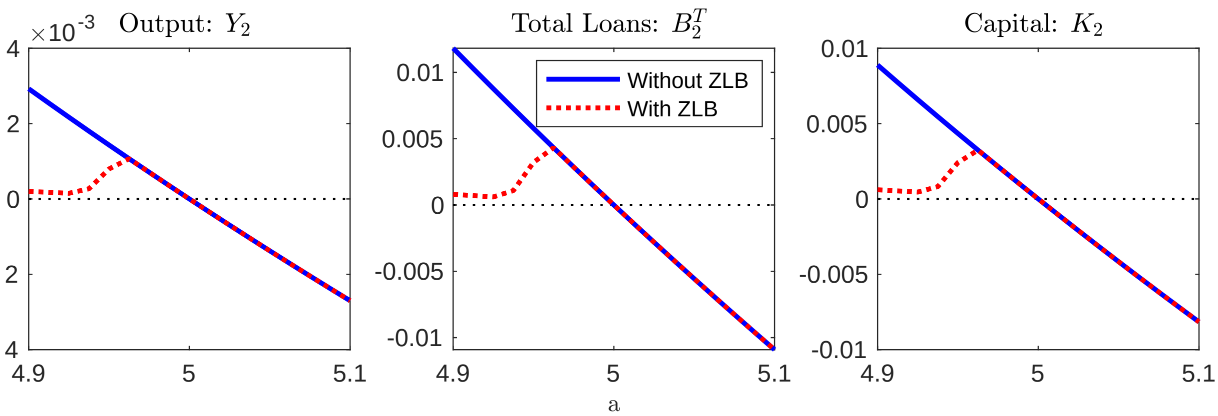


Note: Figure shows the solutions for different values of the productivity level, a. These go from 0.98 to 1.02 times its baseline value. Distance to the $i_{min} = 0$ is 1.23%. Credit policy intervention is "countercyclical". All solutions are identical at baseline calibration. $B_2^T = B_2 + B_2^g$. $\psi_{CB,2} = -3\Delta a$, where Δa is the relative deviation of the productivity level from its baseline.

The reduced effectiveness of the unconventional credit policy when the ZLB binds is explained by two features: (i) When the ZLB is reached, since the policy maker cannot reach the target inflation, it moves above its target level; however, credit policy pushes down inflation. This negative impact on inflation of the credit policy is not observed when the ZLB does not bind (i.e., when the policy maker reaches its target inflation). Then, this negative impact reduces firms' incentives to demand capital and then entrepreneurs' incentives to demand credit. As a result, we observe a relatively stronger shift to the left of the credit demand curve of entrepreneurs (see figure 6) when the ZLB binds. And (ii) the binding ZLB increases the real interest rate and hence the cost of the indirect central bank loans, which in turn pushes up the entrepreneur default probability and reduces the demand of credit of entrepreneurs.[40]

Furthermore, this result highlights the importance of having a proactive central bank. In other words, according to the model central banks have stronger incentives to implement unconventional credit policy before the economy reaches the ZLB.

Figure 5. *Impact of the credit policy with and without ZLB*

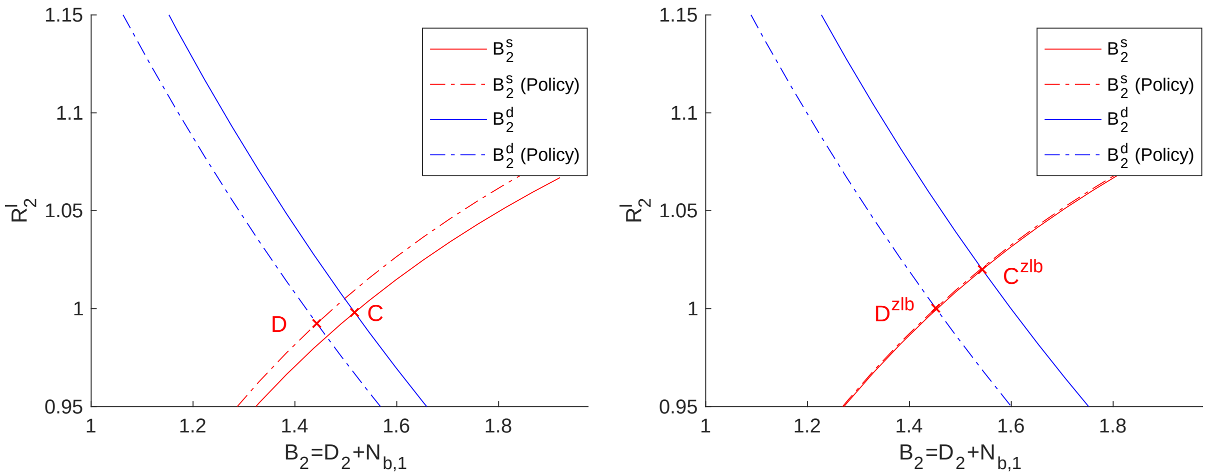


Note: Figure shows the percentage difference between the equilibrium solutions of output, total loans and capital without and with unconventional credit policy for different values of a, the productivity level, for an economy with and without ZLB. Distance to the $i_{min} = 0$ is 1.23%. Credit policy intervention is "counter-cyclical". All solutions are identical at baseline calibration. $B_2^T = B_2 + B_2^g$. $\psi_{CB,2} = -3\Delta a$, where Δa is the relative deviation of the productivity level from its baseline.

[40] Recall that one positive aspect of the unconventional credit policy is to provide of cheaper funding to entrepreneurs. Hence, the ZLB diminishes this channel.

Figure 6. *Traditional bank loans market*

Panel (a) without ZLB　　　　Panel (b) with ZLB



Note: Figure plots supply and demand curves of traditional bank loans at a productivity level 2% below its baseline value. At this point the ZLB binds (even after the credit policy intervention of $\psi_{CB,2} = 6\%$). Point C (C^{zlb}) indicates the equilibrium without policy intervention and without (with) ZLB. Point D (D^{zlb}) indicates the equilibrium with policy intervention and without (with) ZLB. D^s_2 : deposit supply curve. D^d_2 : deposit demand curve. B^s_2 : traditional bank loans supply curve. B^d_2 : traditional bank loans demand curve.

7 Conclusions

In this paper we use a novel model that incorporates credit supply and credit demand frictions together to understand the role of an unconventional credit policy in a ZLB equilibrium. First, The model is stylized enough that in a two periods model can show the main mechanisms that operate in the interaction between deposits markets and credit markets. Second, we show that presence of credit frictions makes more likely that a ZLB equilibrium occurs. Third, the unconventional credit policy has a positive impact on capital and credit by undoing partially the effects of credit frictions in the allocation of resources in the economy. More interestingly, our model suggests that a strong enough policy intervention might take the economy out of the ZLB, and the presence of the unconventional credit policy reduces the likelihood of reaching the ZLB. However, once the ZLB binds (even after the policy intervention) the effectiveness of the credit policy is diminished.

However, since our model is very simple, and involves only two periods, our analysis has limits. First, we cannot respond questions related to the effects of unconventional credit on future expected inflation, or about the duration of ZLB under credit policy actions. Second, our analysis abstracts from optimal credit policy intervention and/or fiscal and monetary policy coordination. Third, a more realistic ZLB environment requires to think about the risks of a deflationary spiral.

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Appendices

A Credit Supply Frictions curve of deposits

Credit Supply friction are modeled a la Gertler and Karadi 2011. A Bank.

Problem of Banks:

$$\max_{D_2} \quad R_2^l(N_{b,1} + D_2) - R_2 D_2$$
$$\text{s.t} \quad \text{Incentive constraint (IC):}$$
$$R_2^l(N_{b,1} + D_2) - RD_2 \geq \lambda(N_{b,1} + D_2)R_2^l$$

The first order condition with respect to D_2 is

$$(R_2^l - R_2) + \nu((R_2^l - R_2) - \lambda) = 0. \tag{A.1}$$

where $\nu \geq 0$ is the Lagrange multiplier associate with the incentive constraint. We calibrate our model so that (3.2.5) binds: Without credit supply frictions, $R_2^l = R_2$ and then $V_1 = R_2 N_{1,b}$ and so we calibrate the model such that $R_2 N_{1,b} < \lambda B_2$. This results in $\nu > 0$ and from (A.1) it arises a credit risk premium $R_2^l - R_2 > 0$:

$$R_2^l - R_2 = \frac{\nu\lambda}{(1+\nu)}. \tag{A.2}$$

And from the binding incentive constraint, we solve for D_2, to obtain the demand curve for deposits:

$$D_2 = N_{b,1}\frac{(1-\lambda)R_2^l}{R_2 - (1-\lambda)R_2^l}. \tag{A.3}$$

B Entrepreneurs: Lending diversification and credit demand frictions

The incentive constraint for bank's loan contract $(\bar{\omega}_2,\ Z_2)$ in equation (3.3.11) can be rewritten by using (3.3.10) and (3.3.11) as follow

$$[\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)]\, R_2^k K_2 = R_2^l(K_2 - N_{1,e}), \tag{B.1}$$

where,

$$\Gamma(\bar{\omega}_2) = \int_0^{\bar{\omega}_2} \omega dF(\omega) + (1 - F(\bar{\omega}_2))\bar{\omega}_2, \qquad G(\bar{\omega}_2) = \int_0^{\bar{\omega}_2} \omega dF(\omega). \tag{B.2}$$

The expected profits to the entrepreneur in equation (3.3.13) by using (3.3.11) is rewritten as,

$$[1 - \Gamma(\bar{\omega}_2]\, R_2^k K_2. \tag{B.3}$$

Entrepreneurs aim to maximize (B.3) optimally choosing K_2 and $\bar{\omega}_2$ subject to the constraint implied by the bank loan contract, equation (B.1). Formally, the optimal problem may be now written as:

$$\max_{K_2,\bar{\omega}_2} \left(1-\Gamma(\bar{\omega}_2)\right) R_2^k K_2 + \eta_2 \left[\left(\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)\right) R_2^k K_2 - R_2 B_2\right],$$

where η_2 is the Lagrange multiplier associated with the loan contract. The first order conditions for $\bar{\omega}_2$ is,

$$-\frac{\partial\Gamma(\bar{\omega}_2)}{\partial\bar{\omega}_2} + \eta_2 \left(\frac{\partial\Gamma(\bar{\omega}_2)}{\partial\bar{\omega}_2} - \mu\frac{G(\bar{\omega}_2)}{\partial\bar{\omega}_2}\right) = 0. \tag{B.4}$$

The first order conditions for K_2 is,

$$\left(1-\Gamma(\bar{\omega}_2)\right) R_2^k + \eta_2 \left[\left(\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)\right) R_2^k - R_2^l\right] = 0. \tag{B.5}$$

The first order condition for η_2 yields the constrain, equation (B.1), where,[41]

$$\frac{\partial\Gamma(\bar{\omega}_2)}{\partial\bar{\omega}_2} = 1 - F(\bar{\omega}_2), \qquad \frac{\partial G(\bar{\omega}_2)}{\partial\bar{\omega}_2} = \bar{\omega}_2 f(\bar{\omega}_2).$$

Combining equations (B.4) and (B.5) yields the credit demand curve of the representative entrepreneurs, given by

$$\left[\left[1-\Gamma(\bar{\omega}_2)\right]\frac{1-F(\bar{\omega}_2) - \mu\bar{\omega}_2 f(\bar{\omega}_2)}{1-F(\bar{\omega}_2)} + \left(\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)\right)\right] R_2^k - R_2^l = 0, \tag{B.6}$$

and equation (B.1).

C Sticky prices: Final goods and intermediate firms

The final goods are produced by competitive firms, takes price, P_2, as given, and combines substitute intermediate domestic goods into a homogeneous good using the following CES technology, by solving the following profit maximization problem:

$$\max_{Y_{i,2}} \; P_2 Y_2 - \int_0^1 P_{i,2} Y_{i,2} di,$$

s.a

$$Y_2 = \left[\int_0^1 Y_{i,2}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}},$$

[41] We assume $ln(\omega) \sim \mathcal{N}\left(-0.5\sigma_\omega^2, \sigma_\omega^2\right)$ so we have $\mathbb{E}(\omega) = 1$ and then $\Gamma(\bar{\omega}) = \Phi(z-\sigma_\omega) + \bar{\omega}[1-\Phi(z)]$, $G(\bar{\omega}) = \Phi(z-\sigma_\omega)$, $\partial\Gamma(\bar{\omega})/\partial\bar{\omega} = 1-\Phi(z)$ and $\partial G(\bar{\omega})/\partial\bar{\omega} = \bar{\omega}\Phi'(z)$, where $\Phi(.)$ and $\Phi'(.)$ are the c.d.f. and the p.d.f., respectively, of the standard normal and z is related to $\bar{\omega}$ through $z = (ln(\bar{\omega}) + 0.5\sigma_\omega^2)/\sigma_\omega$.

where $\theta > 1$. The solution of maximization problem yields the demand schedule for a domestic intermediate i:

$$Y_{i,2} = \left(\frac{P_{i,2}}{P_2}\right)^{-\theta} Y_2, \tag{C.1}$$

and an aggregate price index,

$$P_2 = \left[\int_0^1 P_{i,2}^{1-\theta} dz\right]^{\frac{1}{1-\theta}}. \tag{C.2}$$

Intermediate good producers

Given the Decreasing Return to Scale technology, with $\alpha < 1$:

$$Y_{i,2} = a\,(K_{i,2})^{\alpha}\ ,$$

the inverse demand curve from (3.4.17),

$$p_{i,2} = \frac{P_{i,2}}{P_2} = \left(\frac{Y_{i,2}}{Y_2}\right)^{-\frac{1}{\theta}},$$

and the total cost function $\mathcal{C}(Y_{i,2}) = R_2^k K_{i,2}$ and where $K_{i,2}$ is found in the production function (3.4.19), the problem of an intermediate firm i that has the opportunity to change prices, is to maximize profits, which can be rewritten, in terms of $Y_{i,2}$ defining the relative prices as a function of output,

$$\max_{Y_{i,2}} \left[p_{i,2}(Y_{i,2})Y_{i,2} - R_2^k \left(\frac{Y_{i,2}}{a}\right)^{1/\alpha}\right], \tag{C.3}$$

with F.O.C.,

$$\begin{aligned} \frac{d(.)}{dY_{i,2}}: \qquad & p_{i,2}(Y_{i,2}) + Y_{i,2}\frac{dp_{i,2}}{dY_{i,2}} - \frac{1}{\alpha a^{1/\alpha}} R_2^k\,(Y_{i,2})^{\frac{1-\alpha}{\alpha}} = MR - \mathfrak{c}_{it} = 0 \\ & p_{i,2}\left(1 + \frac{dp_{i,2}}{dY_{i,2}}\frac{Y_{i,2}}{p_{i,2}}\right) - \mathfrak{c}_{it} = MR - \mathfrak{c}_{it} = 0, \end{aligned}$$

where the MR is the marginal revenue and $\mathfrak{c}_{it} := \partial\mathcal{C}(Y_{i,2})/\partial Y_{i,2}$ is the marginal cost. From the demand curve for intermediate firm, (3.4.17), $\frac{dp_{i,2}}{dY_{i,2}}\frac{Y_{i,2}}{p_{i,2}} = -\frac{1}{\theta}$, the optimal pricing is:

$$\frac{P_{i,2}}{P_2} = \mathcal{M}\mathfrak{c}_{i,2}\,,$$

where $\mathcal{M} :\equiv \frac{\theta}{\theta-1}$ denotes the constant markup of the monopolistic firm. As shown in Galí (2015) one can eliminate the markup distortion on prices by considering a capital subsidy for the firm: $\tau = \frac{1}{\theta}$. Thus the optimal price without price distortions is[42]:

$$\frac{P_{i,2}}{P_2} = \mathfrak{c}_{i,2} \,, \tag{C.4}$$

Notice that with $\alpha < 1$, a decreasing return to scale in capital, the marginal cost is itself endogenous,

$$\mathfrak{c}_{i,2} = \frac{1}{\alpha a^{1/\alpha}} R_2^k \left(Y_{i,2}\right)^{\frac{1-\alpha}{\alpha}} \underbrace{=}_{\text{from (3.4.17)}} \frac{1}{\alpha a^{1/\alpha}} R_2^k \left(\left(\frac{P_{i,2}}{P_2} \right)^{-\theta} Y_2 \right)^{\frac{1-\alpha}{\alpha}} \tag{C.5}$$

and by inserting back into (C.4) and solving for $p_{i,2}$ we get that firms that optimally choose a price and denoting P_2^o as the optimal price for those firms that can update prices:

$$\frac{P_{i,2}}{P_2} = \frac{P_2^o}{P_2} = \left(\frac{1}{\alpha a^{1/\alpha}} R_2^k \left(Y_2\right)^{\frac{1-\alpha}{\alpha}} \right)^{\frac{\alpha}{\alpha+\theta(1-\alpha)}} \,. \tag{C.6}$$

In equilibrium, the consistent aggregate price index, (3.4.18), P_2 is

$$P_2 = \left[(1-\gamma) \left(P_2^o\right)^{1-\theta} + \gamma \left(P_1\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \,,$$

where the all firms of measure γ that can not adjust their prices set $P_{i,2} = P_1$.

Appendix C.A Market Clearing

In equilibrium, market clearing in the capital market requires:

$$K_2 = \int_0^1 K_{i,2} di = \int_0^1 \left(\frac{Y_{i,2}}{a} \right)^{1/\alpha} di = \int_0^1 \left(\frac{\left(\frac{P_{i,2}}{P_2} \right)^{-\theta} Y_2}{a} \right)^{1/\alpha} di$$
$$= \left(\frac{Y_2}{a} \right)^{1/\alpha} \int_0^1 \left(\frac{P_{i,2}}{P_2} \right)^{-\theta/\alpha} di$$

Thus, solving the GDP at $t = 2$ we find:

$$Y_2 = \Delta^{-1} a K_2^\alpha,$$

[42]Note that with capital subsidy to the firm,

$$\frac{P_{i,2}}{P_2} = (1-\tau)\mathcal{M}\mathfrak{c}_{i,2}$$

, where $\tau = \frac{1}{\theta}$ and $\mathcal{M} = \frac{\theta}{\theta-1}$

where,

$$\Delta = \int_0^1 \left(\frac{P_{i,2}}{P_2}\right)^{-\theta/\alpha} di = \left[(1-\gamma)\left(\frac{P_2^o}{P_2}\right)^{-\theta/\alpha} + \gamma\left(\frac{P_1}{P_2}\right)^{-\theta/\alpha}\right], \tag{C.A.7}$$

is the price dispersion. **Market clearing in final goods market**,

$$C_1 = y_1 - N_e - N_b - D_2 = y_1 - K_2$$
$$C_2 = Y_2 - \mu G(\bar{\omega}_2) R_2^k K_2.$$

D Gap Representation: Phillips curve & IS curve

We represent the system in terms of gaps as in Woodford (2003). This is a practical representation for models with price rigidities. To do so, we apply a first order log-linearization to our non linear system, where for any X, its log-linear approximation around the natural equilibrium, X^n, is $\hat{x} = logX - logX^n \approx \frac{X - X^n}{X^n}$. .

First, we log-linearize the production function, equation (3.5.24),

$$\hat{y}_2 = logY_2 - logY_2^n = (log\, a + \alpha\, logK_2 - log\, \Delta) - (log\, a + \alpha\, logK_2^n)$$
$$= \alpha \hat{k}_2 - log\, \Delta$$

From the equation (C.A.7) we know that $log\, \Delta$,

$$log\, \Delta = log\left[(1-\gamma)\left(\frac{P_2^o}{P_2}\right)^{-\theta/\alpha} + \gamma\left(\frac{P_1}{P_2}\right)^{-\theta/\alpha}\right]$$

We know that $\Delta = 1$ at $P_2/P_1 = 1$ or at zero inflation, .i.e., $\Delta^n = 1$. Using this fact, and differentiate around the natural equilibrium,

$$log\, \Delta = -\frac{1}{1}\left[(1-\gamma)\frac{\theta}{\alpha}(p_2^o - p_2) - \gamma\frac{\theta}{\alpha}(p_2 - p_1)\right]$$
$$= \frac{\theta}{\alpha}\left[-(1-\gamma)(p_2^o - p_2) + \gamma(p_2 - p_1)\right],$$

where we used the facts of the natural equilibrium, and zero inflation: $\hat{\frac{p_2^o}{p_2^o}} = log\frac{P_2^o}{P_2} - log1 = p_2^o - p_2 \approx \frac{\frac{P_2^o}{P_2} - 1}{1}$ and $\hat{\frac{p_2}{p_1}} = log\frac{P_2}{P_1} - log1 = p_2 - p_1 \approx \frac{\frac{P_2}{P_1} - 1}{1}$, given $P_2^n/P_1^n = 1$ and $P_2^{n,o}/P_2^n = 1$.

And from the aggregate price index, (E.1)

$$log\frac{P_2^o}{P_2} = \frac{1}{1-\theta} log\left(\frac{1 - \gamma\left(\frac{P_1}{P_2}\right)^{1-\theta}}{(1-\gamma)}\right)$$

a log-linearization around the natural equilibrium,

$$p_2^o - p_2 = \frac{\gamma}{1-\gamma}\left(p_2 - p_1\right)\,, \tag{D.1}$$

shows that actual inflation is a constant proportion of the optimal reset price relative to aggregate prices. Now if we use this in the expression for price dispersion we are left with

$$log\Delta = 0\,,$$

which confirms the results from Galí (2015) and price dispersion is a second order phenomenon. Thus in this first order approximation around the zero inflation, $P_2^n/P_1^n = 1$, we can ignore the role of price dispersion, and log-linearized production function is just:

$$\hat{y}_2 = \alpha\hat{k}_2$$

Phillips curve

Using the definition of natural equilibrium, (3.5.26), we can rewrite the equation (3.4.21) which impose a constraint on the output,

$$\frac{P_2^o}{P_2} = \left(\frac{R_2^k}{R^{K,n}}\left(\frac{Y_2}{Y_2^n}\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{\alpha}{\alpha+\theta(1-\alpha)}} \tag{D.2}$$

in which $P_{i,2} = P_2^o$ for all firms of measure $1-\gamma$ that can adjust their prices. A log-linear approximation to (D.2)

$$p_2^o - p_2 = \frac{\alpha}{\alpha+\theta(1-\alpha)}\left(logR^k - logR^{K,n}\right) + \frac{1-\alpha}{\alpha+\theta(1-\alpha)}\left(logY_2 - logY_2^n\right)$$

where $p = log(P)$. By inserting the log-linear version of the aggregate price index (D.1) into the this, we have an aggregate supply equation.

$$p_2 - p_1 = \frac{\alpha(1-\gamma)}{\gamma\left(\alpha+\theta(1-\alpha)\right)}\left(\hat{r}_2\right) + \frac{(1-\alpha)(1-\gamma)}{\gamma\left(\alpha+\theta(1-\alpha)\right)}\left(\hat{y}_2\right)$$

where $\hat{r} = logR_2 - logR^n = logR^k - logR^{K,n}$, where we use the frictionless the credit market equilibrium condition, $R = R^k$, and $\hat{y}_2 = logY_2 - logY_2^n$.

IS curve

The log-linear version of the supply curve of capital, which is a the pricing of deposits or the Euler equation (3.5.25) is

$$logR_2 - logR_2^n = \sigma\left(\left(logC_2 - logC_2^n\right) - \left(logC_1 - logC_1^n\right)\right)$$

where we have replaced the market clearing conditions $C_1 = \Delta^{-1}aK_2 :\equiv Y_2$ and $C_1 = y_1 - K_2$, which have as a log-linear transformation $\hat{c}_2 = \hat{y}_2$ and $\hat{c}_1 = -\frac{K^n}{y-K^n}\hat{k}_2$, respectively. Notice that if we use the log-linear version of the production function, we can further use rewrite $\hat{c}_1 = -\frac{K^n}{\alpha(y-K^n)}\hat{y}_2$. With this, the Euler equation becomes:

$$\hat{r}_2 = \sigma\left(1 + \frac{K^n}{\alpha\left(y - K^n\right)}\right)\hat{y}_2$$

Further, the log of real interest rate, by the Fisher equation is, $logR_2 = i - (p_2 - p_1) = i - \pi_2$, in the later equality we define $\pi_2 \equiv p_2 - p_1$.

Finally, the IS curve is:

$$0 = \sigma\left(1 + \frac{K^n}{\alpha\left(y - K^n\right)}\right)\hat{y}_2 - \left(i - \pi_2 - r^n\right)\ , \tag{D.3}$$

and the Phillips Curve is

$$\pi_2 = \frac{\alpha(1-\gamma)}{\gamma\left(\alpha + \theta(1-\alpha)\right)}\left(i - \pi_2 - r^n\right) + \frac{(1-\alpha)(1-\gamma)}{\gamma\left(\alpha + \theta(1-\alpha)\right)}\left(\hat{y}_2\right) \tag{D.4}$$

where $r^n = log(R^n)$ is the real natural rate and defined in the flexible price equilibrium. Together, the IS curve and Phillips Curve, summarize the equilibrium as the deviation of output from its natural level and interest from its natural level.

E Demand for capital

Notice that from (3.4.22), $\frac{P_2^o}{P_2}$ is a function of the ratio $\frac{P_1}{P_2}$, which is the inverse of inflation,

$$\frac{P_2^o}{P_2}\left(\frac{P_1}{P_2}\right) = \left(\frac{1 - \gamma\left(\frac{P_1}{P_2}\right)^{1-\theta}}{(1-\gamma)}\right)^{\frac{1}{1-\theta}} \tag{E.1}$$

Further, the dispersion of prices, (C.A.7), is also a function of P_1/P_2:

$$\Delta = \Delta\left(\frac{P_1}{P_2}\right) = \left[(1-\gamma)\left(\left(\frac{1 - \gamma\left(\frac{P_1}{P_2}\right)^{1-\theta}}{(1-\gamma)}\right)^{\frac{1}{1-\theta}}\right)^{-\theta/\alpha} + \gamma\left(\frac{P_1}{P_2}\right)^{-\theta/\alpha}\right] \tag{E.2}$$

Then, using (3.5.24) and the two previous computations, one can rewrite (3.4.21),

$$\frac{P_2^o}{P_2}\left(\frac{P_1}{P_2}\right) = \left(\frac{R_2^k}{\alpha a K^{\alpha-1}}\left(\frac{1}{\Delta\left(\frac{P_1}{P_2}\right)}\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{\alpha}{\alpha+\theta(1-\alpha)}}. \tag{E.3}$$

After some algebra, the demand for capital equation becomes,

$$R^k = \mathcal{W}\left(\frac{P_1}{P_2}\right)\alpha a K^{\alpha-1} \tag{E.4}$$

where

$$\mathcal{W}\left(\frac{P_1}{P_2}\right) = \left(\frac{1-\gamma\left(\frac{P_1}{P_2}\right)^{1-\theta}}{(1-\gamma)}\right)^{\frac{\alpha+\theta(1-\alpha)}{(1-\theta)\alpha}} \Delta^{\frac{1-\alpha}{\alpha}} \tag{E.5}$$

where Δ is defined in (E.2). $\mathcal{W}$ is a wedge between the return on capital. Notice that if $\gamma = 0$ (i.e., non sticky prices), $\mathcal{W}\left(\frac{P_1}{P_2}\right) = 1$.

F Optimal policy and loss function

In this section we develop an optimal monetary policy rule in an economy without financial frictions. As in Woodford (2003) an optimal MP is one that minimizes a loss function:

$$\min_{y_2,\hat{p}_2,\hat{r}} L = \frac{1}{2}\left(\hat{y}_2\right)^2 + \kappa\frac{1}{2}\left(\hat{p}_2 - \hat{p}^e\right)^2$$

subject to (D.4). After replacing (D.4) into the loss function and solving for y , $\hat{r}$, the first order conditions are:

$$\begin{aligned} \hat{y}_2 + \Psi\left(\hat{p}_2 - \hat{p}^e\right) &= 0 \\ r &= r^n \end{aligned}$$

where $\Psi = \kappa\frac{(1-\alpha)(1-\gamma)}{\gamma(\alpha+\theta(1-\alpha))}$.

From the this it is clear that optimal MP requires that the real interest rate must be equal to the natural real interest rate. and that inflation is negatively related to output gap.

G Maximization problem of entrepreneurs with credit policy

Recalling the bank loan contract, equation (5.2.8),

$$[1 - F(\bar{\omega}_2)]Z_2B_2 + (1-\mu)\int_0^{\bar{\omega}_2} \omega R_2^k K_2 x_2 dF(\omega) = R_2^l B_t. \tag{G.1}$$

Recalling Z_2 is obtained in equation (5.2.6). Then,

$$x_2 = (\bar{\omega}_2 R_2^k K_2 - R_2 B_2^g)/(\bar{\omega}_2 R_2^k K_2) = 1 - \frac{R_2 B_2^g}{\bar{\omega}_2 R_2^k K_2}, \tag{G.2}$$

and so equation (G.1) becomes,

$$[1-F(\bar{\omega}_2)](\bar{\omega}_2 R_2^k K_2 - R_2 B_2^g) + (1-\mu)\int_0^{\bar{\omega}_2}\left(\omega R_2^k K_2 - \frac{\omega}{\bar{\omega}_2}R_2 B_2^g\right)dF(\omega) = R_2^l B_2.$$

For convenience, this is written as,

$$-\Psi(\bar{\omega}_2)R_2 B_2^g + \left(\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)\right) R_2^k K_2 = R_2^l\left(K_2 - B_2^g - N_{1,e}\right), \tag{G.3}$$

where,

$$\Gamma(\bar{\omega}_2) = \int_0^{\bar{\omega}_2}\omega dF(\omega) + (1-F(\bar{\omega}_2))\bar{\omega}_2, \qquad G(\bar{\omega}_2) = \int_0^{\bar{\omega}_2}\omega dF(\omega).$$

$$\Psi(\bar{\omega}_2) = (1-\mu)\frac{1}{\bar{\omega}_2}G(\bar{\omega}_2) + (1-F(\bar{\omega}_2)).$$

The optimal contracting problem may be now written as:

$$\max_{K_t,\bar{\omega}_2}\mathbb{E}_t\{\left(1-\Gamma(\bar{\omega}_2)\right)R_2^k K_2 + \eta_2\left[-\Psi(\bar{\omega}_2)R_2 B_2^g + \left(\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)\right)R_2^k K_2 - R_2^l B_2\right]\},$$

where $B_2^j = K_2 - B_2^g - N_{1,e}$. The first order condition for $\bar{\omega}_2$:

$$-\frac{\partial\Gamma(\bar{\omega}_2)}{\partial\bar{\omega}_2}R_2^k K_2 + \eta_2\left[-\frac{\partial\Psi(\bar{\omega}_2)}{\partial\bar{\omega}_2}R_2 B_2^g + \left(\frac{\partial\Gamma(\bar{\omega}_2)}{\partial\bar{\omega}_2} - \mu\frac{G(\bar{\omega}_2)}{\partial\bar{\omega}_2}\right)R_2^k K_2\right] = 0. \tag{G.4}$$

The first order condition for K_2:

$$\mathbb{E}_t\left\{\left(1-\Gamma(\bar{\omega}_2)\right)R_2^k + \eta_2\left[(\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2))R_2^k - R_2^l\right]\right\} = 0. \tag{G.5}$$

The first order condition for η_2 yields equation (G.3), where,

$$\frac{\partial\Gamma(\bar{\omega}_2)}{\partial\bar{\omega}_2} = 1-F(\bar{\omega}_2), \quad \frac{\partial G(\bar{\omega}_2)}{\partial\bar{\omega}_2} = \bar{\omega}_2 f(\bar{\omega}_2).$$

$$\frac{\partial\Psi(\bar{\omega}_2)}{\partial\bar{\omega}_2} = (1-\mu)\left(-\frac{G(\bar{\omega}_2)}{(\bar{\omega}_2)^2} + \frac{1}{\bar{\omega}_2}\frac{\partial G(\bar{\omega}_2)}{\partial\bar{\omega}_2}\right) - f(\bar{\omega}_2) = -(1-\mu)\frac{G(\bar{\omega}_2)}{(\bar{\omega}_2)^2} - \mu f(\bar{\omega}_2) < 0.$$

Combining equations (G.4) with (G.5) yields,

$$\mathbb{E}_t\left\{\left(1-\Gamma(\bar{\omega}_2)\right)R_2^k + \frac{1-F(\bar{\omega}_2)}{\Upsilon + 1 - F(\bar{\omega}^j) - \mu\bar{\omega}_2 f(\bar{\omega}_2)}\left[\left(\Gamma(\bar{\omega}_2) - \mu G(\bar{\omega}_2)\right)R_2^k - R_2^l\right]\right\} = 0.$$

where,

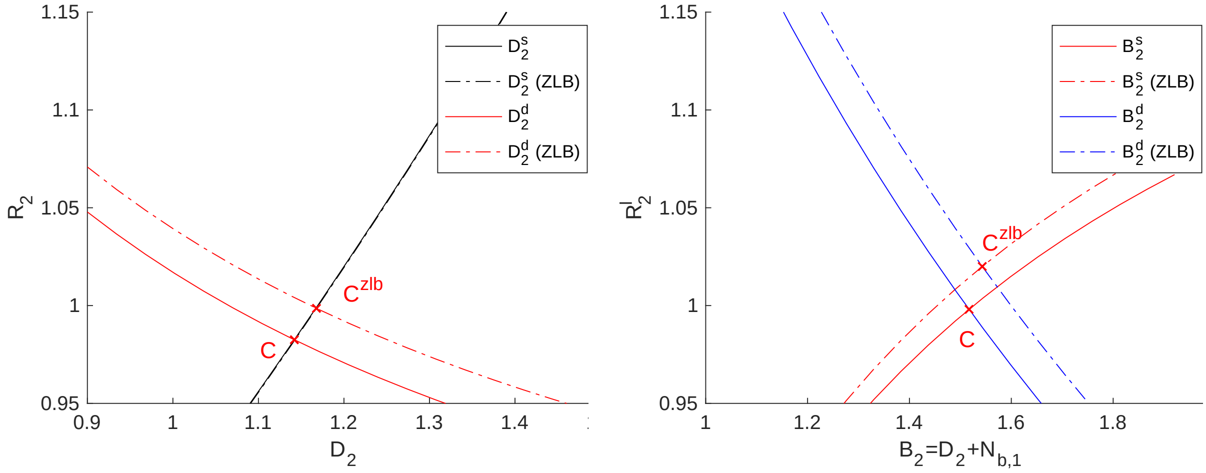
$$\Upsilon = -\frac{\partial\Psi(\bar{\omega}_2)}{\partial\bar{\omega}_2}\frac{R_2 B_2^g}{R_2^k K_2} > 0.$$

H Zero Lower Bound

Figure 7 reports the effects of the ZLB on demand and supply curves in the deposit and credit market and hence their impact on the real interest rate and the return of traditional bank loans. The ZLB avoids a lower nominal interest rate and hence a lower real interest rate, so it pushes up this latter. Since the nominal interest rate cannot adjust, inflation moves. In particular, since the central bank cannot longer achieve its target inflation, and inflation moves above its target value. This higher inflation produces a shift to the right of the credit demand of entrepreneurs. This raises the return of loans and increases the demand curve of deposits of banks, which in turn explains the higher real interest rate in equilibrium.

Figure 7. *Deposits and traditional bank loans market*

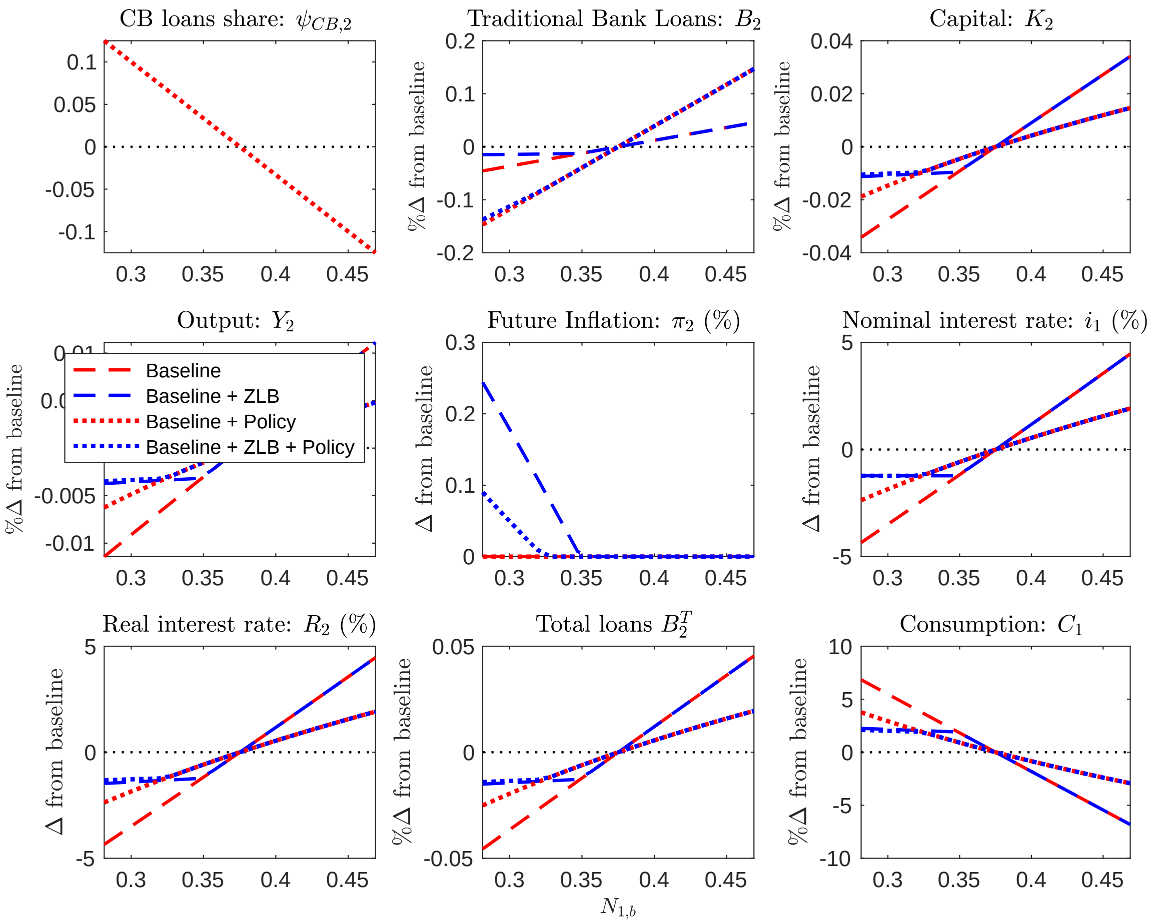
Panel (a) Deposit market | Panel (b) Credit Market



Note: Figure plots the demand and supply curves of deposits and traditional bank loans at a productivity level 2% below its baseline value. At this point the ZLB binds. Point C (C^{zlb}) indicates the equilibrium without policy intervention and without (with) ZLB.

I Figure

Figure 8. *ZLB and Bank Net Worth:*



Note: Figure shows the solutions for different values of $N_{b,1}$, the bank net worth. These go from 0.75 to1.25 times its baseline value. Distance to the $i_{min} = 0$ is 1.23%. Credit policy intervention is "countercyclical". All solutions are identical at baseline calibration. $B_2^T = B_2 + B_2^g$.