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Forecasting Inflation in a Macroeconomic Framework: An Application to Tunisia

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Abstract

The aim of this paper is to demonstrate the relative performance of combining forecasts on inflation in the case of Tunisia. For that, we use a large number of econometric models to forecast short-run inflation. Specifically, we use univariate models as Random Walk, SARIMA, a Time Varying Parameter model and a suite of multivariate autoregressive models as Bayesian VAR and Dynamic Factor models.

Results of forecasting suggest that models which incorporate more economic information outperform the benchmark random walk for the first two quarters ahead. Furthermore, we combine our forecasts by means and the finding results reveal that the forecast combination leads to a reduction in forecast error compared to individual models.

Key words: Short-run forecasting, Dynamic Factor Models, Forecast combination.

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1. Introduction

Tunisia is gradually moving toward full flexibility of its exchange rate and an inflation targeting framework. A successful transition to the regime of inflation targeting depends not only on the perquisites for adopting this strategy, but also on the ability to predict inflation. Forecasting inflation will become a key task for the Central Bank of Tunisia (BCT). Because of the time lags between monetary policy and its effects on the economy, particularly on inflation, the BCT will need to base its monetary policy decisions not on past inflation outcomes but on inflation forecasts. The precision with which inflation can be forecasted is a critical element of the inflation targeting framework.

The BCT uses a large information set coming from expert judgments, which is derived using both now-casting tools, and a variety of models ranging from simple traditional time series models to theoretically well-structured dynamic stochastic general equilibrium models to predict inflation. Our object in this paper is to base medium-term forecasts on more accurate and well-performing short-term projections, which rely on the maximum information set available. To this end, we use different modelling approaches in order to improve the performance of short term projection.

Inflation in Tunisia has been moderately volatile, it outperforms a number of other Middle Eastern, North African countries, Afghanistan and Pakistan in terms of low inflation and it compares favorably to comparator countries, as indicated in Table I. In fact, inflation in Tunisia was always below the line representing the average inflation of Middle East, North Africa, Afghanistan and Pakistan.

In this study, we use different modeling approaches in order to provide a rich set of short - term model based inflation forecasts and we compare the forecasting performance of the various models of inflation. Performance is measured at different forecast horizons (mainly one or two quarters ahead).

We employ various time series models: Bayesian VAR models, Time Varying parameters models, unobserved components model and data intensive factors models (FAVAR). In addition to the individual forecasting models, we also provide evidence on the performance of a simple forecast combination. This forecast combination is computed as the simple root mean squared errors weighted average (RMSE). In this methodology, the weights are based on the forecast error performances measured by RMSE and a final forecast combination is computed by summing the forecasts of individual models multiplied by their weights.

The paper is organized as follows. In the second section, we develop the block of model to use for forecasting inflation and the empirical study in which we compare the performance of these estimated models generating pseudo out of sample forecast in Tunisia and for different horizons. In the third section, we explain the forecast combination procedure used in our short term forecasting practice. In fourth section, we present our results and conclude.

Table I: Consumer Price Index Evolution in Tunisia and sum other comparable countries (1980-2016)

	Mean	*Standard-Deviation*	*Min*	*Max*
Algeria	*9.054*	**8.375**	**0.3**	**31.7**
Egypt	*11.464*	**6.171**	**2.4**	**25.2**
Jordan	*4.808*	*5.152*	*0.9*	*25.7*
Morroco	*3.989*	**3.465**	**0.4**	**12.5**
Tunisia	*5.259*	**2.685**	**1.9**	**13.7**
Middle East, North Africa and Afghanistan	*8.637*	**3.235**	**2.7**	**16.5**

2. Models

In this section, we use several types of models to forecast short-term inflation for Tunisia.

Standard VAR models are useful since they allow for the interaction of different related macroeconomic variables. However, in VAR models, the number of parameters to be estimated increases geometrically with the number of variables and proportionally with the number of lags included. The BVAR approach limits the dimensionality problem by shrinking the parameters via the imposition of priors (the coefficients are shrunk towards prior values, reducing the ‘curse of dimensionality’ issue that afflicts classical VAR when the number of variables increases).

In our study, we impose Minnesota-style priors where the priors are specified to follow a multivariate normal distribution. The means of the coefficients on first own lags are one and the coefficients on the cross lags are zero.

For our exercise to forecast short-run inflation via BVAR models, we apply pseudo out of sample forecasting. In the first step, we divided our sample period: 2000Q1 to 2015Q4 into two parts. The first period is the training sample period (2000Q1:2010Q4). The training sample is used to estimate the models throughout the forecasting sample, one to four quarters ahead.

We extend the estimation one period ahead and we collect the forecast at each step which are obtained for one to four quarters ahead. This process is repeated until the end of pseudo out of sample period.

We measure the performance of our forecasting models by calculating the Root Mean Squared Error (RMSE):

$$RMSE_h^i = \sqrt{\frac{\sum_{t=2010Q4+h}^{2015Q4}(f_t - r_t)^2}{T}}$$

Where $h = 1, \ldots .4$ quarters, i represents the model, T is the out -of-sample size. f_t denotes the forecast and r_t is the realized annual inflation rate.

2.1. Empirical study:

2.1 .1 ARIMA specification model:

The first step –as a benchmark –is to assume that inflation cannot be forecasted. Thus, no other model can beat a random walk, which implies that the best forecast for future is current inflation. The second benchmark is an ARMA model that uses only past inflation observations to forecast inflation. Then we use the forecast from ARMA models allowing the disturbances to follow ARMA specification. We estimate the following ARMA (p ,q) model that includes both autoregressive and moving average terms:

$$\pi_t = c + \sum_{i=1}^{p} \emptyset_i \pi_{t-i} + \sum_{j=0}^{q} \theta_j \, \varepsilon_{t-j} \qquad (1')$$

Where **P** is the number of lags of autoregressive process and **Q** is the number of lags of Moving average process.

The choice of data sample for forecasting inflation is dictated by data availability. The data sample analyzed here comprises quarterly observations ***of consumer price index (CPI) from 2000Q1 to 2010Q4.*** This variable is tested in logarithmic form for nonstationary using Phillips-Perron and Augmented Dickey-Fuller. The results of these tests confirm the non-stationary in level of CPI but it's integrated in order (1).

The SARIMA model selection is based on Schwarz criterion to determine the number of ARMA terms. Determining the number of ARMA terms is done by specifying a maximum number of AR or MA coefficients, then estimating every model up to those maxima, and we evaluate each model using its information criterion.

The best model's transformation differencing an ARMA length has been selected through information criteria, the model is used to calculate the forecasts.

The best specification is an SARIMA (4, 0, 1, 3) and the actual inflation is shown by Graphs:

Figure 1: actual inflation and inflation forecasting for a one quarter ahead

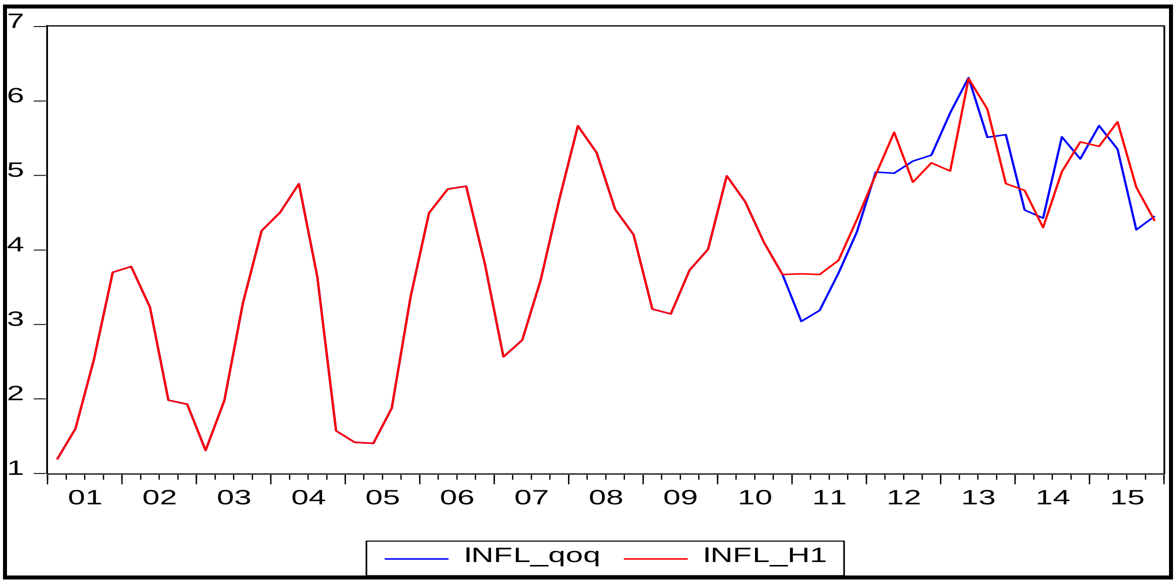


Figure 2: Root Mean Square Error for a one quarter ahead

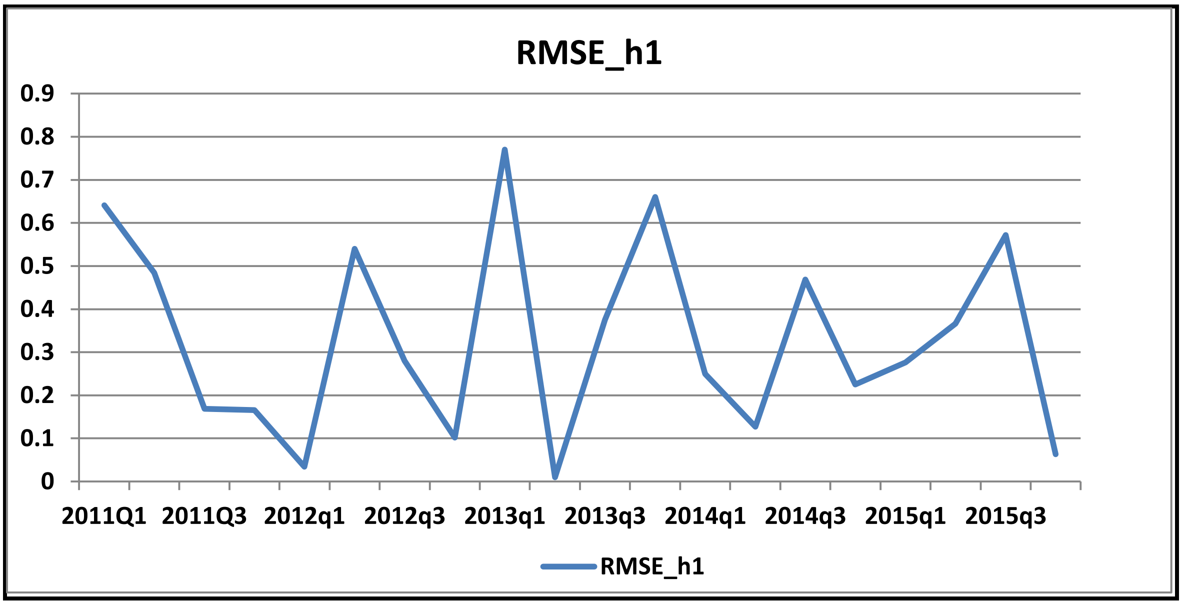
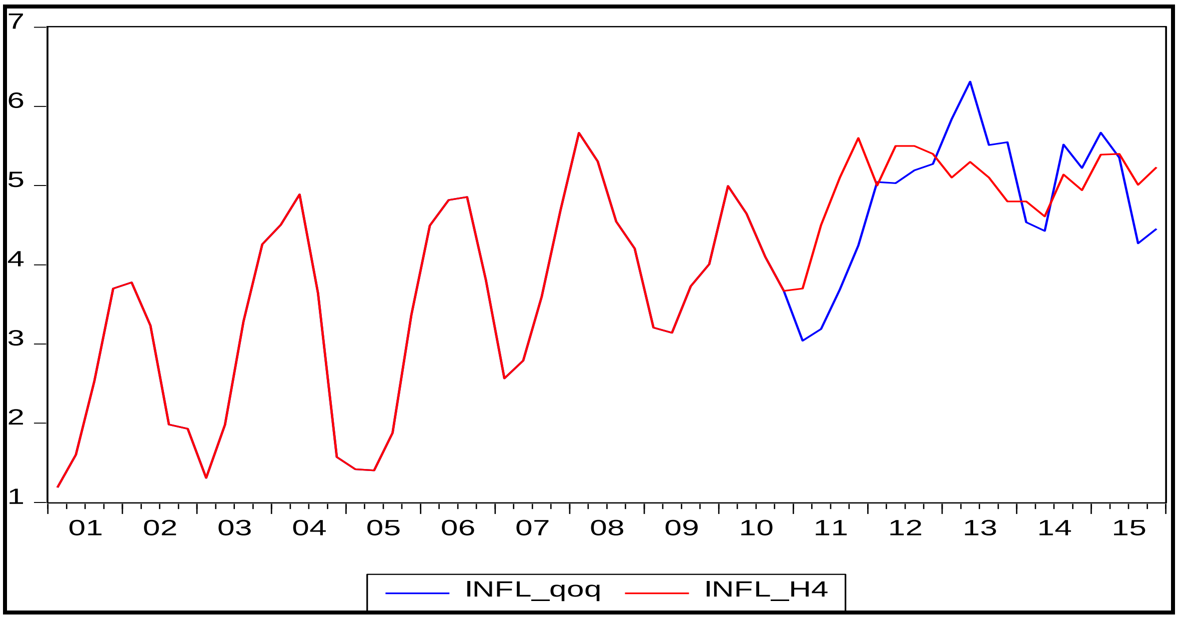
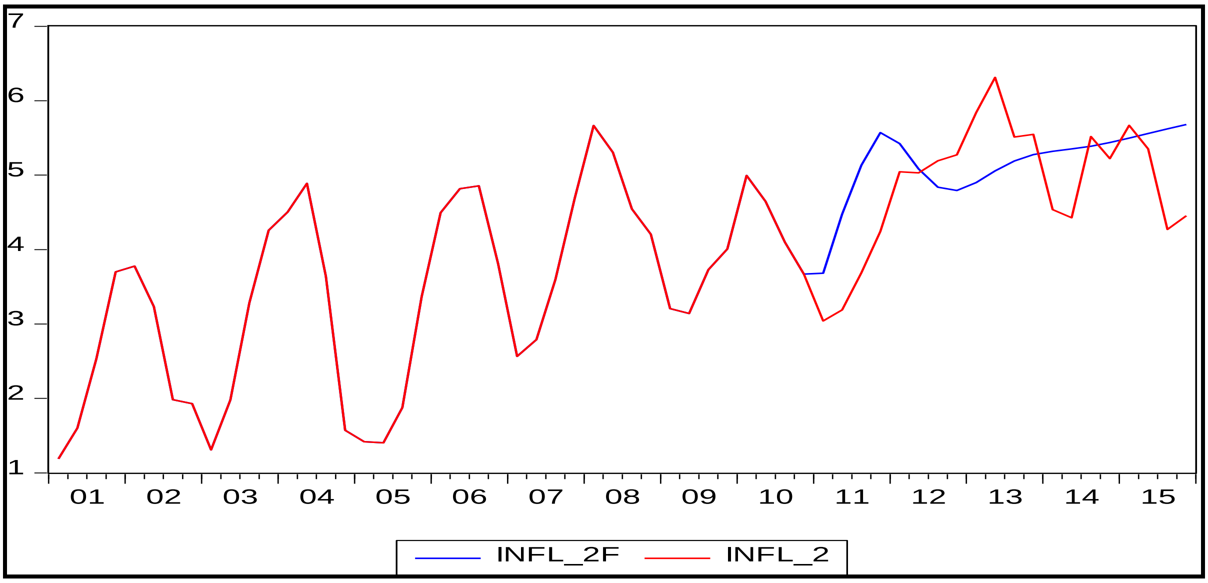


Figure 3: actual inflation and inflation forecasting for two quarters ahead



And the forecasting inflation for a long horizon (5 years ahead) is shown by **figure 4**:



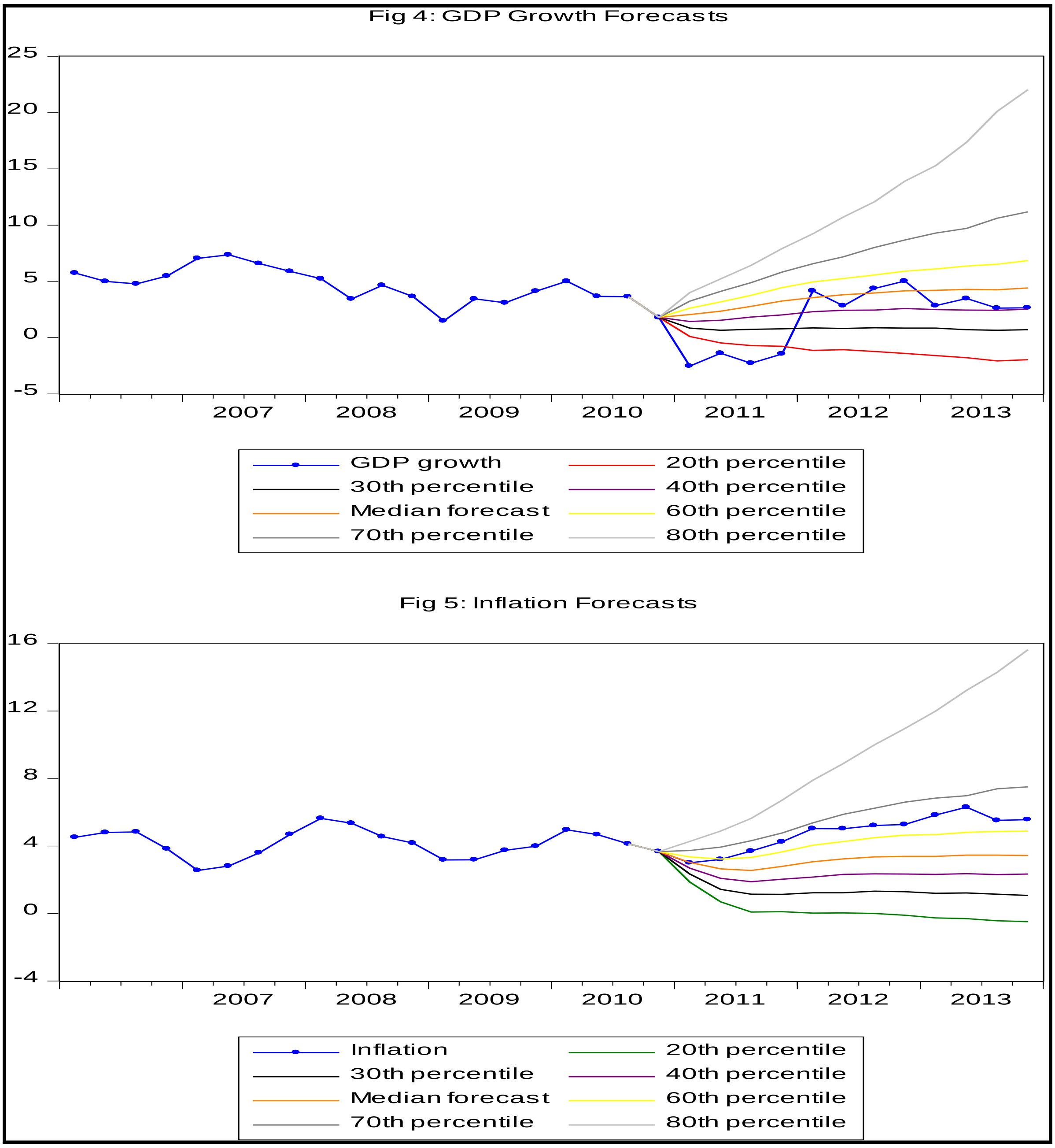
Generally, performance of traditional univariate models is not promising for a long horizon. Particularly, SARIMA displays a poor performance for a long horizon (4 quarters ahead and 5 years ahead), as shown in **Figure 4**. However, for Tunisia poor performance of SARIMA model does not come as a surprise given the lack of information coming from macroeconomic variables that are especially important for the inflation dynamic in emerging market economies. Moreover, inflation in Tunisia does not present such a stable dynamic.

2.1 .2 Estimation of BVAR model:

We consider the estimation of a bi-variate **VAR (2)** model using quarterly data on annual GDP growth and CPI inflation of Tunisia from **2000Q1 to 2010Q4 (training sample) and construct one, two, three and four-step-ahead forecasts.** Then the sample is extended one period and models are re-estimated. New forecasts are obtained **until 2015Q4. Out of sample forecast accuracy is measured in terms of RMSE**.

For estimation, we employ a **Minnesota prior** which incorporates the belief that both variables follow a random walk. While annual CPI inflation is non-stationary and hence the random walk prior is reasonable. The model is estimated using the Gibbs sampling algorithm and the quantiles of the predictive density are shown in the **figures 5 and 6:**

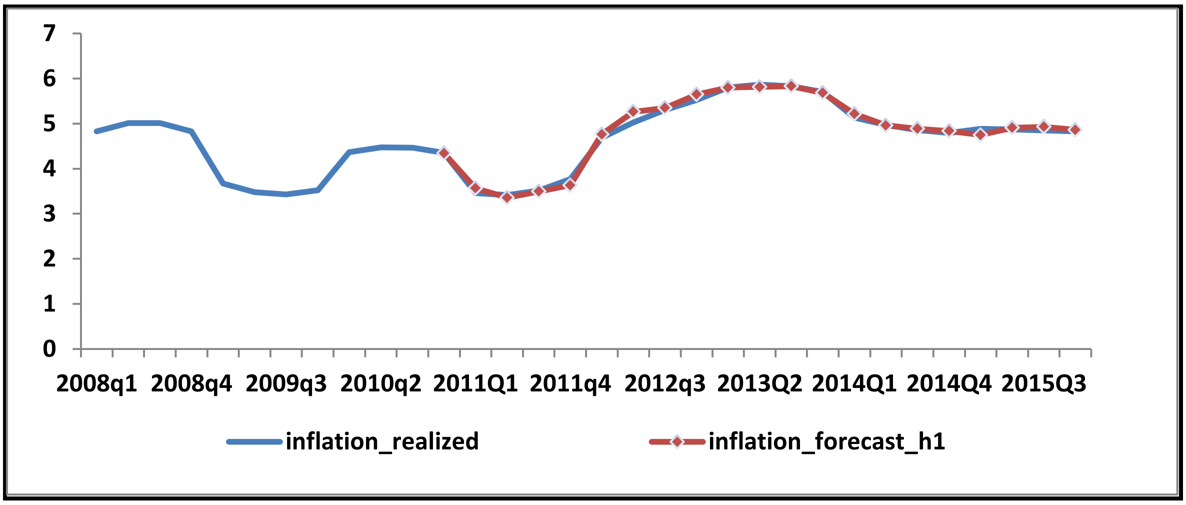
<u>Figures 5 and 6: Forecasting GDP growth and Inflation with Bayesian VAR</u>



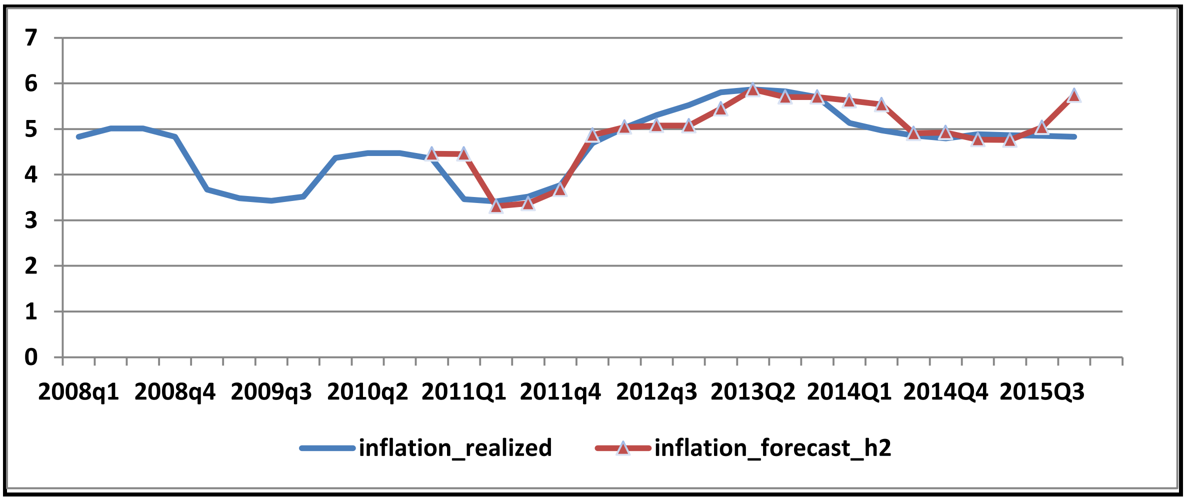
The ***Figure 7*** displays the inflation forecasts via ***BVAR approach*** and their corresponding realizations respectively at horizons one-quarter and two-quarters ahead. This approach produces a more accurate forecasts when forecasting one-quarter ahead inflation compared to two-quarter ahead. **Besides, one-quarter-ahead forecasts are strikingly close to the realizations.**

Figure 7: Forecasts via Bayesian VAR and realizations

For h=1



For h=2



One argument developed for these finding results, concerning the accurate forecasts for two-quarters ahead, is related with the problem of dimensionality (only two variables as Real GDP and inflation) are used as regressors in this estimation. For this reason and in the purpose of avoiding this problem of dimensionality, we consider a model of time varying parameters model, including much larger set of variables.

2.1 .3 Estimation of a Time varying parameter model (VAR)

We model the behavior of quarterly consumer prices index inflation, $\Delta \ln CPI$, the quarterly growth rate of **unit value prices index, $\Delta lnIPM$, quarterly nominal exchange rate** ,$\boldsymbol{Euro/TND}$ and quarterly exchange rate $\boldsymbol{Dollar\ /TND}$ .analysis on the stationarity of our quarterly series precedes the estimation process. Our sample includes the period from 2000Q1 to 2015Q4. ***The augmented Dickey-Fuller unit*** root test results show that non-stationarity is rejected at ***1%*** significance level for quarterly inflation.

Specifically, we consider the following reduced form time varying parameter **(TVP) VAR**:

$$Y_t = c_t + \sum_{j=1}^{p} \beta_{j,t}\, Y_{t-j} + v_t$$

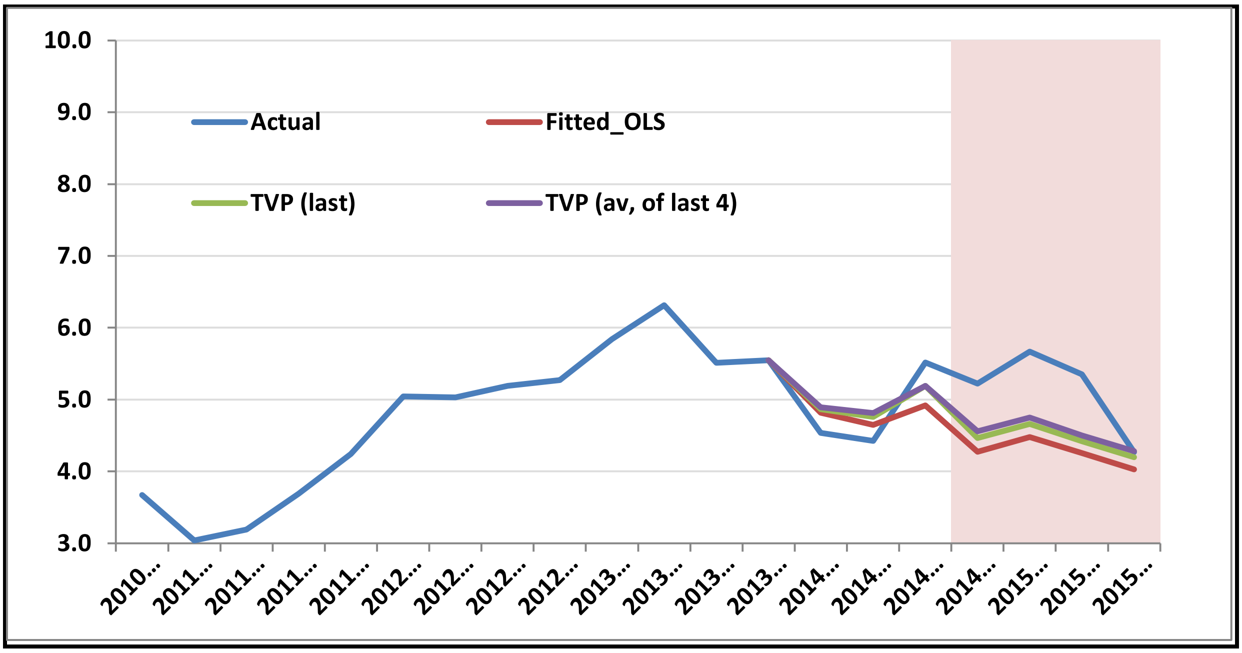
$$E(v_t' v_t) = R_t$$

$$E(v_t' v_s) = 0 \ if\ t \neq s$$

$$\beta_t = \mu + F\beta_{t-1} + e_t, \qquad VAR(e_t) = Q$$

Where Y_t is the 4×1 vector $(\Delta \ln CPI \quad \Delta lnIPM \quad Euro/TND \ \ Dollar\ /TND\ )'$ v_t is a vector of reduced-form errors, c_t is a vector of constants and the $\beta_{j,t}$ s are matrices of coefficients. We assume that Tunisia is 'small' in the sense that movements in TU variables have no effect on world variables. The Gibbs sampling algorithm can be discerned by noticing that if the time-varying **coefficients $\boldsymbol{\beta_t}$** are **known, the conditional posterior distribution of $\boldsymbol{R}$ is inverse Wishart. Similarly, conditional on $\boldsymbol{\beta_t}$** the distribution of Q is inverse Wishart. Conditional on R and Q and with assumption that $\mu = 0$ and $F = 1$ the model is a linear Gaussian State space model **(appendix B).**

Figure 8: forecasts via Time varying parameters models



When forming the forecasts, we compare the simple OLS (characterized by a constancy of the parameters of the model) with last estimates of β_s and with the last 4 quarter estimates of β. We prefer using averages of the last four quarter estimates of β_s to ensure some persistency; the approach of TVP is very robust to some form of structural change, such as intercept shifts.

2.1.4. Estimation of a FAVAR model

Factor models exploit the fact that macroeconomic and financial time series are characterized by strong correlations. Under the assumption that most of fluctuations are driven by relatively limited set of common sources. Factor models offer a parsimonious representation by summarizing the information from large number of data series in a few common factors. Dynamic factor models parameterize the dynamics of the factors further, typically assuming VAR process. The estimation of factor models generally requires the data to be stationary. Assuming that stationarity is achieved via tacking the first differences.

Our model is based on the ***Factor Augmented VAR introduced in Bernanke et al (2005).*** The FAVAR model can be written as:

$$X_{it} = b_i F_t + \gamma_i TMM_t + v_{it} \quad (1')$$

$$Z_t = c_t + \sum \beta_j Z_{t-j} + e_t \quad (2')$$

$$Z_t = \{F_t | TMM_t\} \quad (3')$$

$$var(v_{i,t}) = R, var(e_t) = Q \quad (4')$$

Where $X_{i,t}$ is a $(t \times m)$ matrix containing a panel of macroeconomic and financial variables. TMM_t denotes the market interest rate and F_t are the unobservable factors which summarize the information in the data $X_{i,t}$. the first equation (1') is the observation equation of the model while the second one is a transition equation. **Bernanke et al (2005)** consider a shock to the interest rate in the transition equation and calculate the impulse response of each variable in X_{it} .

We estimate a FAVAR model using Tunisian Data over the period 2000Q1 to 2015Q4. We use 30 Macroeconomic and Financial time series to estimate and predict inflation. (Real GDP, Real Consumption, Government Consumption, Real Exports, Real Imports, commodity prices, consumer prices index, components of prices index, Nominal exchange rates and Monetary market rate) (Table 2 provides the details of the data).

Table2: Variables in Dynamic Factor FAVAR model

Category	Variables
Real activity measures	Real exports(sa), Real imports(sa), investment(sa), Private consumption(sa), Governement consumption(sa), RGDP(sa), index prices_clothes, index prices_goods_services, index prices_communication,
Infaltion components	index prices _energy, index prices_education, consumer prices index_all, consumer prices without food, core inflation, consumer prices without energy, prices_culture, prices_health, prices_hotel, prices_transports.
Monetary Indicators	Monetry aggregates(M3)(sa), credit to economy(sa), foreign assets, Reserve money(sa), interest rate.
Exchange rates	Euro/TND, USD/TND.
Note:"sa" refers to seasonally adjusted series.	

We include three common factors in the FAVAR, which are meant to capture roughly the information on real developments, prices and interest rates. Furthermore, impulse responses of principal components are obtained in figure 9.

Figure 9: Impulse responses of principal components

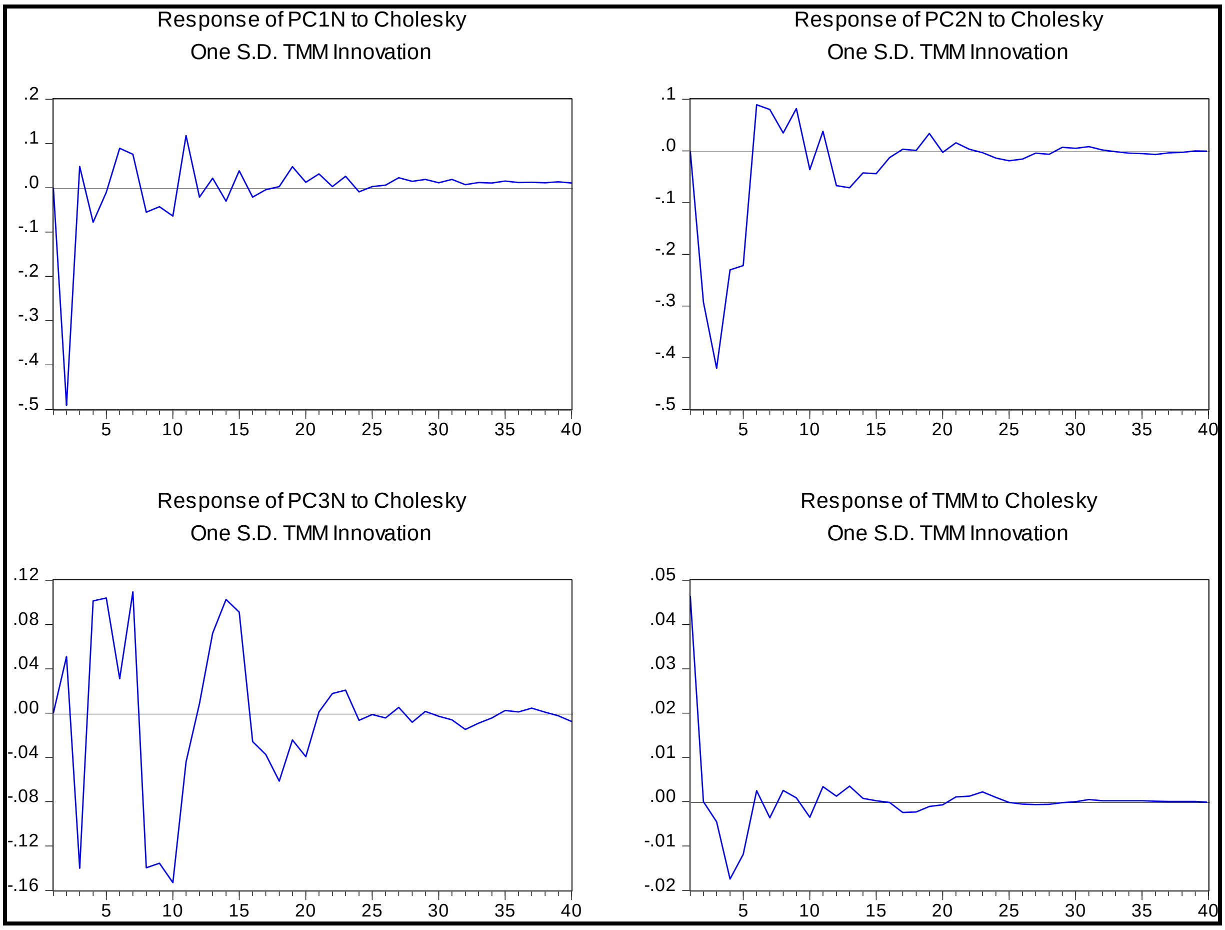


Table3

	Correlation coefficients		
Principal Components	**Variables**	**Correlations**	**Bloc**
PC1	CPI_inflation	0.36	**Inflation components**
	Core_inflation	0.36	
	IPC_food	0.36	
	IPC_Energy	0.35	
	IPC_Meubles	0.29	
	IPC_alimentaires	0.2	
	ipc hotelerie	0.24	
	ipc _clothes	0.24	
PC2	exports	0.42	**Real activity economy**
	imports	0.42	
	investment	0.42	
PC3	M3	0.46	**Monetary indicators**
	M0	0.49	

Table 3 presents the correlation between the principal components and some of the variables in the dataset. From the table, the first component (PC1) tends to describe the inflation components, while the second component (PC2) is related to the real variables as exports, imports and investment. The third one describes the monetary indicators (nominal variables).

Note that one of advantages of FAVAR approach is that the impulse response functions can be constructed for any variable in the informational data set, that is, for any variable of matrix X_t. This gives both more information and provides more comprehensive check on the empirical plausibility of the specification.

The Figure 9 shows the impulse responses with 95 percent confidence intervals of a selection of principal components to a monetary policy shocks. The responses are generally with expected sign and magnitude. Following a contractionary monetary policy shock, prices eventually go down, real activity measures decline and monetary aggregates decline. After one year, an increase of 1% of money market rate (TMM) results a decrease of inflation about 0, 8%. While real variables, react negatively. Finally, monetary aggregates respond negatively at the short run.

In the next step, we use these factors to forecast quarterly inflation. For the FAVAR forecasting, we build a FAVAR model and we name it FAVAR01 with the lag order of one by using these repetitive factors and inflation forecasts are formed from the projection of the linear single equation where quarterly inflation rate is a function of projected factors:

$$\boldsymbol{\pi_{t+h} = \mu + \beta(L) f_{t+h} + \alpha(L) X_t + v_{t+h}} \qquad \mathbf{(5')}$$

Where $\boldsymbol{\mu}$ is a constant, $\boldsymbol{f_t}$ is estimated factors, $\boldsymbol{\beta_L}$ and $\boldsymbol{\alpha_L}$ are vectors of lag polynomials and X_t is the vector of exogenous variables (i.e. seasonal dummies). Then we obtain $\boldsymbol{h - step\ ahead}$ predictions for inflation. Therefore, FAVAR forecasting is based on the same properties introduced in the VAR approach. The only difference is that we used only the estimated factors as endogenous variables while exogenous variables remain the same.

3. Forecast combination

Timermann (2006) argues that it is critical to identify whether or not the information sets underlying the individual forecasts are observed by the forecast user. If so, it would be appropriate to pool all the information and construct a “super” model nesting each of the individual forecasting model Also, Bjorland et al (2008) argue that usual analytical techniques may not be suitable for combined information set since the number of regressors may be large relatively to the sample size. Under these conditions, the best way to exploit information from different forecasters is to combine their forecasts. Therefore, combination methods have gained even more ground in the forecasting literature.

Empirical evidence suggests that combining forecasting systematically perform better than alternative based on forecasts from a single model. Different forecasting models are combined using equal, fit-based weights and compared with the multivariate and random walk benchmarks.

As contrary to trimming approach, root mean squared averaging model is based on the forecast error performance measured by (RMSE). A model with lowest RMSE receives the highest weight of this approach. The former method use RMSE of each model calculated for the Pseudo out of sample period.

3.1. Forecast Evaluation

The quality of the forecasts is evaluated by the relative RMSE (RRMSE), which is the ratio of the RMSE of a model or combination method to the RMSE of the benchmark. The (RRMSE) is calculated at each forecast horizon h as follows.

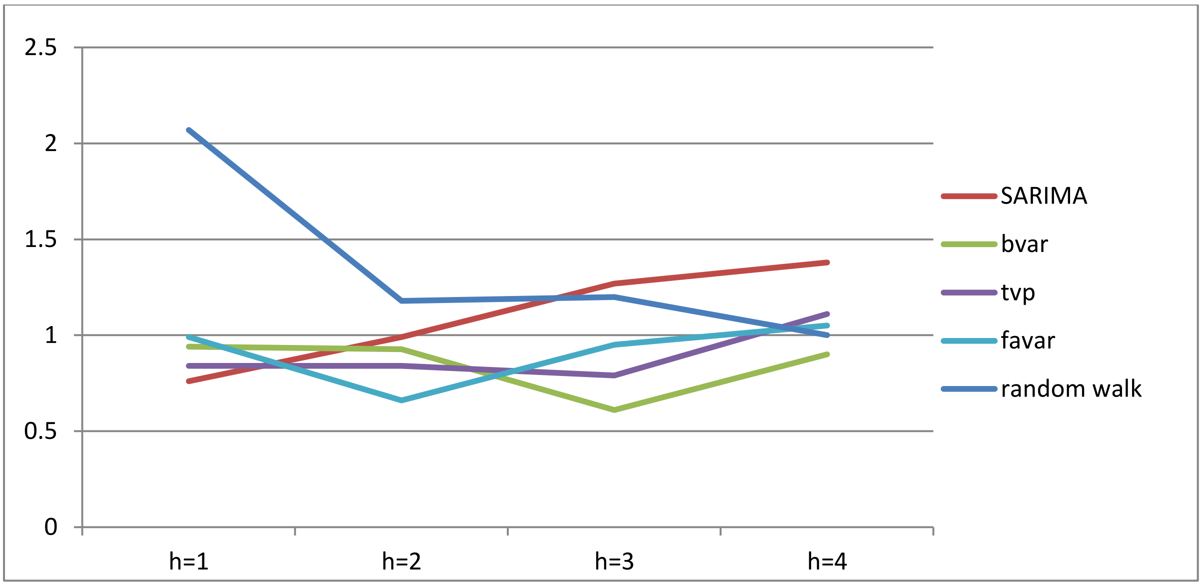
$$RRMSE_h = \sqrt{\frac{\sum_{t=2010Q4+h}^{2015q4}(f_t^m - r_t)^2}{\sum_{t=2010Q4+h}^{2015Q4}(f_t^b - r_t)^2}}$$

Where $h = 1\ldots4$ quarters, f_t^m represents the forecast of a model or combination method, f_t^b shows the forecast of the benchmark and r_t stands for the realized value of quarterly inflation rate.

Table 4: RMSE relative to the Random Walk Benchmark

	h=1	***h=2***	***h=3***	***h=4***
Individual Model Forecasts				
Random walk	2.07	1.18	1.2	1
SARIMA	0.96	0.99	1.27	1.38
BVAR	0.73	0.79	0.81	0.9
TVP	0.64	0.74	0.79	1.11
FAVAR	0.59	0.66	0.65	1.05
Forecast combination				
RMSE	0.34	0.54	0.55	0.62
Trimmed average	0.47	0.61	0.68	0.75

Figure 10: RMSE for individual models



T**able 4** suggests that individual models provide better inflation forecasts relative to the benchmark. Therefore, the performance of the benchmark is difficult to beat for most of the individual models at 4 quarters ahead. **Kapetanios et al. (2007)** point that the random walk performs relatively well for longer horizons given the fact that inflation is mean-reverting in the long run and inflation targets form a natural anchor in low inflation economies. The gains are clearly evident for the FAVAR, **TVP-VAR and BVAR**, which forecast **1 quarter and 2-quarter** ahead inflation are better compared to **3 and 4 quarters ahead.**

The best forecasts are provided by the **FAVAR** model. However performances of this factor-based model change across horizons and the gains are not quantitatively noticeable most of the time.

On the other hand, combining forecasts improves the forecast accuracy compared to the benchmark. Then forecast combination yields a superior performance. All forecasts combinations have Relative RMSE less than 1 for the four quarters ahead. The poor performance decreases as the horizon grows. In fact, the best combination scheme is the RMSE weights since it gives lowest relative root mean square at all horizons.

4. Conclusion

In this paper we propose modelling and forecasting inflation in Tunisia for short-run by using a large number of econometric models.

We proceed with a panel of models including univariate models, a Philips curve motivated time varying parameters model, a suite of BVAR, FAVAR models. Furthermore, root mean squared weights methods are implemented to combine individual model forecast.

The findings of the study illustrate that individual models incorporating more economic information perform better than the benchmark random walk model at least up to two quarters ahead forecasts. Those models exploit larger data sets, which are likely to involve more information about inflation compared to a data set used by any single equation model. In particular, FAVAR model appears to fit the data well, it consistently outperforms the benchmark at all forecasting horizons.

Despite the favorable gains under individual models, there is a scope for improvement from combinations strategies. Forecast combination reduces forecast error compared to individual models and slightly improves on the FAVAR when RMSE weighting scheme is adopted**.**

References

Bai, J.(2003). Inferential theory for factor models of large dimensions". Econometrica, 71(1) 135-171.

Bernanke, B. and J.Boivin (2003), "Monetary policy in a data-rich environment", Journal of Monetary Economics, vol.50 (3), pp. 525-546.

Bernanke, B, Boivin, J & Eliasz, P(2005)' Measuring the effects of monetary policy: A factor-augmented vector autoregressive (FAVAR) approach' Quarterly JournaL Of Economics, 120(1),387-422.

Banbura, M., D. Giannone, and L.Reichlin (2008),"Largge Bayesian VARs,"ECB Working Paper, No.966.

Carter, C., & Kohn, P.(1994). On Gibbs sampling for state space models.Biometrica, 81,541-553.

Clark, T E and McCracken , M W (2001), "Tests of equal forecast accuracy and encompassing for nested models', Journal of Econometrics, Vol .2 Part B, Amesterdam, Elseiver, pages 1107-1201.

Clark, T E and McCracken, MW(2013), 'Advances in forecast evaluation', Chapter 20 in Elliott, G and Timmermann, A(eds), Handbook of economic forecasting, Vol.2, Part B, Amesterdam, Elseiver, pages 1107-1201.

Christiano, L.J. Eichenbaum,M., & Evans, C.L.(1999. Monetary policy shocks : What have we learned and to what end ?. In J.B.Taylor, & M.Woodford (Eds), Handbook of macroeconomics: vol.1 (pp.65-148).Elseiver.

Forni, M, Hallin, M, Lippi, M & Reichlin, L (2000), 'The generalized dynamic factor model: identification and estimation , Review of Economics and Statistics, 82(4), 540-554.

Giannone, D; Lenza, M.Moferatou, D;& Onorante, L,(2014); Short –term inflation projections: a Bayesian vector autoregressive approach. International Journal of forecasting , 30(3) 635—644.

Stock, J.H., &Watson (2011). Dynamic factor models . In M.p Clements& D.F Hendry (Eds).th handbook of economic forecasting.oxford .

Stock, J.H; &Watson. (2004) Combination forecasts of output growth in a seven country data set. Journal of forecasting, 23,405-430.

Timmermann, A.(2006) Forecat combinations . In G.Elliot, C.W;J Granger , Handbook of Economertrics.

Appendix (A)

Bayesian estimation, forecasting and Fancharts:

Bayesian VAR model (BVAR) with Gibbs sampling algorithm:

For the BVAR models, we consider the following linear regression:

$$Y_t = \beta X_t + v_t, v_t \sim N(0, \sigma^2) \qquad t = 0, \dots \dots \dots . . T \quad (1)$$

The aim of this section is to estimate model coefficients, β. In the classical world, we use information contained in data by maximizing the following likelihood function:

$$F(Y/B) = (2\pi\sigma^2)^{-T/2} \exp(\frac{-1}{2\sigma^2}(Y_t - \beta X_t)'(Y_t - \beta X_t)) \quad (2)$$

And we obtain the classical OLS estimator:

$$\hat{\beta}_{ols}=(X'_t X_t)^{-1} X'_t Y_t \quad (3)$$

$$\widehat{\sigma_2} = \frac{v'_t v_t}{T} \quad \text{(4) and} \quad var(\hat{\beta})=\sigma^2 (X'_t X_t)^{-1} \text{ (4)}$$

In the Bayesian approach, we simply combine the information contained in data with our beliefs. We are interested in the posterior distribution, which is defined by Bayes theorem:

$$H(B/Y) = \frac{G(Y,B)}{F(Y)} = \frac{F(Y/\beta)\, P(\beta)}{F(Y)} \qquad (5\,)$$

- $F(Y)$ is the density of the data (marginal data density), which is a scalar. Therefore, we can write the following:

$$H(B/Y) \propto F(Y/\beta)\, P(\beta) \qquad (6)$$

The posterior distribution is proportional to the likelihood (information contained in data) times the prior (our beliefs). In practice, the estimation is based on three steps:

Gibbs sampling algorithm for the VAR model:

The Gibbs sampling algorithm for the VAR model consists of the following steps:

✓ ***Step1***: Set priors for the VAR coefficients and the covariance matrix. The prior for the VAR coefficients is normal and given by :

$$p(b) \sim N(b_0, H)$$

The prior for the covariance matrix of the residuals $\sum$ is inverse Wishart and given by $IW(\bar{S}, \alpha)$. Set a starting value for $\sum$ $(by\ ols\ estimation)$.

- ✓ ***<u>Step2:</u>*** We combine our prior belief, $P(B)$, with information about the model parameters contained in the data $F(Y/B)$, so we obtain the posterior distribution.

In reality, B and σ^2 are unknown parameters, so we have to calculate joint posterior:

$$H(B,\sigma^2 /Y) \propto F(Y/\beta,\sigma^2)\, P(\beta,\sigma^2)$$

The joint prior:

$$P(\beta,\sigma^2) = P(\beta/\sigma^2) \times P(\sigma^2)$$

To make inference, we need the marginal distributions, for example make inference about β we need:

$$H(\beta/Y_t) = \int_0^{\infty} H(\beta,\sigma^2/Y_t)\, d\sigma^2$$

Obtaining marginal posterior distribution requires integration. For integration step, we use a simulation method "Gibbs sampling" «that uses draws from conditional distribution to approximate the marginal one.

Appendix (B)

The time varying parameter model

In the case of structural change, statistical forecasting methods that incorporate parameter instability such as rolling regressions or time varying parameters **(TVP)** models might perform better than other models.

In the TVP models, we allow the model parameters to vary over time contrary to the standard models. The linear regression model with time varying parameters:

$$y_t = \mu_t + \beta_t x_t + e_t \quad e_t \sim N(0, \sigma_e^2)$$

$$\mu_t = \mu_{t-1} + w_{\mu,t} \qquad w_{\mu,t} \sim N(0, \sigma_\mu^2)$$

$$\beta_t = \beta_{t-1} + \omega_{\beta,t} \qquad \omega_{\beta,t} \sim N(0, \sigma_\beta^2)$$

Once, the model has been put in state space form, the Kalman filter may be applied to get the time varying μ_t and β_t state space model for an y_t consists of two equations , namely measurement and transition equations.

Measurement equation relates the observed data to unobserved state vector α_t where transition equation describes the evolution of the state vector over time:

$$y_t = Z_t \alpha_t + d_t + e_t \qquad t = 1, \ldots \ldots \ldots \ldots T$$

Where y_t is containing N elements, α_t is $m \times 1$ vector, Z_t is a $N \times m$ matrix, d_t is an ($N \times 1)$ vector of serially uncorrelated disturbances with zero mean and covariance matrix Q_t, that's :

$$E(w_t) = 0 \quad \text{and} \quad Var(w_t) = Q_t.$$

$E(e_t w_s') = 0$ for all s, *t=1 ...T.*

The initial state vector, α_0, has a mean of a_0 and covariance matrix P_0. The matrices Z_t, d_t , H_t, C_t, R_t and Q_t are called the system matrices. For the linear regression model, we define $\alpha_t = (\mu_t, \beta_t)'$, then state space form of the time varying parameter regression model can be written as:

$$\underbrace{\begin{pmatrix} \mu_t \\ \beta_t \end{pmatrix}}_{\alpha_t} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{T} \underbrace{\begin{pmatrix} \mu_{t-1} \\ \beta_{t-1} \end{pmatrix}}_{\alpha_{t-1}} + \underbrace{\begin{pmatrix} w_{\mu,t} \\ w_{\beta,t} \end{pmatrix}}_{w_t}$$

$$\text{With} \begin{cases} var(w_t) = Q = \begin{pmatrix} \sigma_\mu^2 & 0 \\ 0 & \sigma_\beta^2 \end{pmatrix} \\ y_t = (1 \quad x_t) \begin{pmatrix} \mu_t \\ \beta_t \end{pmatrix} + e_t, \; var\,(e_t) = H = \sigma_e^2 \end{cases}$$

If $\sigma_{\mu}^2 = \sigma_{\beta}^2 = 0$ then it's just a fixed coefficient linear regression model. For computing the optimal estimator of the state vector at time t, we use the Kalman filter defined as a recursive algorithm based on the information available at time t. This information consists of the observations up to and including $Y_t = (y_1, \dots \dots .. y_t)$ (Harvey, 1990).

In fact, the system matrices together with a_0 and P_0 are assumed to be known in all time periods and there are two sets of equations in the Kalman filter, these are prediction and updating equations.

Then the optimal estimator of α_t is given by the prediction equations are:

$$\left.\begin{aligned} a_{t/t-1} &= T_t a_{t-1} + c\,t \\ P_{t/t-1} &= T_t P_{t-1} T_t' + R_t Q_T R_t' \end{aligned}\right\} \quad \text{Prediction equations}$$

Let $a_{t/t-1} = E(\alpha_t | Y_{t-1})$ and $P_{t/t-1}$=$var(\alpha_t / Y_{t-1})$. Once the new observation, y_t,becomes available, we can correct or update the estimator of α_t, $a_{t/t-1}$.

$$\left.\begin{aligned} a_t &= a_{t/t-1} + K_t \underbrace{(y_t - Z_t a_{t/t-1} - d_t)}_{v_t \quad \textit{Prediction error}} \\ P_t &= P_{t/t-1} - K_t Z_t P_{t/t-1} \\ K_t &= P_{t/t-1} Z_t' F_t^{-1} \; and \; F_t = Z_t P_{t/t-1} Z_t' + H_t \end{aligned}\right\} \text{Updating equations}$$

Where K_t is the Kalman gain whereas F_t is a prediction error variance, that's $F_t = var(v_t)$. So $a_t = E(\alpha_t / y_t)$ and $p_t = var(\alpha_t / y_t)$.

- **<u>Step 1</u>**: Set a prior for R and Q and starting values of the Kalman filter. The prior for Q is inverse Wishart $p(Q) \sim IW(Q_0, T_0)$. this prior is quite crucial as it influences the amount of time variation allowed for in the VAR model. In fact, a large value for the scale matrix Q_0, would imply more fluctuations in β_t . This prior is set using a training sample. The first T_0 observations of the sample are used to estimate a standard fixed coefficients VAR via OLS such that $\beta_0 = (X_{0t}' X_{0t})^{-1} ((X_{0t}' Y_{0t})$ with a coefficient covariance matrix given by $p_{0/0} = \sum_0 \otimes (X_{0t}' X_{0t})^{-1}$ where X_{0t}=$\{Y_{0t-1}, \dots \dots . Y_{0t-P,} 1\}$, $\sum_0 = \frac{(Y_{0t} - X_{0t}\beta_0)'(Y_{0t} - X_{0t}\beta_0)}{T_0 - K}$ and the subscript 0 denotes the fact that this is the training sample. The scale matrix Q_0 is set equal to $p_{0/0} \times T_0 \times \tau$ where τ is a scaling factor.
- ***<u>Step2:</u>*** sample $\widetilde{\beta_t}$ conditional on R and Q from its conditional posterior distribution $H((\tilde{\beta}_T | R, Q, \widetilde{Y_t})$ where $\tilde{\beta}_T$=$[vec(\beta_1)' \quad vec(\beta_2)' \dots \dots \dots \quad vec(\beta_T)']$ and $\widetilde{Y_t} = [Y_1, \dots \dots \dots Y_T]$. This is done via the Gibbs sampling algorithm.

- ***Step3:*** sample Q from its conditional posterior distribution. Conditional on $\tilde{\beta}_t$ the posterior of Q is inverse Wishart with scale matrix $(\tilde{\beta}_t^1 - \tilde{\beta}_{t-1}^1)' \, \widetilde{(\beta}_t^1 - \tilde{\beta}_{t-1}^1) + Q_0$ and degrees of freedom $T + T_0$ where T denotes the lenght of the estimation sample and $\tilde{\beta}_t^1$ is the previous draw of the state variable $\widetilde{\beta}_t$.
- ***Step 4:*** Sample R from its conditional posterior distribution. Conditional on $\tilde{\beta}_t{}^1$ the posterior of R is inverse Wishart with scale matrix $(Y_t - (c_t^1 + \sum_{j=1}^{P} \beta^1{}_{j,t} \, Y_{t-j}))'$ $(Y_t - (c_t^1 + \sum_{j=1}^{P} \beta^1{}_{j,t} \, Y_{t-j})) + R_0$ and degrees of freedom $T + v_{R0}$.
- ***Step 5:*** Repeat steps 2 to 4 M times and use the last L draws for inference. This state space model requires a large number of draws for convergence.(e .g $M \geq 100000$).