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Evaluating Growth-at-Risk as a tool for monitoring macro-financial risks in the Peruvian economy

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Evaluating Growth-at-Risk as a tool for monitoring macro-financial risks in the Peruvian economy*

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Abstract

Growth at Risk (GaR) methodology developed by Adrian et al. (2019) has been of special interest by policymakers since it provides a measure of the relationship among macro-financial variables. GaR requires estimating a set of predictive quantile regressions (QR) where future economic activity (GDP growth) is linked to current financial conditions, measured through a set of alternative market or bank related indicators.

As GaR methodology increased in popularity among policymakers, recent literature has stressed the need of model evaluation of GaR results. For instance, Reichlin et al. (2020) evaluate the out-of-sample performance of a GaR model and find little evidence of predictability beyond what can be achieved using timely indicators of the real economy. Moreover, Brownlees and Souza (2020) use a Garch-type model to forecast the distribution of future economic growth, and compare their forecasting power against GaR model, finding that a Garch-type model outperforms a GaR model.

Taking into consideration the need for a proper evaluation of GaR results, our work implements several model evaluation techniques to increase the accuracy of a Growth at Risk model for the Peruvian Economy. Considering a broad sample of parametric and nonparametric distributions to fit the GaR results, we use log scoring, probability integral transform and entropy tests as model evaluation tools to select the best density forecast that fits Peruvian data. Once we obtain a more reliable GaR results, we use this model to implement a counterfactual analysis to evaluate the impact of Reactiva Peru, a government program that support the credit to firms during the lockdown due the Covid-19 crisis. Our results show that Reactiva Peru had a sizable impact in macroeconomic and financial stability, since it avoided a much deeper decrease in economy activity during the covid-19 crisis.

JEL: C21, C22, C32, C38, C52
Keywords: Growth-at-risk, financial stability, quantile regression

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1 Introduction

In the wake of the Global Financial Crisis (GFC) there has been an increasing interest in understanding the relation between financial conditions and real activity. Beyond academic research, many policy institutions have developed empirical models to identify early signals of financial crises and subsequent large output losses. These models are of great relevance considering the high output losses a financial crisis can generate. For example, Hoggarth et al. (2002) find that, on average, crisis periods result in cumulative output losses of 15-20% of annual GDP. Laeven and Valencia (2013) estimate that output losses during past financial crises across a large sample of countries worldwide amounted on average to 23% of GDP. Lo Duca et al. (2017) estimate that output losses during past systemic financial crises in EU countries amounted to 8% of GDP on average.

The large body of literature on the macro-financial interactions in recessions has traditionally found few robust results on the predictability of real economic activity using financial predictors (such results can be found in Stock and W.Watson (2003), Forni et al. (2003) and Hatzius et al. (2010)). These authors consider different explanations for this finding: for example, financial innovation may cause some financial indicators to lose or acquire predictive power over time; or the relation between financial and real variables may be nonlinear, so that financial variables' predictive power is activated during extreme events as many macro-finance models indeed suggest.

The mechanisms through which financial markets can trigger or amplify business cycle fluctuations have been investigated by economists for a long time. The experience of the last decade, however, has drawn attention to the possibility that the financial accelerator operates in a highly non-linear way, and that financial markets might be prone to 'crises' that generate sharp, long-lasting recessions rather than ordinary business cycles. After the introduction of Basel III, this possibility has become extremely relevant for macro-prudential authorities tasked with preserving the resilience of the financial sector and the stability of credit markets.

For this reason, recently there has been a renewed effort to develop new methods for assessing the risk of large output losses, given financial conditions, rather than focusing exclusively on the prediction of expected growth (mean). In a recent contribution, Adrian et al. (2019) have pioneered this research and suggested an easily implementable method for this purpose, known as Growth-at-Risk (GaR). Focusing on U.S. data, they found that the lower quantiles of GDP growth vary with financial conditions while the upper quantiles are stable over time, thereby pointing to an asymmetric and non-linear relationship between financial and real variables.

GaR methodology developed by Adrian et al. (2019) requires estimating a set of predictive quantile regressions (QR) where future economic activity is linked to current financial conditions, measured through a set of alternative market or bank related indicators. The quantile regression setup allows modeling the relation between financial markets and real economy in a flexible way, allowing for the possibility of a stronger correlation arising in bad times. One of its key advantages is that no restrictions are imposed a priori on the nature of these non-linearities.

Building on this work, several recent papers have explored the idea, while policy institutions have adopted the methodology to monitor risk in different countries. In particular, the IMF uses this method in its Financial Sector Assessment Program for countries under monitoring. In addition to, a growing number of coutries have implemented this methodology for financial stability purposes. Moreover, Gondo (2019) and Superintendency of Banking and Insurance (2019) have estimated GaR with Peruvian data.

However, as Reichlin et al. (2020) pointed out, in practice the value of this policy framework rests on whether the dynamics of the moments of the conditional distribution of GDP can be captured with some degree of accuracy and on whether there is some out-of-sample predictability for moments other than the mean. Reichlin et al. (2020) evaluate the out-of-sample performance of a GaR model and find little evidence of predictability beyond what can be achieved using timely indicators of the real economy. Moreover, as an alternative to using quantile regression as in Adrian et al. (2019), Brownlees and Souza (2020) use a Garch-type model to forecast the distribution of future economic growth and compare the forecasting power among these two methodologies, finding that a Garch-type model outperforms a QR model. These results raise concerns about using a specific methodology without proper evaluation of its reliability as a tool for monitoring financial risks.

The purpose of this paper is to develop a reliable GaR model that can be included in the BCRP's toolkit for monitoring financial stability risks (which already includes a financial conditions index and a heat map for the financial system, among others), incorporating several model validation techniques to assess the correct specification of the forecasted densities from QR results.

Once we obtain a more reliable GaR results, we use this model to implement a counterfactual analysis to evaluate the impact of Reactiva Peru, a government program that support the credit to firms during the lockdown due the Covid-19 crisis. Our results show that Reactiva Peru had a sizable impact in macroeconomic and financial stability, since it avoided a much deeper decrease in economy activity during the covid-19 crisis.

The remainder of this paper is organized as follows: section 2 explain the methodology used in the paper, Section 3 presents the data used and results from the GaR model, section 4 presents a policy analysis of an credit program using the GaR results and section 5 concludes.

2 Methodology of Growth-at-risk Model

Following Prasad et al. (2019) we can summarize the Growth-at-risk in three steps:

1. Using dimensionality reduction techniques to obtain a group of factors that summarize a broad set of macrofinancial variables.

2. Using Quantile Regression estimation to forecast the quantiles of the distribution of GDP growth using the factors from previous step as regressors.

3. Using density estimation techniques to obtain a distribution that fits the quantiles estimated in the previous step.

In addition to these steps, we included a fourth step that need to take into account for a more reliable results.

4 Implementing different model evaluation criteria for selecting the density that best fit the Peruvian data.

2.1 Reduction of dimensionality

The first step implies using a dimensional reduction technique to summarize the information of a broad set of macrofinancial variables into a small number of factors. In particular, given a set $Z_{i,t}$ that includes a broad set of macrofinancial variables, we use Orthogonal Projection for Latent Structures (O-PLS) to estimate a small set of factors. Unlike calculating the factors through standard principal component model (PCA), the O-PLS model allows the correlation between financial variables and a target variable to be used for determining the factors, thus increasing their predictive power.

Given a set of financial variables Z_i represented in matrix form as Z, which are used to explain a target variable W, we want to solve the following problem:

$$\boldsymbol{Z} = \boldsymbol{X}\boldsymbol{\lambda}^{\top} + A$$

$$\boldsymbol{W} = \boldsymbol{X}\boldsymbol{\beta}^{\top} + B$$

O-PLS proposed by Trygg and Wold (2002) developed an algorithm to solve for the factors X and loadings λ that maximize the covariance between X and W. These method made possible to reduce the dimension of the set of explanatory variables into a reduce set of factors to be used in the Quantile Regression estimation.

2.2 Quantile Regressión estimation

Quantile Regression (Koenker and Bassett, 1978; Koenker, 2005) allow to map a set of regressors $X_{i,t}$ to the quantiles of the distribution of a dependent variable Y_{t+h}. In particular Quantile Regression solves the following optimization problem:

$$\hat{\beta}_q = \underset{b}{\operatorname{argmin}} \sum_{t=1}^{T-h} \rho_q(Y_{t+h} - X_t' b)$$

$$\rho_q(u) = u(q - \mathbf{1}_{\{u<0\}})$$

Where $\hat{\beta}_q$ is referred as the $p-th$ regression quantile and $\rho_q(u)$ is known as the check function. Once the optimization problem is solved we can obtain the quantiles of the distribution of Y_{t+h} as $X_t'\hat{\beta}_q$.

Following Adrian et al. (2019) GAR model requires to estimate the following equation by Quantile Regression (QR):

$$Y_{t+h}^q = \alpha^q + \beta_1^q X_{1,t} + \beta_2^q X_{2,t} + \beta_3^q X_{3,t} + \beta_4^q X_{4,t} + \beta_5^q X_{5,t} + \beta_6^q Y_t$$

where:

- Y_{t+h}^q corresponds to the p-percentile of the projected cumulative GDP growth in the period $t+h$.
- $X_{i,t}$ corresponds to the factors obtained using O-PLS
- Y_t correspond to GDP growth at period t.
- β_i^q represents the contribution of factor i in the q-percentile projection of cumulative GDP growth distribution.

2.3 Density forecast

Once obtained the quantiles of future GDP growth Y_{t+h}^q, we proceed to fit a density function at each forecasting period h. Different from Adrian et al. (2019) who are fitting conditional quantiles to a parametric density function. here we work at the sample level, so we have many more choices of fitting.

We start interpolating the QR estimation results to obtain a continuous quantile function with uncrossing property, following Schmidt and Zhu (2016) and Chernozhukov et al. (2010) using a gaussian interpolation method. following this approach, we obtain a large sample of the distribution of Y_{t+h} (i.e. $\{Y_{t+h}^{(i)}\}$ with i in $\{1, ..., N\}$). we use the this new sample to fit alternative of family distributions (non parametric, parametric and mixture of normal) aiming to obtain the distribution of conditional forecast of GDP growth at different horizons.

Nonparametric density fitting includes kernel density estimation (KDE). Let $(Y_{t+h}^{(1)}, Y_{t+h}^{(2)}, ..., Y_{t+h}^{(N)})$ be the sample obtained by interpolation of QR estimation, we assume this sample has an unknown density function $f(Y_{t+h})$. The Kernel Density Estimator (KDE) of $f(Y_{t+h})$ is:

$$f(Y_{t+h}; g) = \frac{1}{Ng} \sum_{i=1}^{N} K(\frac{Y_{t+h} - Y_{t+h}^{(i)}}{g})$$

where $K()$ is the kernel function (i.e., gaussian function) and g is a smoothing parameter named *bandwidth*, which can be estimated optimally using a cross validation criteria.

Parametric density fitting implied using a family of density function to fit the interpolated sample of QR estimation. The broad set of Parametric densities considered here includes normal, non centered t-student, skew normal, beta, Weibull, Gumbel, among others. Here, we obtain the parameters of each density by MLE.

Finally, we included mixture of normal density as an alternative to fit the sample of QR estimation.

$$f(Y_{t+h}; \theta) = \sum_{k=1}^{K} p_k \mathcal{N}(Y_{t+h}; \mu_k; \sigma_k)$$

where

$$\mathcal{N}(Y_{t+h}; \mu_k; \sigma_k) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{1}{2}\sum_{i=1}^{N}\left(\frac{Y_{t+h}^{(i)} - \mu_i}{\sigma_i}\right)^2}$$

Parameters of this mixture of normal $(p_k; \mu_k; \sigma_k)$ densities are estimated by MLE, considering

K=2.

2.4 Model Evaluation

Density forecasts from the Growth at Risk model play such an important role in providing information on the uncertainty related to the economic growth forecast, and therefore it is crucial to evaluate whether Growth at Risk models are well specified. If density forecasts from GaR model are not correctly specified, then the measure of uncertainty that they provide is incorrect.

As was pointed out earlier, to make the Growth at risk results more reliable we need to include some additional evaluation tools that make possible to select the best among several alternatives we estimated.

2.4.1 Log score comparison via Diebold and Mariano test

Following Amisano and Giacomini (2007) and Diks et al. (2011), let $\hat{f}(Y_{t+h})$ and $\hat{g}(Y_{t+h})$ two different density forecasts and define $S(\hat{f}, Y_{t+h})$ as the score rule of the form:

$$S(\hat{f}, Y_{t+h}) = log[\hat{f}(Y_{t+h})]$$

then the log score difference is define as

$$d_{t+h} = S(\hat{f}, Y_{t+h}) - S(\hat{g}, Y_{t+h})$$

with the mean score difference as:

$$d_{m,n} = \frac{1}{n} \sum_{t=m}^{T-h} d_{t+h} \;\; with \;\; n = T - m$$

therefore, it is possible to implement a Diebold-Mariano type of test (Diebold and Mariano, 1995):

$$t_{m,n} = \frac{d_{m,n}}{\sqrt{\frac{\hat{\sigma}^2_{m,n}}{n}}} \sim \mathcal{N}(0, 1)$$

2.4.2 Probability Integral Transform

Diebold et al. (1998) initiated the use of Probability Integral Transform to evaluate a correct specification of a density forecast model. A probability integral transform (PIT) is the cumulative probability evaluated at the realized value of the target variable. It measures the likelihood of observing a value less than the actual realized value, where the probability is measured by the density forecast.

Let $f_t(Y_{t+h})$ the forecasted density function of a random variable Y_{t+h} from Growth at Risk model, then the cumulative density function (CDF) can be represented as:

$$F_t(Y_{t+h}) = \int_{-\infty}^{Y_{t+h}} f_t(z)\, dz$$

Using this CDF, the Probability Integral Transform (PIT) is defined as the transformation of the random variable Y_{t+h}:

$$U_{t+h} = F_t(Y_{t+h})$$

Diebold et al. (1998) demonstrate that the PIT is uniform, independent and identically distributed if the density forecast is correctly specified. Therefore, Diebold et al. (1998)

propose to test the correct specification of density forecasts by testing whether the PIT is uniformly distributed and independent,i.e., the sequence of all U_{t+h} is *iid* Uniform (0,1) and its cumulative distribution is the 45-degree line.

Moreover, (2019) consider testing how close is the CDF of the density forecast to an uniform distribution via a Kolmogorov-Smirnov (KS) test. This is the approach we will implement here.

2.4.3 Downside and Upside Entropy

Policymakers are often concerned with the downside and upside risks when forecasting a key economic variable. In this context, they are interested on how vulnerable the predicted path of GDP growth is to unexpected shocks. Following Adrian et al. (2019) we quantify downside and upside vulnerability of future GDP growth as the additional probability mass that the conditional density assigns to extreme right and left tail outcomes relative to the probability of these outcomes under the unconditional density (time-invariant density of GDP Growth), i.e., downside and upside entropy.

Let $\hat{g}(y_{t+h})$ the unconditional empirical distribution and $\hat{f}(y_{t+h})$ the estimated conditional distribution, we define the downside, $\mathcal{L}_t^D$, and upside, $\mathcal{L}_t^U$ entropy as:

$$\mathcal{L}_t^D = -\int_{-\infty}^{F_t^{-1}(0.5)} (log\hat{g}(y) - log\hat{f}(y))dy$$

$$\mathcal{L}_t^u = -\int_{F_t^{-1}(0.5)}^{\infty} (log\hat{g}(y) - log\hat{f}(y))dy$$

3 Data and results

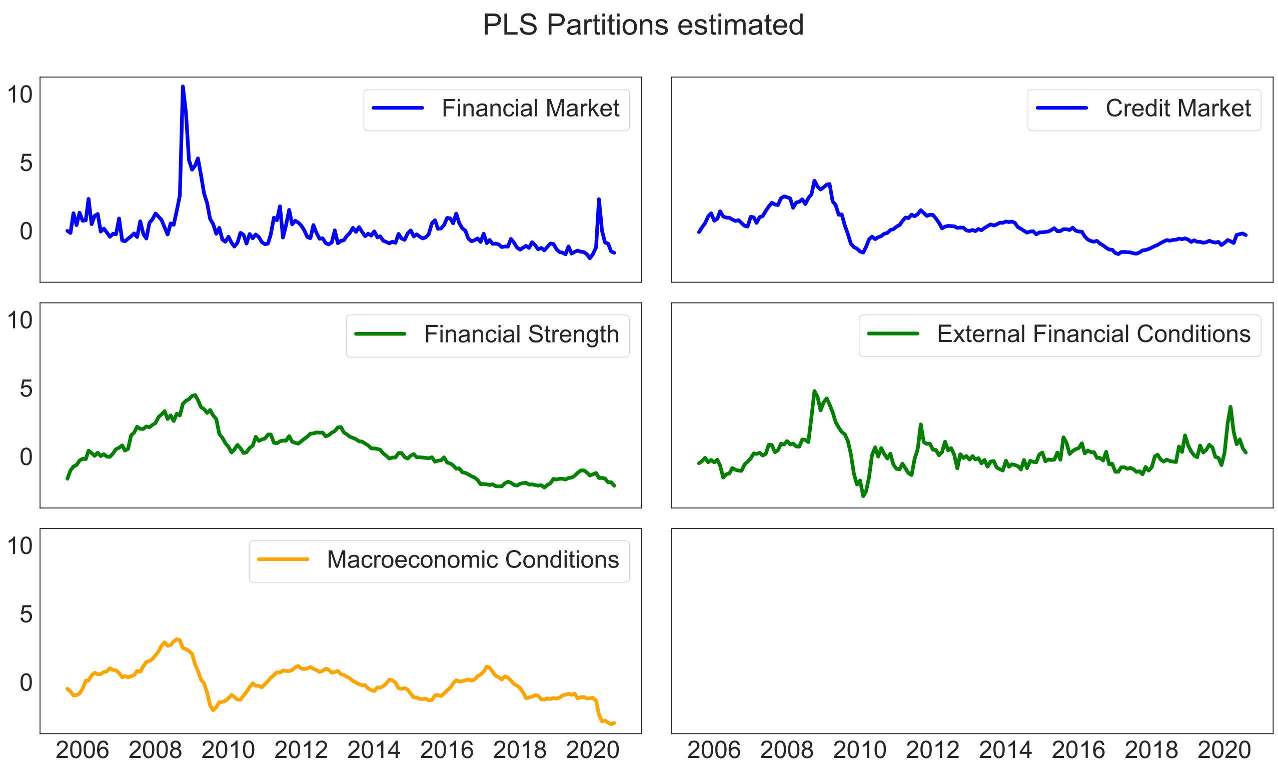
We use macrofinancial variables using Peruvian data at a monthly basis from from August 2005 to August 2020. Table 1 includes the list of variables used for GaR estimation grouped within 5 sectors of the Peruvian financial system.

Table 1: Partition groups (factors) and target variables

Factor	**Credit Market**	**Financial Market**		**Financial Strength**		**External Financial Conditions**		**Macroeconomic Conditions**
Target	**Credit to businesses**	**Credit to businesses**		**Credit to businesses**		**Credit to businesses**		**GDP**
Variables	Credit to businesses Household credit	EMBIG Return IGBVL Volatility BVL Liquidity BVL	CDS Pension fund ret Spread node 10 Non-resident holdings	Financial income Opex Net profit Equity capital	Dependence on external funding Liquid assets/ Short-term Liab	Copper S&P500 USA Spread	VIX Global spread Index	Terms of trade Inflation Exchange rate Monetary stimulus

As Table 1 shows, each group includes macrofinancial variables that will have an impact on GDP growth at different horizons in the future. Under the methodology implemented in this paper, the first step is use O-PLS estimation to obtain a factor that summarize the information in each group. As it was pointed out before O-PLS required a target variable to increase the forecasting power. Therefore, for the credit markets, financial markets financial strength and external financial conditions, the target variables is credit to businesses while for macroeconomic conditions the target variable is GDP growth.

Figure 1: Evolution of the O-PLS Partition groups, $X_{i,t}$



In step 2, GAR model requires to estimate the following equation by Quantile Regression (QR) for cumulative GDP growth at period $t+h$ using the estimated factors as regressors plus the GDP growth in time t:

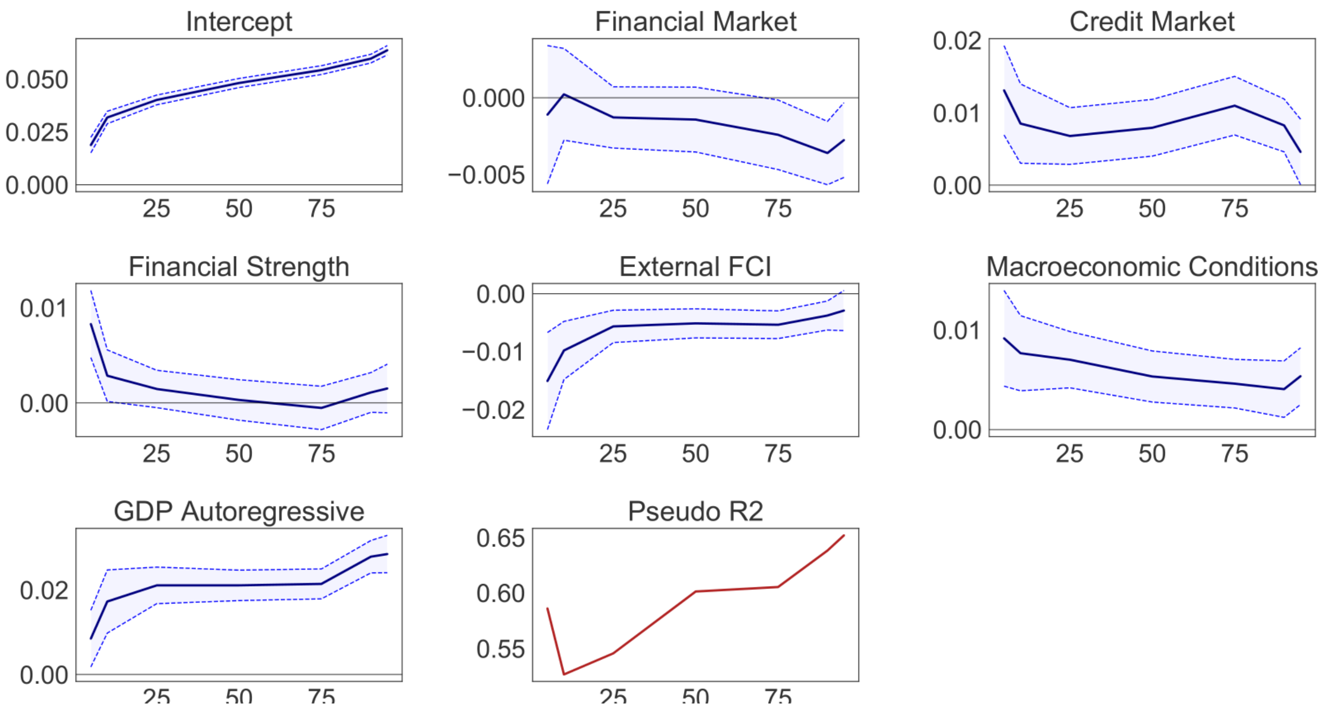
$$Y^q_{t+h} = \alpha^q + \beta^q_1 X_{1,t} + \beta^q_2 X_{2,t} + \beta^q_3 X_{3,t} + \beta^q_4 X_{4,t} + \beta^q_5 X_{5,t} + \beta^q_6 Y_t$$

Where the set of regressors are as following:

$$\begin{bmatrix} X_{1,t} \\ X_{2,t} \\ X_{3,t} \\ X_{4,t} \\ X_{5,t} \\ Y_t \end{bmatrix} = \begin{bmatrix} \text{Credit market} \\ \text{Financial market} \\ \text{Financial strength} \\ \text{External financial conditions} \\ \text{Macroeconomic conditions} \\ \text{Current GDP growth} \end{bmatrix}$$

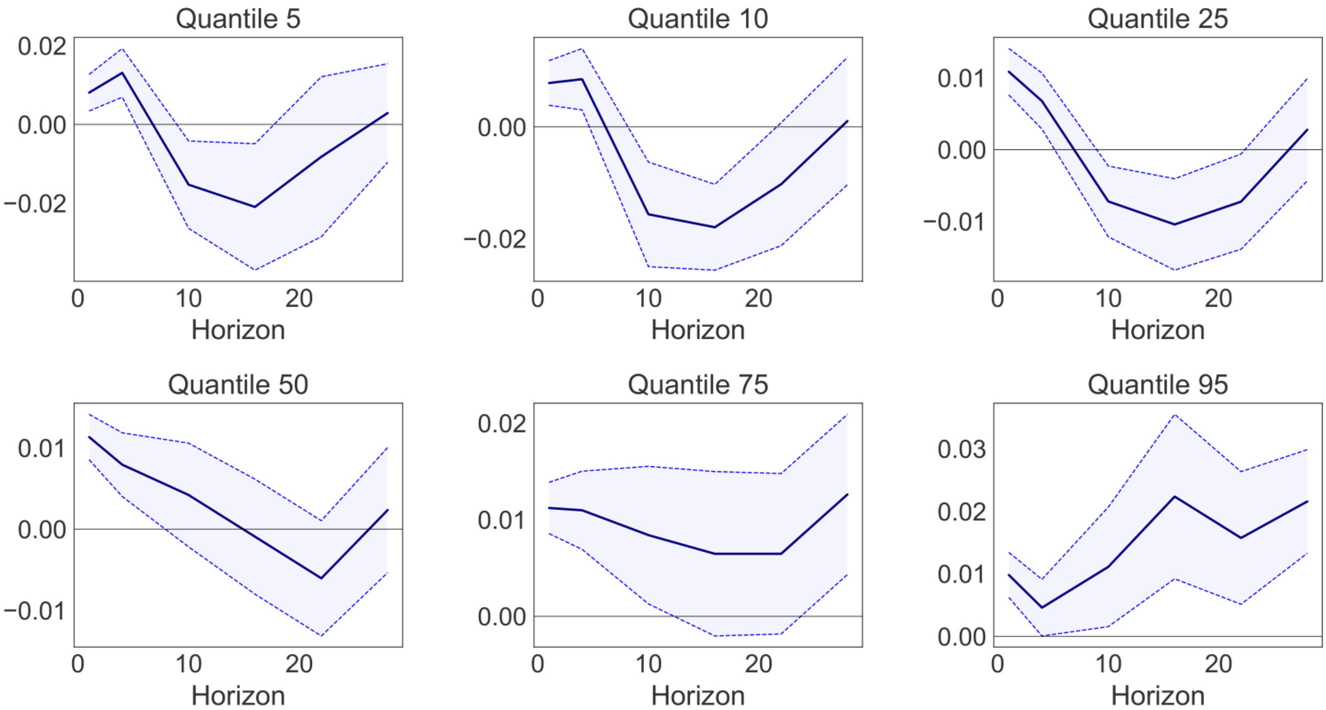
Figure 2 show the results of QR estimation at horizon $t + 4$ corresponding to the GDP growth of year 2020. Overall, there is a heterogeneous effect of the macrofinancial factors on the different quantiles of the distribution of future GDP growth. Interestingly, the negative impact of external financial conditions is more extreme in the lower quantile of the distribution of GDP growth, which is consistent with the literature of amplification effect of foreign shocks on emerging markets.

Figure 2: Quantile coefficients of the O-PLS factors 4-month horizon (Confidence interval at 5%)



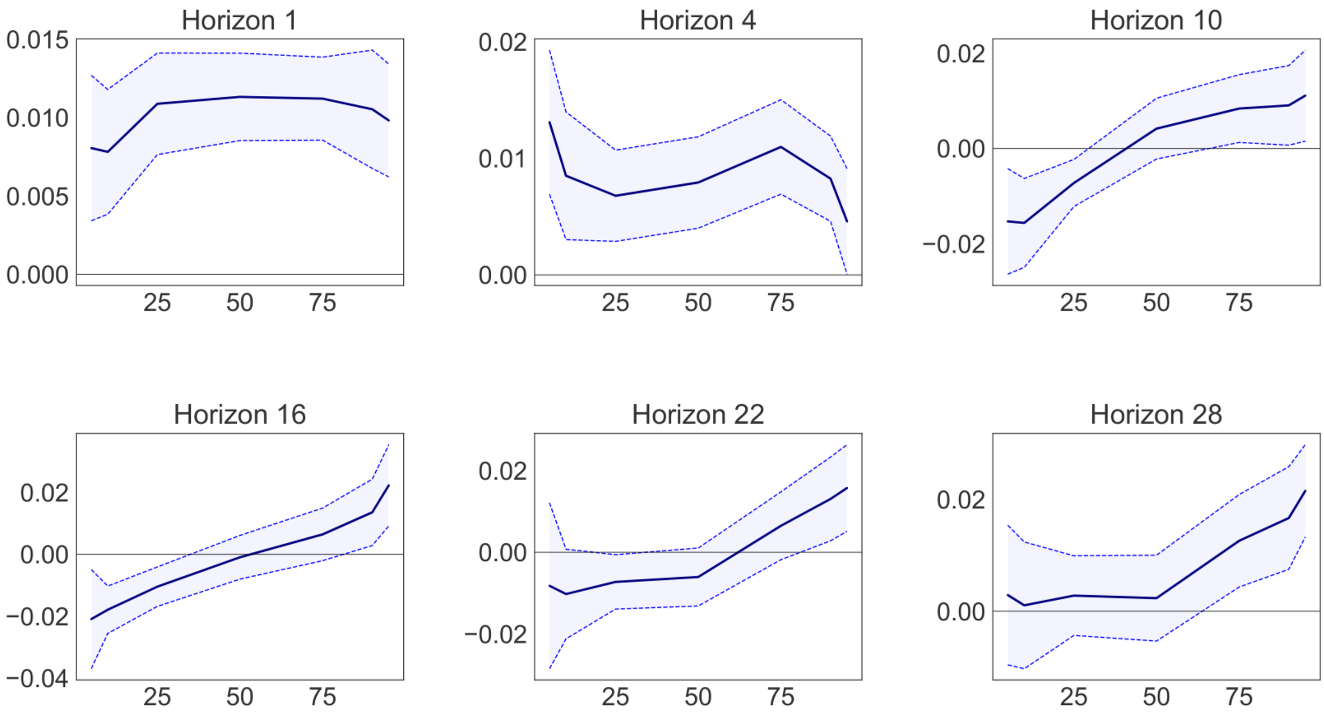
From the QR estimation results we can also highlight the differentiated effect of the macrofinancial factors on future GDP growth at different horizons. For instance, Figure 3 shows the quantile coefficients at different horizon for the credit market factor. From here, we can observe a positive impact of credit market in the near future, but there is a negative impact at a longer horizon. This result is consistent with the literature of leverage cycles. For lower quantiles there is a positive effect of credit markets on future GDP, so during episodes where GDP is weak, more credit can help to support the economy, but at longer horizon the effect on GDP is negative, which can be interpreted as building up more risks as larger level of credit can results in an increase of overindebtedness.

Figure 3: Term structure of Quantile Coefficients for credit market factor (Confidence interval at 5%)



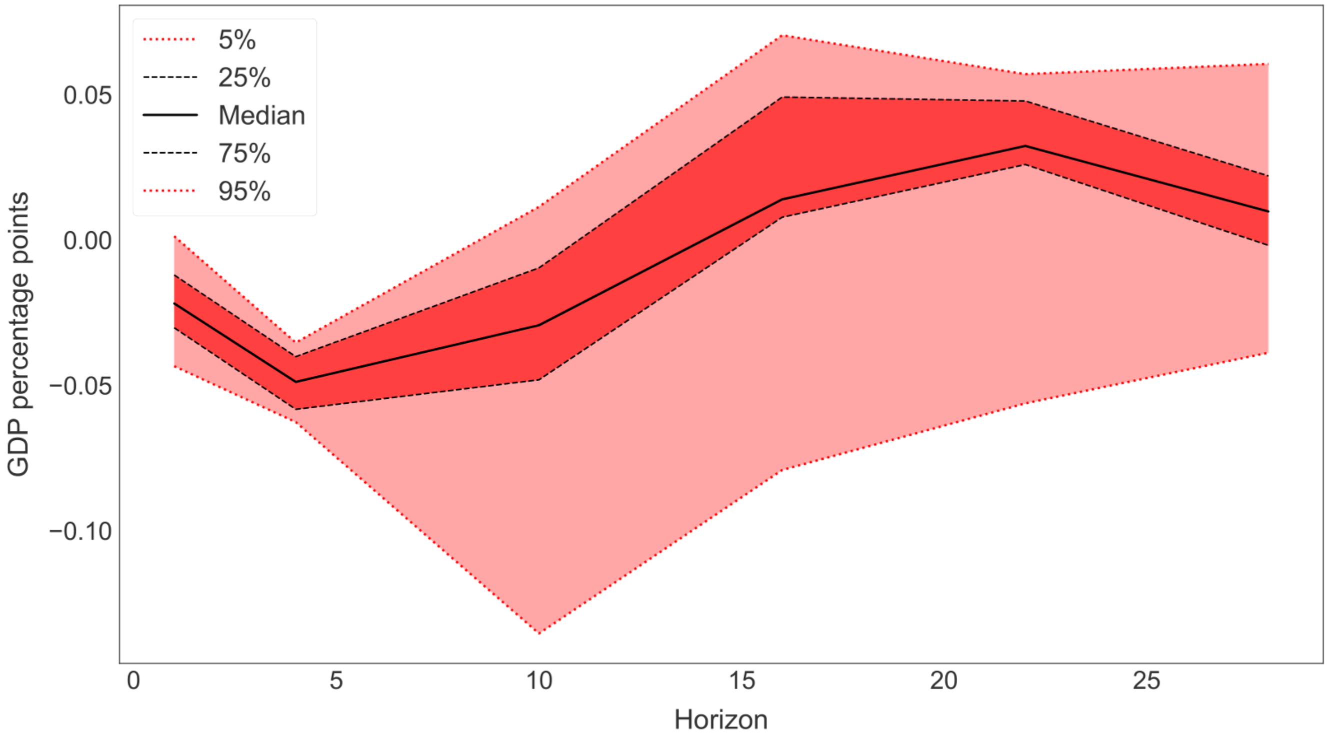
Regarding the heterogeneous effect of an increase in domestic credit, Figure 4 shows a positive effect in the short run with no much difference across quantiles. However, when we analyze longer horizons (above 10 months) the effect of increasing credit depends on the levels of GDP Growth. For lower quantiles this effect is negative, but for larger quantiles the effect is positive and significant. These results suggest that increasing domestic credit can offer support to the economy in short run, but also it increase the vulnerability of financial institutions that can pose risks to economic activity later on.

Figure 4: Quantile coefficients at different horizons (Confidence interval at 5%)



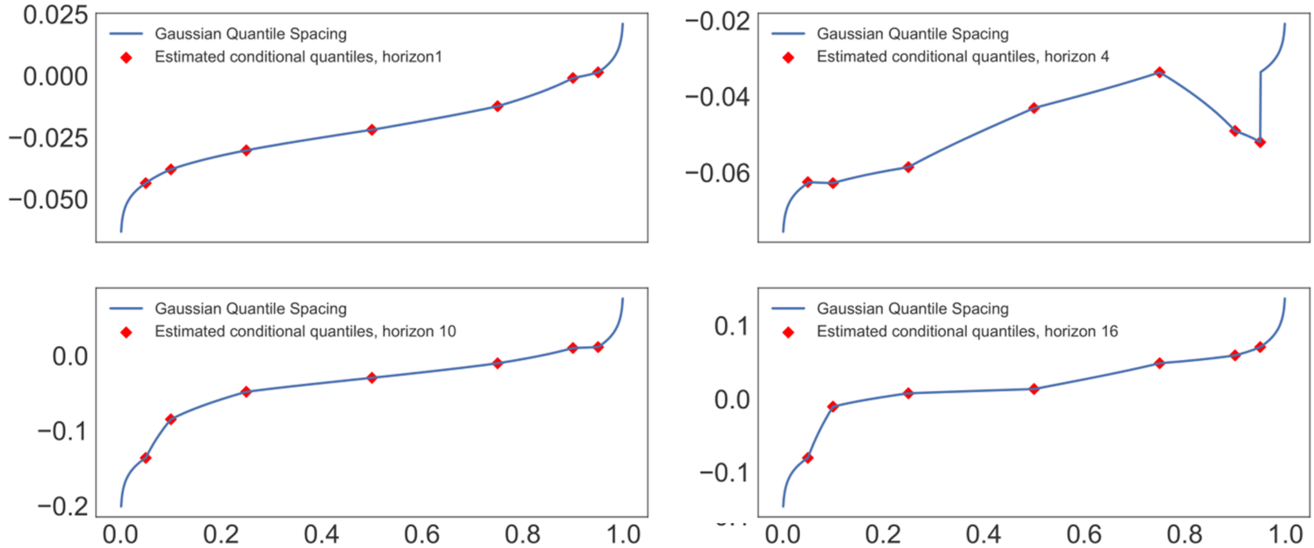
From the QR estimation we can obtain predictions of GDP growth at different horizons, which can be summarized in a fan chart as showed in Figure 5. Fan chart for GDP growth shows a negative growth rates up to ten months ahead related to the effect of Covid-19 crisis. as the forecasting period increases growth rates become positive but with higher uncertainty. Specially, lower tail increases, which can be interpreted as increasing downside risks to economic recovery.

Figure 5: Fan chart of QR results



Different from Adrian et al. (2019) that fit a parametric distribution to the QR results, we obtain a large sample from the estimated quantiles of GDP growth following Schmidt and Zhu (2016) and then we use this large sample to fit a broad set of probability distributions, such as non-parametric distributions (KDE), several parametric distributions and mixture of normal distribution. Figure 6 shows the quantile interpolation for different horizons, which is needed to obtain a large sample of GDP growth used to fit the different pdf we use in this paper.

Figure 6: Quantile interpolation and sampling



3.1 Nonparametric density fitting

This first family of distributions we used to fit the draw from the QR results is Gaussian Kernel. Figure 7 show the sample histogram of the draw from QR results and its corresponding Kernel Density Estimator (KDE) for h periods ahead (equivalent to year 2020 GDP growth). We can see that KDE show a pdf which is bimodal. However, as can be shown in Figure 8, the shape of the pdf from KDE depends on the bandwidth parameter. A larger bandwidth parameter for the Gaussian KDE will result in a unimodal pdf. Therefore, we use a cross validation criteria for selecting the right bandwidth for the KDE.

Figure 7: Histogram and Gaussian Kernel for GDP 4 months ahead

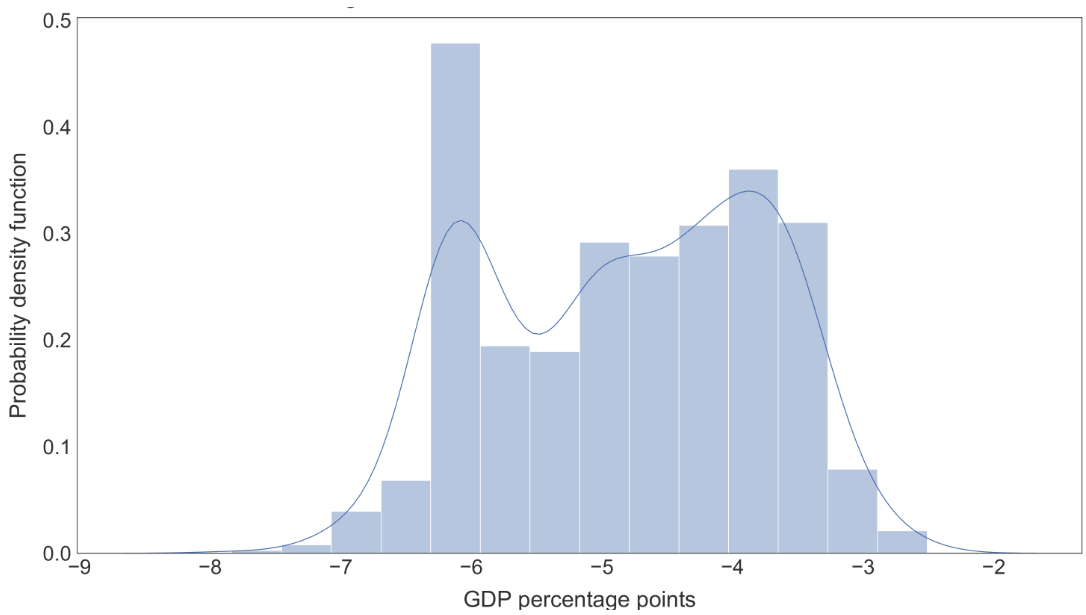
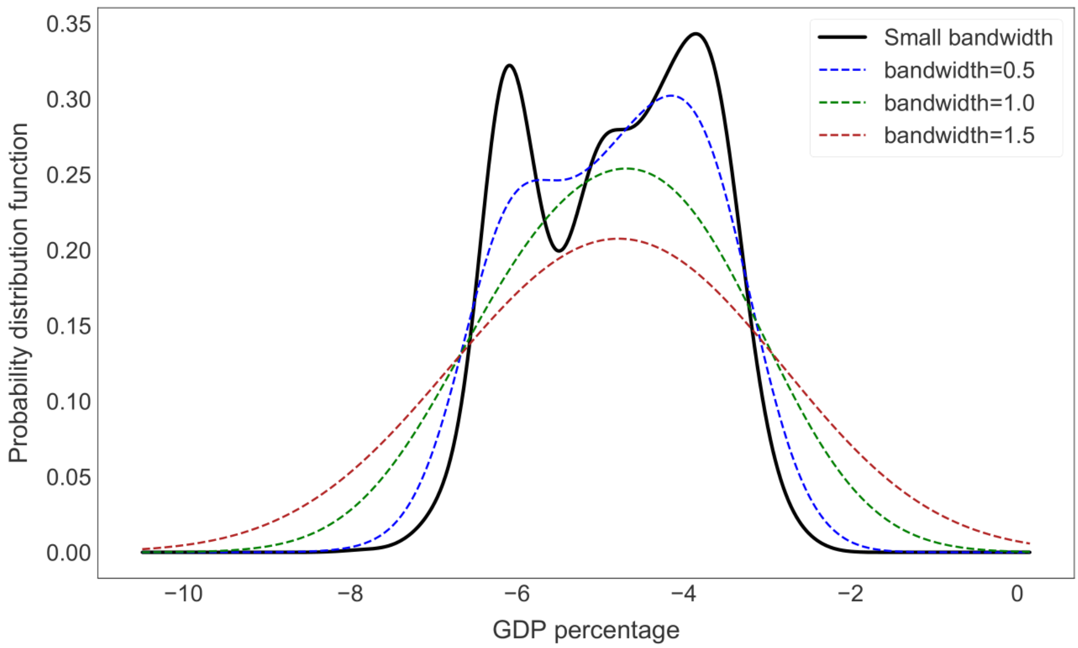
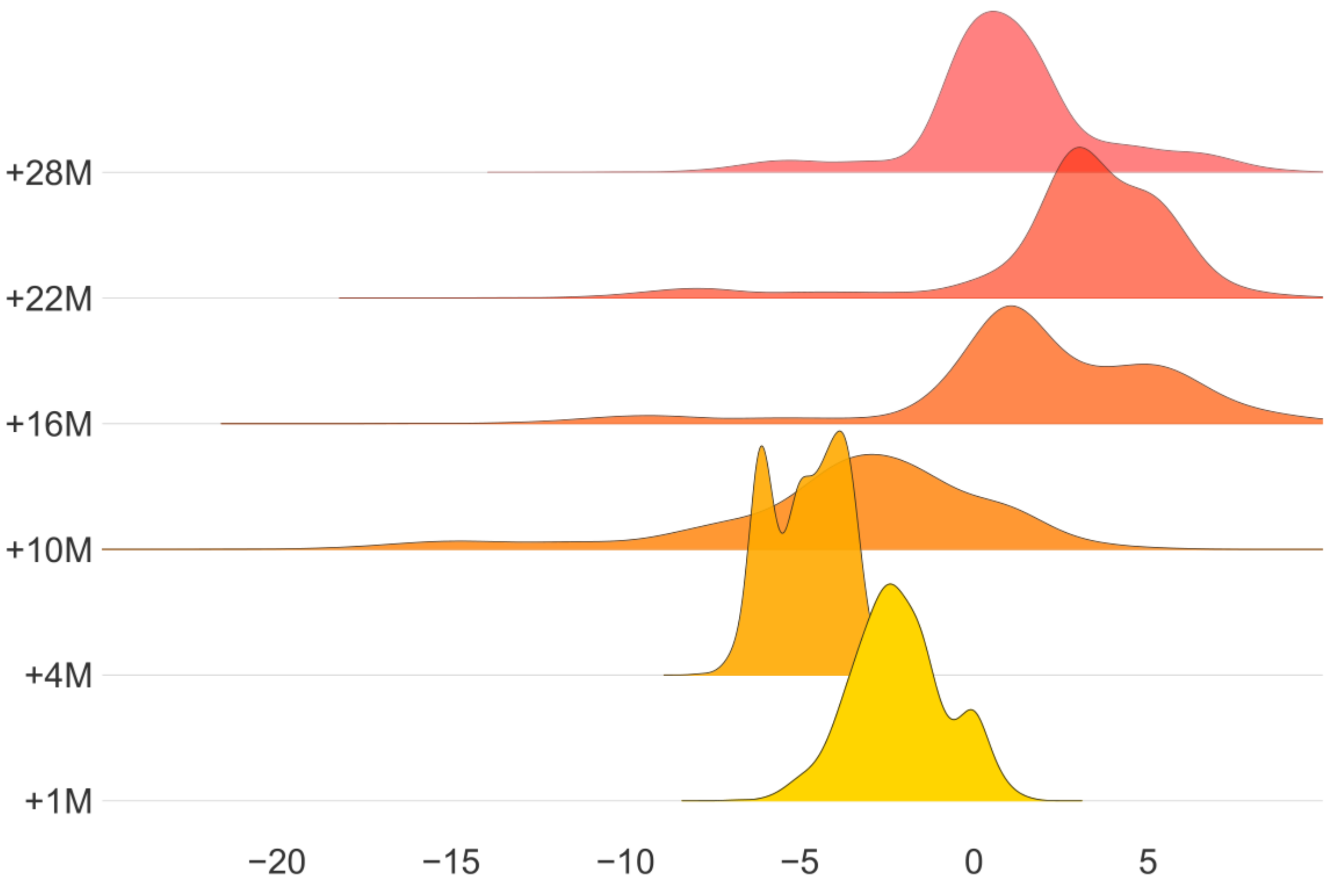


Figure 8: Gaussian Kernel Fit with Different Bandwidths 4 months ahead



After fitting the gaussian kernel distribution for each forecasting horizon, Figure 9 shows the term structure of the distribution of GDP growth, which shows that GDP growth is in negative territory for most of 2020 and recovering faster after a year. This is consistent with an economy heavy hit by the pandemic and the lockdown measures during 2020 and an expected recovery once most of the businesses will fully reopen supporting the economic recovery.

Figure 9: Term Structure of GDP at Risk in Peru, Gaussian Kernel Fitting



3.2 Parametric density fitting

A different approach respect to the previous result is to fit parametric distributions to the sample from QR results. Figure 10 shows the main pdf used to fit the sample of QR results, and includes Normal, Beta, Wilbull, Skew Normal, Noncentered t-student and Gumbel distribution. In order to select from the group of parametric distributions, we need a selection criteria. Figure 10 shows a comparison of the main parametric distributions that fit the draw from QR results including Residual Sum of Square (RS) as a selection criteria. We can see from the results that according with RS criteria beta distribution es the best parametric distribution for horizon h=4. In addition to, we also used additional selection criteria such as Aiken information criteria and bayesian information criteria (Figure 11), which also show that beta distribution is the best pdf that describe the draw of QR results.

Figure 10: Parametric distribution fitting: comparisons among different parametric pdf (4-month ahead)

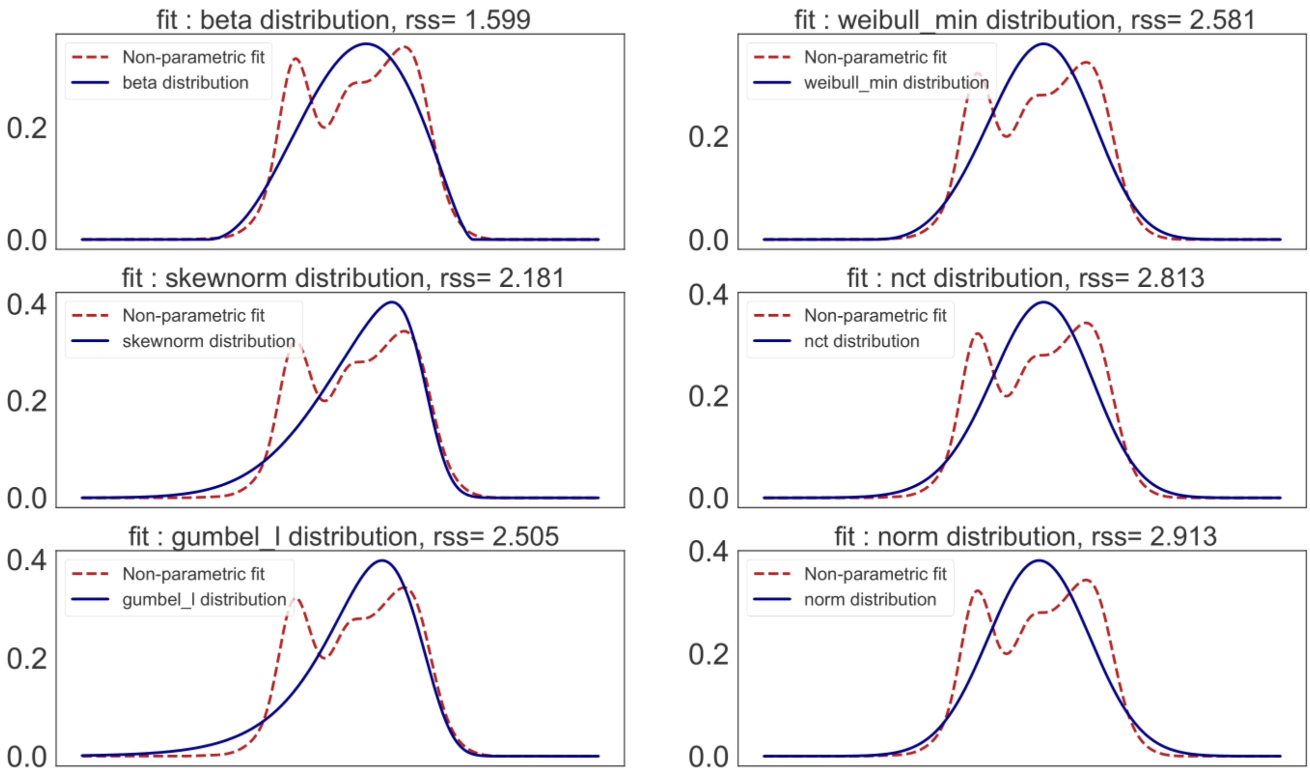


Figure 11: Parametric distribution fitting: selection criteria (4-month ahead)

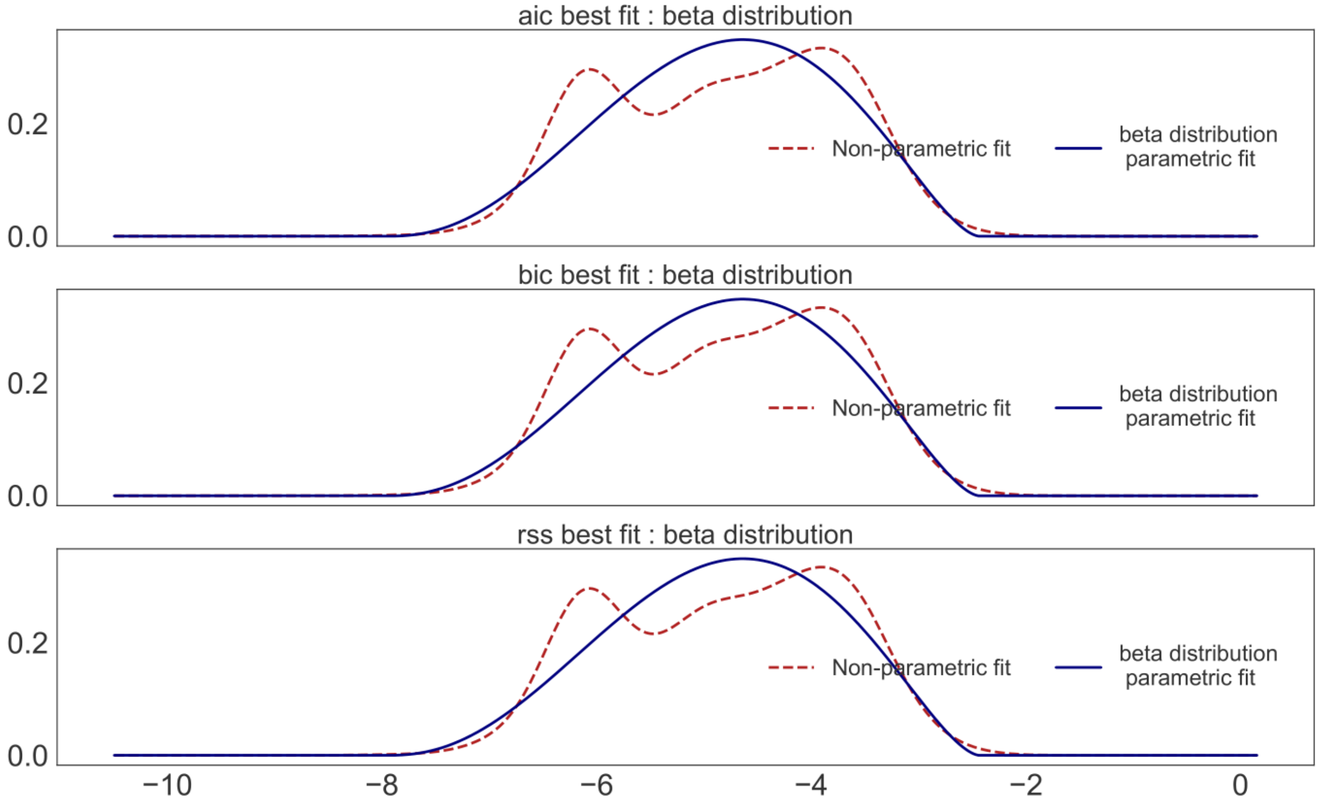
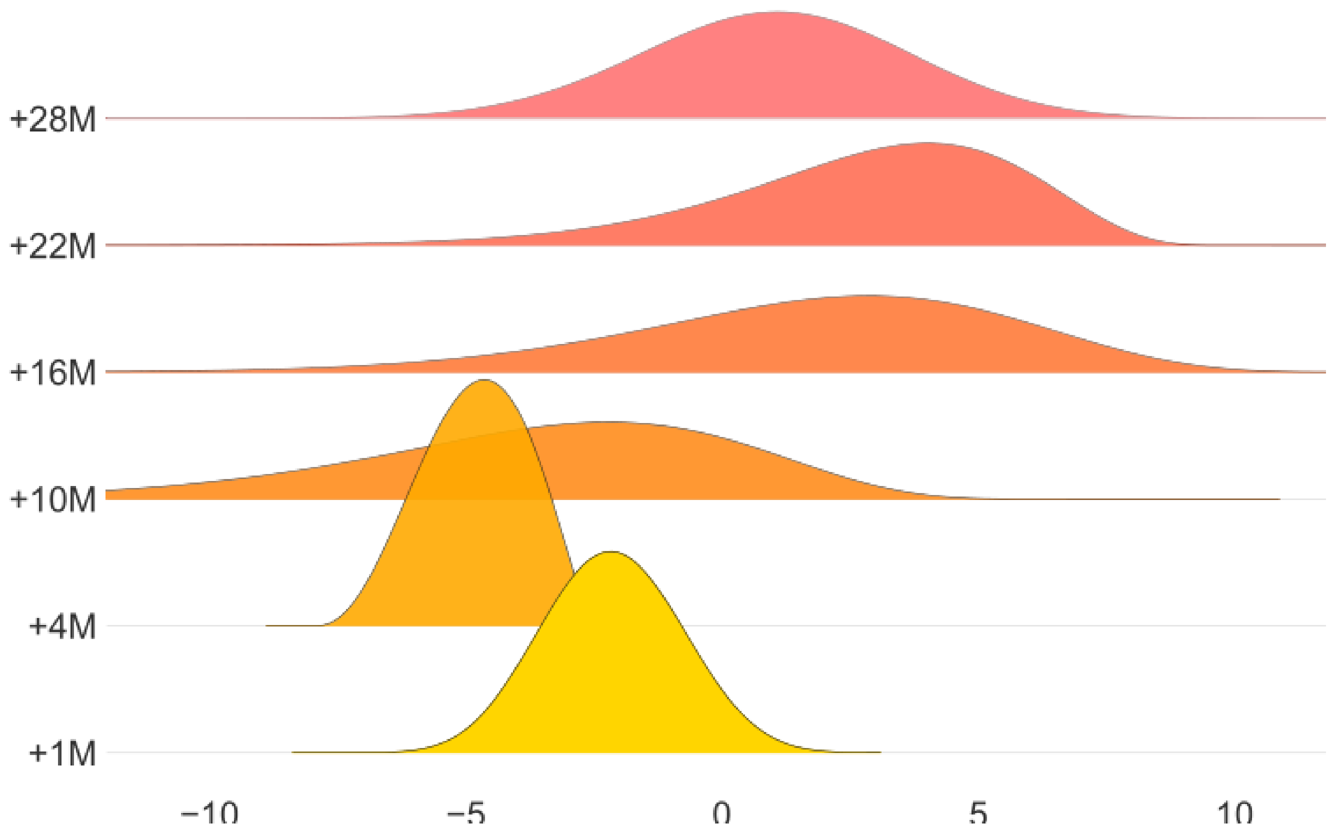


Figure 12 shows the term structure of GDP at risk across different horizons, which shows again that GaR model is forecasting GDP growth in negative territory during the 2020 and a economic recovery since 2021, although results also show there is a increase in uncertainty at longer horizons.

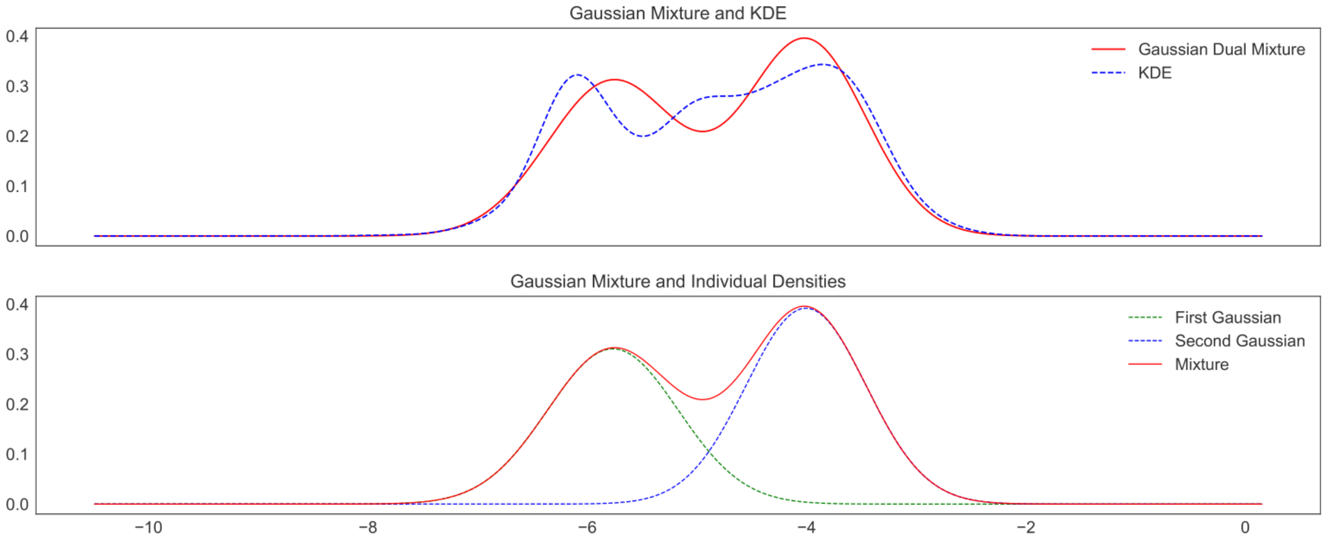
Figure 12: Term Structure of GDP at Risk in Peru,Parametric Fitting



3.3 Mixture of normal density fitting

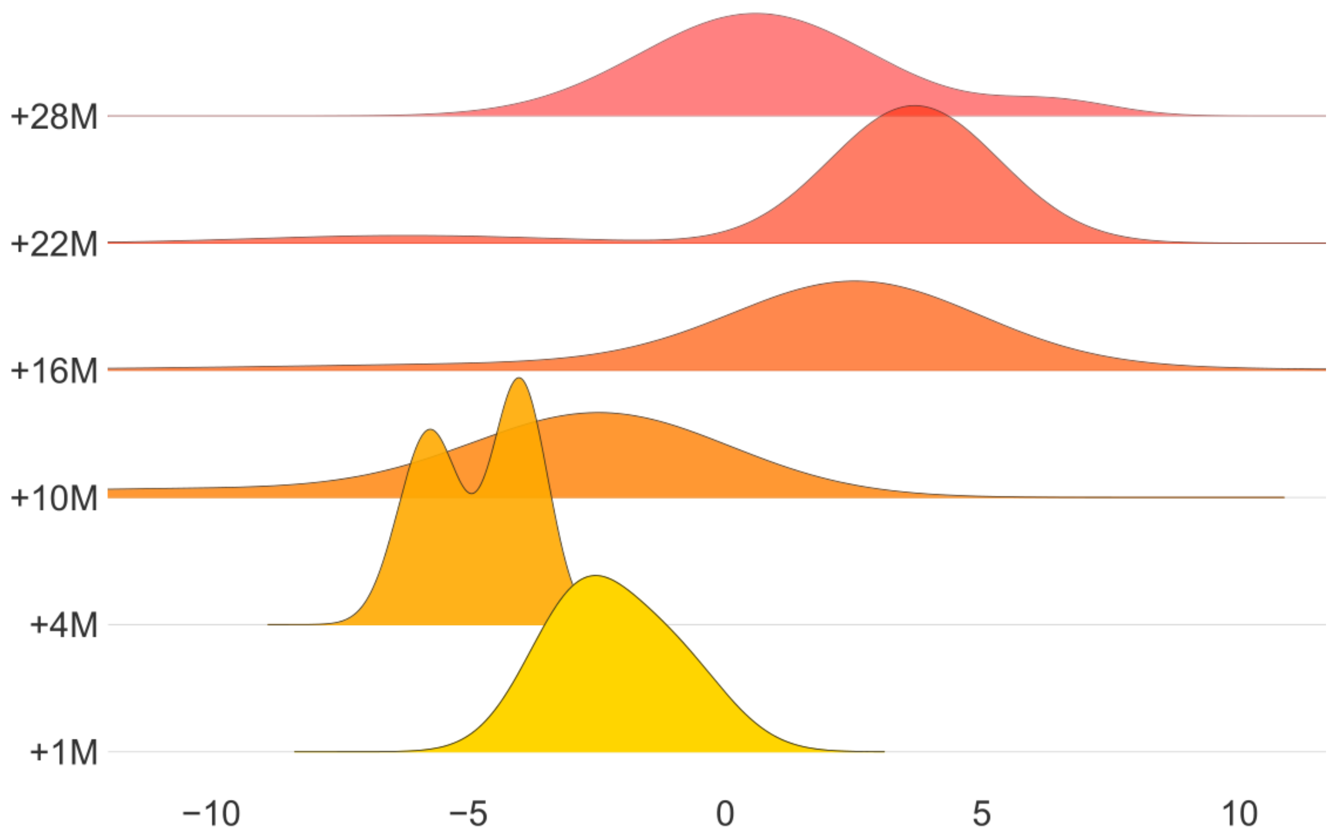
As an alternative to a single parametric distribution representing the distribution of future GDP, we include a mixture of normal distribution. This will help to capture multimodality in the sample from QR results. For this analysis we only included a mixture of two normal distribution to obtain additional flexibility when fitting to a sample of GDP growth. Figure 13 shows that mixture of normal pdf can be useful when the empirical distribution is multimodal and asymmetric, which is the case for GDP growth at $h = 4$ (cumulative year 2020).

Figure 13: Mixture of Normal density fitting (4-month ahead)



Using the Mixture of normal pdf, Figure 14 shows that during 2020 economy growth was in negative territory , consistent with the previous results. Moreover, from 2021 Peruvian GDP is going to experience a recovery path. Comparing the results among these different distributions we estimated, they show similar results qualitatively, there are some differences regarding the shape and location among them. Therefore, it is crucial to implement an optimal criteria to choose the pdf that best represent the Peruvian data.

Figure 14: Mixture of Normal density fitting (4-month ahead)



3.4 Model evaluation results

As was pointed out earlier, in order to obtain reliable results from density fitting, we implement three model evaluation techniques: Log Score, probability integral transform and entropy.

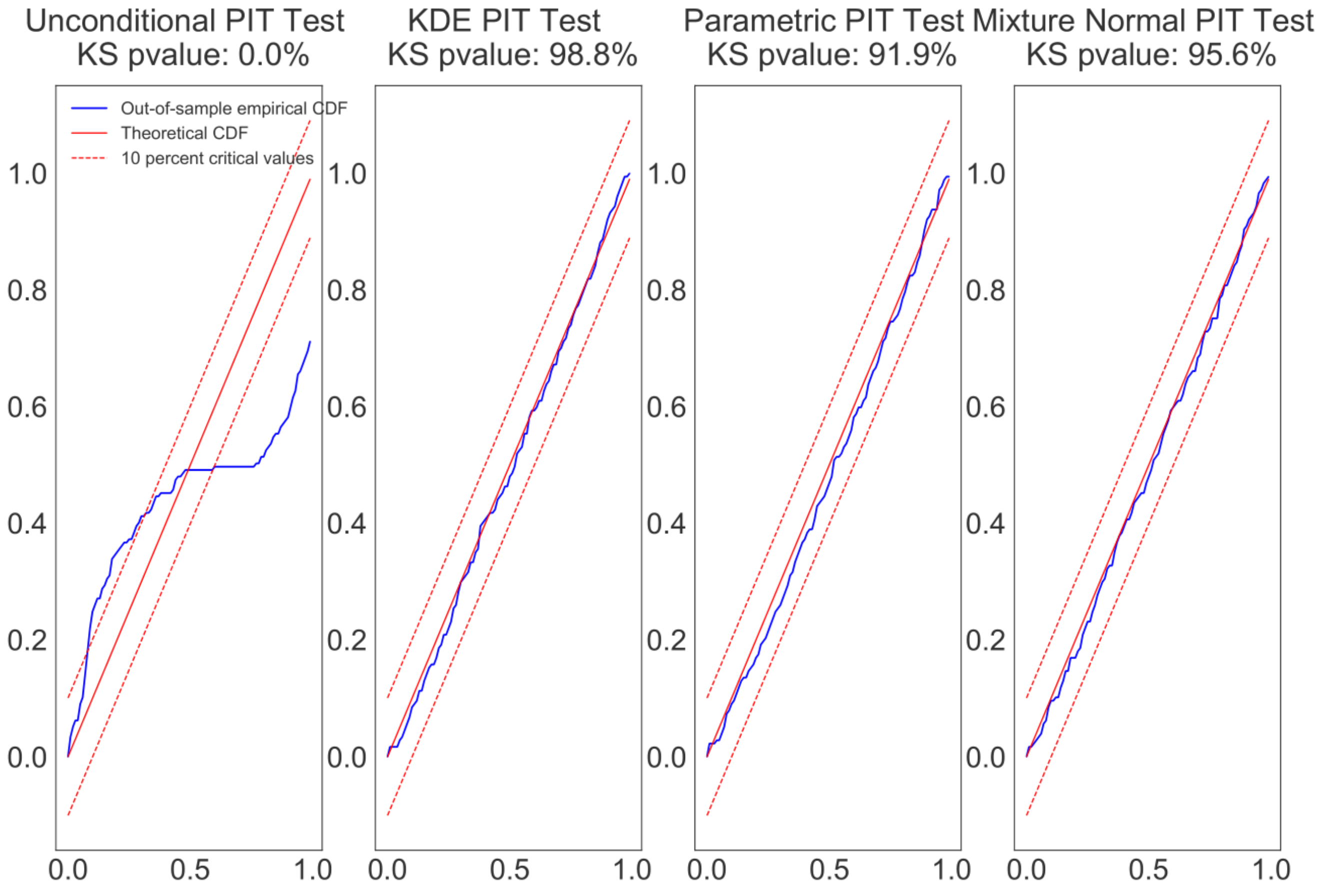
Regarding Log Score, we compare the non parametric (KDE), the best parametric pdf and the mixture of normal pdf via a Diebold-Mariano test. Basically, we compare each par of pdf on a one-sided hypothesis, where the null hypothesis is that both pdf are not different than each other versus an alternative hypothesis of one pdf is better than the other. Table 2 show the results of Diebold-Mariano test of comparison of each part of pdf. The fist three rows show that conditional distributions are superior than unconditional distribution and therefore the QR estimation and density fitting are useful for determining the future distribution of GDP growth. Moreover, row 4 and 5 indicates that KDE and Mixture of Normal pdf are superior than the best parametric distribution, which could be indicative that parametric distribution is too restrictive to represent Peruvian data. Finally, row 5 show indicates we can not select a best pdf among KDE and Mixture of Normal distributions Both are good at representing Peruvian data.

Table 2: Log score comparisons via Diebold-Mariano test statistic

Log Score diff	test statistic	p-value
KDE against Unconditional	3.733	0.000
Parametric against Unconditional	3.722	0.000
Dual Mixture against Unconditional	3.730	0.000
KDE against parametric	2.672	0.004
Dual Mixture against paramteric	2.988	0.001
KDE against Dual Mixture	0.215	0.415

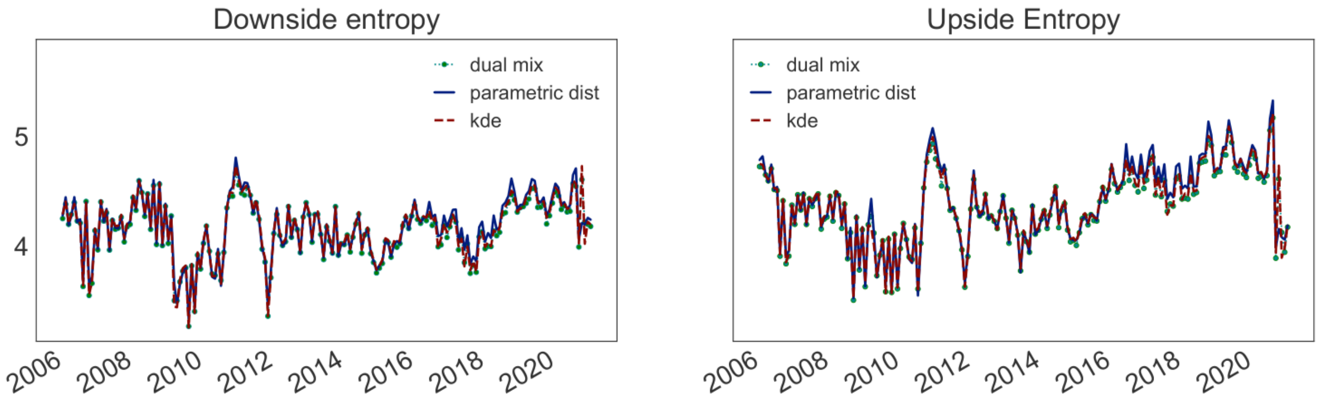
Regarding the Probability Integral Transform, as pointed out earlier we need to verify the correct specification of density forecasts we estimated. PIT evaluate the out-sample performance comparing the cumulative distribution of our fitted pdf with an uniform distribution. if corrected specified, the density forecast from GaR model should have a cumulative distribution that lies on the 45 degree line. Figure 15 show the PIT for unconditional, KDE, parametric and Mixture of Normal cummulative pdf. We can see that unconditional PIT do a poor job since its cumulative distribution fail to follow a uniform distribution. Moreover, the Kolmogorov-Smirnov (KS) test reject that unconditional PIT follows and uniform distribution. Moreover, all three conditional pdf from GaR results show a PIT consistent with a uniform distribution, with the KDE, parametric and Mixture of Normal Cumulative are statistically similar to the Uniform distribution according to the Kolmogorov-Smirnov (KS) test.

Figure 15: Probability Integral Transform test



Finally, we also included Entropy metrics, which is a measure of average information uncertainty. Downside and Upside Entropy represents the uncertainty of GDP growth in the extreme left and right tail. Figure 16 show results for both downside and upside metrics for Nonparametric, Parametric and Mixture of Normal pdf. we can notice that there is not much differences among them which suggest that the fitting parametric distributions preserve the same quantity of information as normal distributions. however, Interestingly, Upside entropy fluctuates more than downside entropy, which is consistent with the fact that Peru did not experience a severe crisis during the sample period, and therefore most of the variation is on the upside.

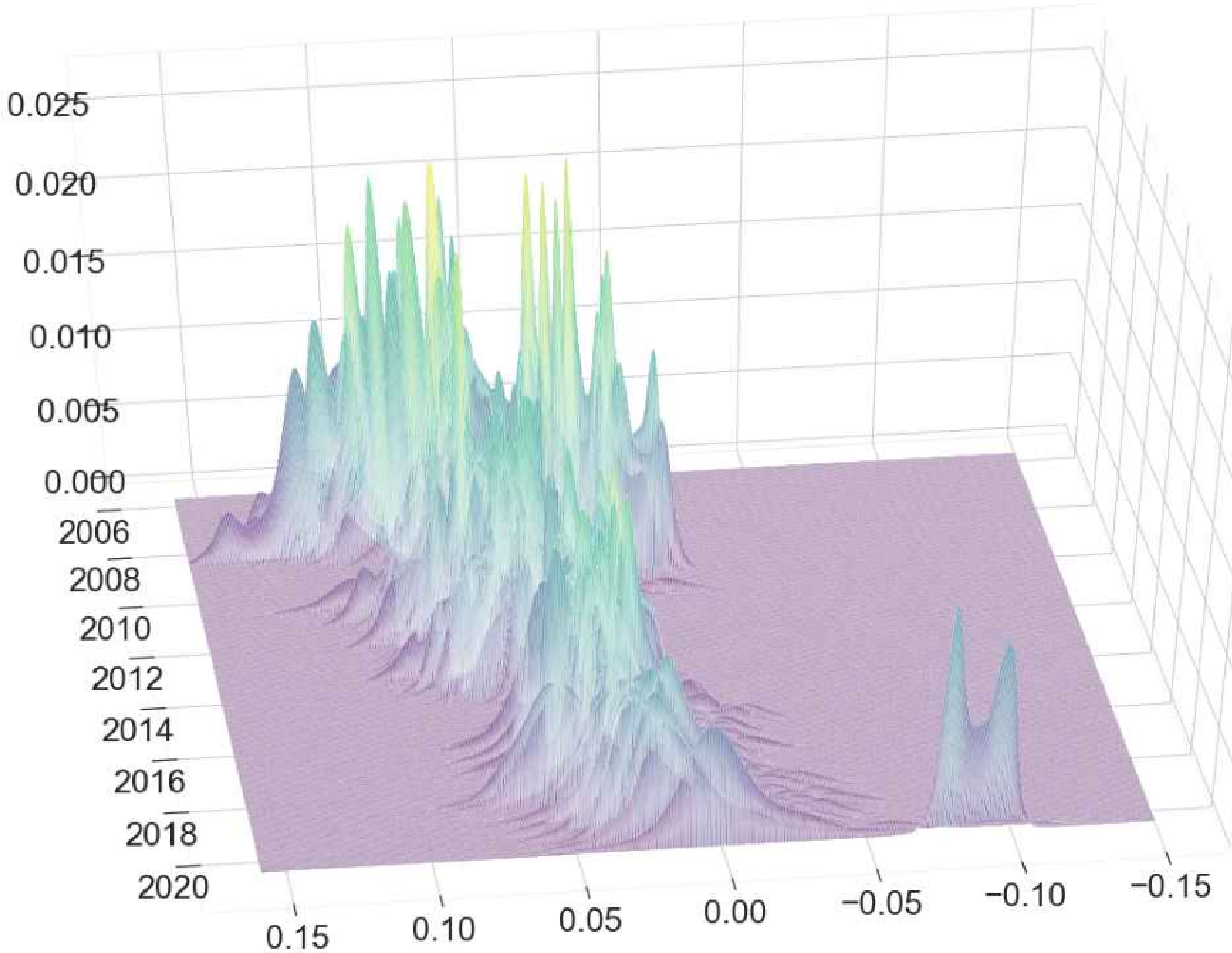
Figure 16: Downside and upside entropy metrics comparison



Summarizing the results from this section, we have selection criteria that will allow us to select the distribution that best represent the Peruvian data on GDP Growth. It is important to consider that on selecting the best pdf at each forecasting horizon we need to balance the statistical fit (which favor KDE and mixture of normal pdf) with consistency across all horizon (which favor simple yet somehow less accurate parametric pdf), since for an specific sample we can have a series of forecasted pdf that varies from parametric to no parametric ones and which can change considerably as the sample increase with new observations, generating challenges for policy analysis.

Having a reliable estimation of density forecast from QR results allow us to track density forecast of GDP growth across time and identify the building up of vulnerabilities to economic growth in the Peruvian economy, as shown in Figure 17.

Figure 17: Historical evolution of density forecast of GDP Growth



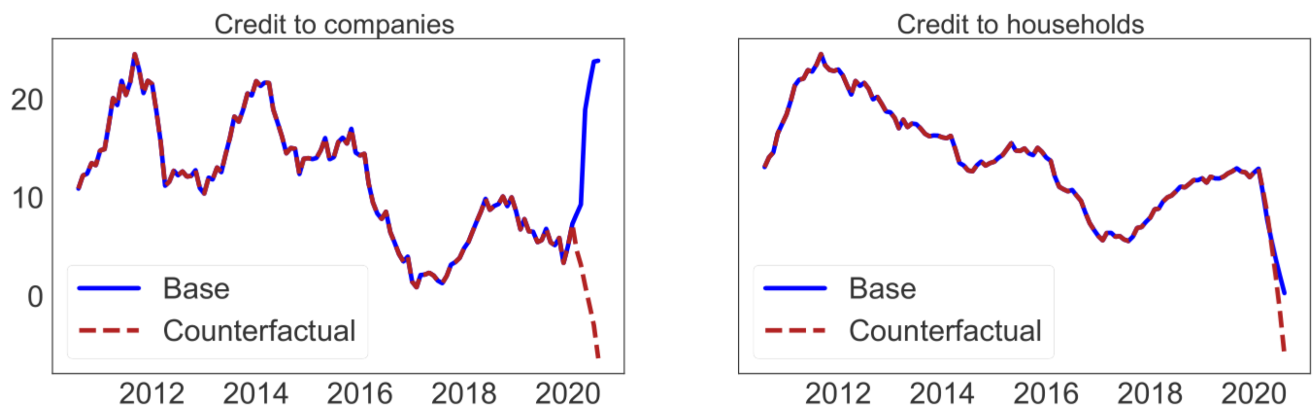
4 Policy analysis

Once the obtain a reliable methodology to forecast the distribution of future GDP, we proceed to implement a countrafactual exercise to evaluate the impact to a credit stimulus set in Peru as a response to the Covid-19 crisis called Reactiva Peru.

"Reactiva Perú" is a Guarantee Program designed by the Central Bank and the Ministry of Finance, which allow Central Bank to provide low cost liquidity to banks to supply loans to businesses while those loans are guarantee by the Treasury. By providing a large supply of low cost credit to firms, specially SME, during the lockdown, this program reduced the impact of the Covid-19 shock to the Peruvian economy.

To test this argument, we implement a counterfactual scenario using the GaR Model. We start building this scenario considering what would it be the likely path of the credit to firms and households if Reactiva Peru were not in place. Figure 18 show the observed path of credit to firm and households (Base) and the counterfactual path for these two variables. For constructing this, we use the VAR-X model used in the Stress Test analysis of the Central Bank of Peru (for firms, this is also consistent with the evolution of credit growth not related to Reactiva Peru). We can see that without implementing Reactiva Peru credit growth would decrease significantly which could have a sizable impact on GDP growth.

Figure 18: Counterfactual scenario for credit market variables



To map this counterfactual scenario the the GaR model we need to estimate the counterfactual scenario for the credit market factor that include the credit variables. We run a OLS regression with the credit market factor against the two credit variables (credit to firms and to households) and use the estimated coefficients and the counterfactual values of these two variables to obtain a counterfactual path for the credit market factor, which is shown in Figure 19.

Figure 19: Counterfactual scenario for credit market factor

In order to transfer the shock of the Credit Market factor to the rest of factors, we followed Kilian (2016) to simulate counterfactual outcomes using a SVAR model. To do that the following steps are needed:

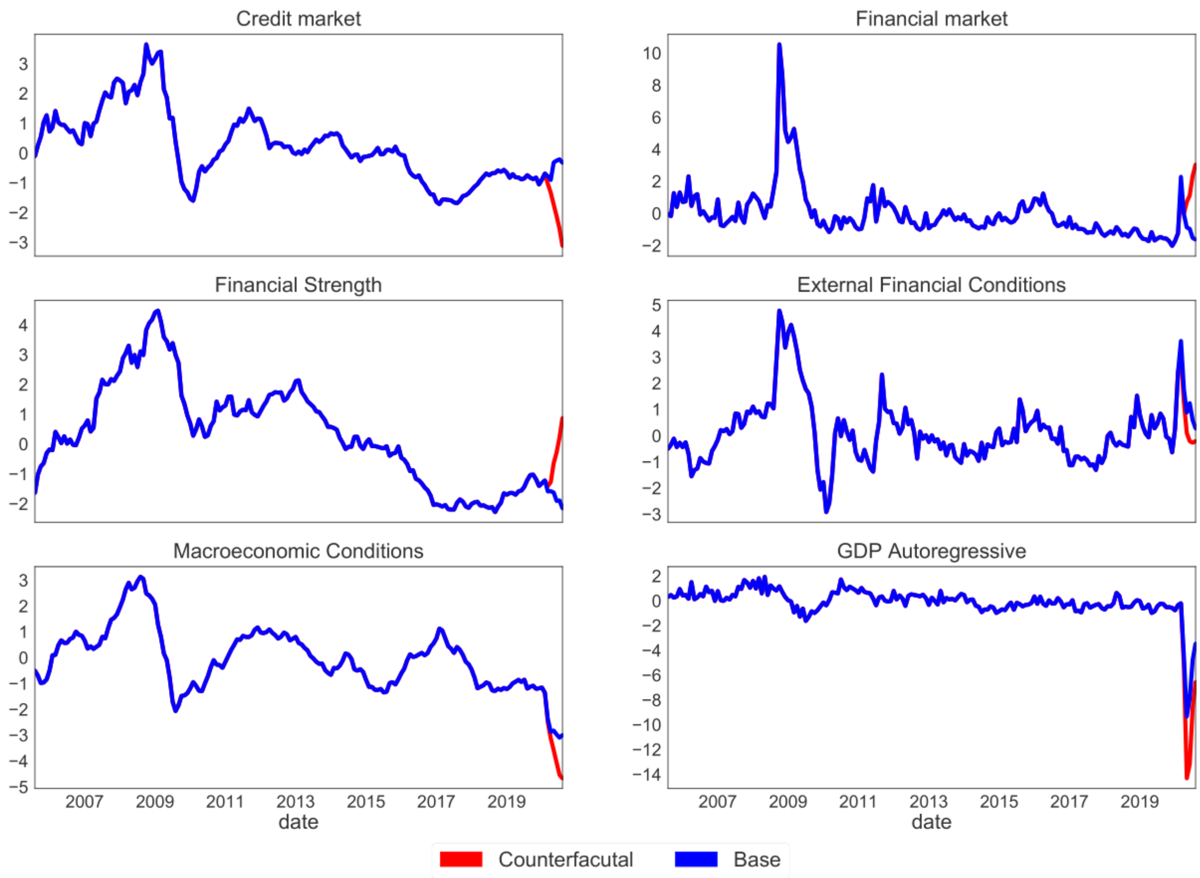
1. Have a time series of factor in the counterfactual scenario: in our case Credit Market factor obtained previously.

2. Estimate a SVAR model for the complete sample of factors and decompose each factor by a sequence of structural shocks (historical decomposition).

3. Construct a sequence of shocks for the Credit Market factor needed to replicate the time series of this factor in the counterfactual scenario.

4. Replace the structural shocks by the counterfactual shocks for the Credit Market factor and keep the structural shocks for the rest of factors in the SVAR model, and simulate the evolution of the factors under this new sequence, resulting the counterfactual scenario shown in Figure 20.

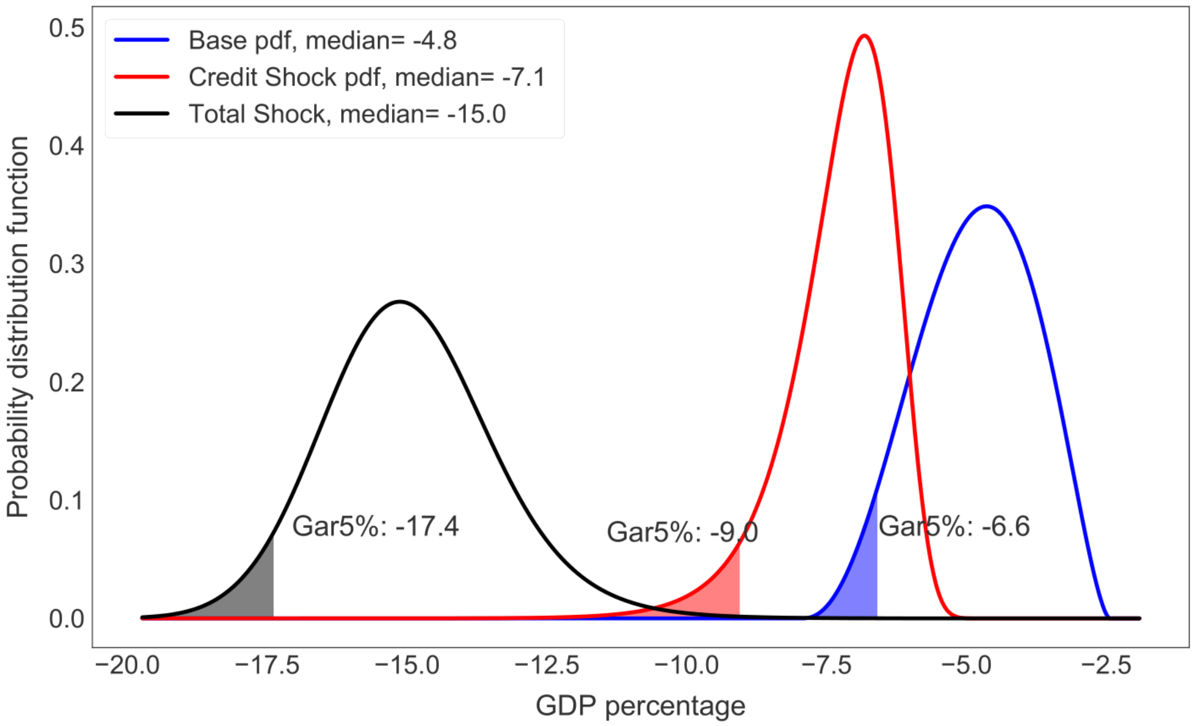
Figure 20: SVAR: Counterfactual Analysis



Then, with this counterfactual scenario and the GaR model estimated we can build density forecasts of GDP growth under this counterfactual scenario, which is shown in Figure 21. Results show that by using GaR model for the counterfactual scenario (without

"Reactiva Peru") we obtain a significant worse impact in economy activity, no only in terms of lower expected growth but also in terms of increased risk (percentil 5% is -17.4% in the contrafactual scenario instead of -6.6% in the baseline scenario).

Figure 21: Density forcast for GDP growth: Counterfactual Analysis



5 Conclusions

Growth at risk is a important tool for monitoring macrofinancial risk since it allow to measure the link between macrofinancial conditions and future GDP growth distribution. However, for the accuracy of the GaR results it is crucial to implement model evaluation techniques to avoid misleading interpretation. Moreover, flexibility of the GaR methodology allows to perform counterfactual scenario analysis that can help to identify sources of risks and communicate policy actions.

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