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The Effect of Equity Market Liberalization on the Transmission of Monetary Policy. Evidence from Australia

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This paper investigates the effects of equity market integration on the transmission of monetary policy shocks. Based on the assumption that financial market liberalization and integration lead to falling portfolio holding costs, we analyze its effect on a two-country DSGE model with staggered prices and endogenous portfolio choice under incomplete markets. The model predicts that the reaction of stock prices, output and RER becomes muted upon impact and less persistence with falling portfolio holding costs. To test for a similar pattern in the data, we estimate a VAR with rolling coefficients for Australia, which provides a good case study. We identify a monetary policy shock with the sign restriction approach. The impulse responses generated by the data are consistent with the prediction of the model and imply that equity market liberalization seems to weaken the impact of monetary policy, at least on stock prices

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This paper investigates the effects of equity market integration on the transmission of monetary policy shocks. Based on the assumption that financial market liberalization and integration lead to falling portfolio holding costs, we analyze its effect on a two-country DSGE model with staggered prices and endogenous portfolio choice under incomplete markets. The model predicts that the reaction of stock prices, output and RER becomes muted upon impact and less persistence with falling portfolio holding costs. To test for a similar pattern in the data, we estimate a VAR with rolling coefficients for Australia, which provides a good case study. We identify a monetary policy shock with the sign restriction approach. The impulse responses generated by the data are consistent with the prediction of the model and imply that equity market liberalization seems to weaken the impact of monetary policy, at least on stock prices .

Key Words: Endogenous portfolio, Monetary Policy, Equity market liberalization, (S)FAVAR

JEL Classification: E52, C32, F21, F36

1 Introduction

The past two decades have witnessed profound changes in financial markets, and equity markets in particular, in form of greater international financial integration as documented, among others, by Lane and Milesi-Ferretti (2006). Naturally, financial market integration needs market liberalization in order to be accomplished in terms of assets

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substitutability conditions and perfect mobility. The mobility is perfect in the absence of capital and transactions control, institutional barriers and transaction cost. In this paper we focus on equity markets and model financial liberalization, and the consequent market integration, through a fall in portfolio holding costs. The latter can be associated with the drop in fixed banking commissions and the effects of mergers among stock exchanges observed in advanced countries during the last 20 years.

We maintain that understanding how financial integration alters the transmission of monetary policy shocks is a relevant avenue of research for both policy makers and scholars to the extent *i)* an increasing number of countries lift the strains of regulation on stock exchanges across the world and *ii)* changing transmission channels of monetary policy have important implications for the inflation-output trade-off and for the appropriate monetary policy response to asset prices, if any. We devote particular attention to stock prices because they play an important role in the transmission of monetary policy to the real economy, but the investigation of the consequences of financial integration on the monetary transmission channels is still scant in the literature.

One could argue that because financial integration makes markets more complete changes in the official interest rate are more readily transmitted to the whole term structure with a strenghtening of the asset price channel of monetary policy (see for instance Visco 2007). While this seems reasonable, in principle we can not exclude that portfolio adjustments and risk-sharing considerations could lead to a different conclusion.

Having this in mind, we attempt to address the question of whether and how equity market liberalization and international integration can influence the effects of monetary policy shocks on equity prices and real variables. In this study we first investigate the issue theoretically and then attempt to find evidence in the case of Australia.

We proceed to build a two-country DSGE model with endogenous portfolio choice in the spirit of the recent contributions by Devereux and Sutherland (2006, 2007). We introduce incompleteness of financial markets by assuming that investors face portfolio holding costs, which depend on the degree of market liberalization and on the market where equities are purchased. In particular, we argue that when markets are strongly regulated and the level of competition among trading firms is low, investors incur a cost on asset holdings. Similarly to Martin and Rey (2004), we think of such a cost in terms of banking commissions and variable fees. Not only, if markets are only partially open to international trade, purchasing abroad entails an extra cost related to the acquisition of information on an unfamiliar market.[1] As consequence, we assume that a domestic agent faces a certain cost on his/her holdings of domestic assets and an higher cost on holdings of foreign assets. As noted in Tille and van Wincoop (2008), the presence of these costs implies that financial markets are incomplete, even if the number of assets equals the number of shocks. To solve the model we follow their approach and assume that the cost is of second order, i.e. small enough to conduce to a well behaved portfolio allocation. Similarly to their model, since investing across border entails a (extra) cost with respect to domestic investment, the model predicts home bias in portfolio holdings.

To study the effect of financial integration on the monetary policy transmission, we generate impulse responses to a monetary policy shock before and after a fall in portfolio holding costs, which can be thought as consequence of the liberalization of the domestic equity market and, more in general, of a process of global financial integration.

An implication of the model is that the reaction of equity prices and some real variables, like output and real exchange rate, to domestic monetary shocks becomes weaker, in terms of impact and persistence, once equity markets are liberalized. Since the dynamics of the model are driven by consumers' behavior, we conjecture that the fall in the portfolio holding costs, by bringing the model closer to an environment of complete markets, widens the opportunities of risk-sharing. This reduces the need for portfolio reallocation in the face of the shock and therefore lowers the impact on equity prices.

To investigate whether a similar pattern is present in the data, we estimate rolling Vector Autoregressions (VARs) on Australian data. We identify monetary policy shocks with the sign restriction identification strategy developed recently by Canova and De Nicolo (2002).

[1] For more detailed discussion see Martin and Rey (2004).

The particular choice of the country is suggested by the fact that Australia has gradually liberalized its equity market industry. From the relaxation of restrictions on banking institutions in the early 1980's, which increased competition among trading firms, to the abolishment of fixed commission in 1983 and the government enforcement of merger of all stock exchanges in 1987, the liberalization took lace at a gradual pace through out the 1980's. Last but not least, Australia has high quality data during all that period.

The contribution of this paper is twofold. It provides empirical evidence that the effect of monetary policy shocks on asset prices has changed over time and suggests a theoretical explanation for it relying on the increased equity market liberalization and integration.

The remainder of the paper proceeds as follows: Section two provides a brief literature review of two-country DSGE models with endogenous portfolio choice. Section three describes the main building blocks of the open economy model (the detailed derivation is in Appendix A) with the solution for optimal portfolio and the model's impulse responses to a monetary policy contraction under falling portfolio holding costs. Section four describes the estimation strategy and the evidence from Australia. Section five concludes.

2 Related Literature

Related to this paper are two different strands of literature. Theoretical studies on portfolio composition and financial markets structure in DSGE models and empirical works on VAR to investigate the effect of monetary policy. We briefly introduce here the theoretical literature which is closely related to this paper, while the main references for the empirical application will be discussed in Section four.

In recent times the class of two-country DSGE models has been widely used to explore different aspects of trade integration but the contributions to modelling realistic financial liberalization/integration to assess the effect of monetary policy have been scant. Most contributions either assume only international trade of a real risk free bond (see for instance Svensson, 1989) or assume that financial markets are completely integrated, thus permitting agents to fully diversify consumption risk (see for instance Engel and Matsumoto (2006), Evans and Hnatkovska (2006), and Kollmann (2006)). Reality is probably somewhere in between. The main reason of this lack of realism is technical. The standard approach to solve DSGE models requires first order approximation of the model around a non-stochastic steady state, yet introducing equity markets requires the consideration of the riskiness of individual assets, which in turn requires information about the covariance of an asset with consumption. Obviously, considerations of risk are absent in a first order approximation and agents result indifferent about choosing one asset versus another. The portfolio choice problem is therefore indeterminate. To account for risk, (at least) second order Taylor expansion is necessary. Recent developments of the literature in this direction have permitted researchers to include more realistic structures of financial markets across countries in general equilibrium models.

Devereux and Sutherland (2006) provide an approximation method for computing equilibrium financial portfolios, which include any type of assets, in DSGE models. This contribution is mainly methodological. In 2007, however, the same authors incorporate in a standard two-country DSGE model, with staggered prices, the optimal portfolio choice to investigate the effect of monetary policy. The general set up of this paper is very similar to ours. The crucial differences are in the way market incompleteness is modelled and in the spirit of the investigation. Devereux and Sutherland are concerned about the link between monetary policy and national asset portfolios. Their emphasis is on the impact of market incompleteness on the distribution of returns on nominal assets and on the role of price stability in the optimal monetary policy rule. Our focus is on how financial liberalization/integration affects monetary policy transmission mechanisms when portfolio choice is endogenous and financial markets incomplete. Tille and van Wincoop (2007), independently and simultaneously, develop a solution method for DSGE models with portfolio choice that is essentially the same as Devereux and Sutherland (2006), yet they go one step forward. They investigate the implications of portfolio choice for both gross and net international capital flows and the time-variation in portfolio allocation following shocks. By adopting the distinction between steady state portfolio shares and change in optimal portfolio, they show how endogenous time-variation in expected returns and risk affect capital flows. In doing so, they incorporate a small second order portfolio holding cost to study the effect of an increment in financial market completeness. Similarly to them, we are interested in the effect of increasing market completeness, and we borrow from them the way of modeling it. Yet, we are concerned about changes in the monetary policy transmission mechanism rather than the time dimension of portfolio determinants.

3 The model

In this paper, we extend the model of Devereux and Sutherland (2007) by introducing market incompleteness, as in Tille and van Wincoop (2008), through a small cost on equity holdings. This will permit to study the effects of changes in portfolio holding costs on the transmission of monetary policy shocks to equilibrium portfolio shares, equity prices and the real side of the economy.

In the following a two-country dynamic general equilibrium model with price rigidities and portfolio choice is derived. The two countries, Home and Foreign, have equal size and are symmetric. Agents maximize utility over an infinite horizon and consume both domestic and imported goods, but they have preference for locally produced goods, implying home bias in consumption. They supply homogeneous labor and hold a portfolio that includes four different assets, Home and Foreign bonds as well as Home and Foreign equities. Equities are claims to firms' profits. Each firm produces a single differentiated good.

As in Devereux and Suterland (2006, 2007) the model is split in two parts: the portfolio part and the non-portfolio part. We start with the former.

3.1 Consumers

We assume a standard CRRA utility function, where infinitely lived agents derive utility from consumption and incur disutility from labor as follows:

$$U = E_0 \sum_{\tau=t}^{\infty} \eta_t \left[\frac{C_t^{1-\rho}}{1-\rho} - KL_t \right] \tag{1}$$

where E is the expectation operator, C is a consumption index defined across Home and Foreign goods, $\rho > 0$ is a measure of risk aversion, L is labor supply and η_t the endogenous discount factor defined by the following recursion:

$$\eta_t = \beta_t \eta_{t-1}$$

One caveat of DSGE models with incomplete financial markets is that they are non-stationary in the sense that the dynamics may not be independent of initial conditions. It is possible therefore that temporary shocks have permanent effects and one country eventually owns the whole world wealth. In order to rectify this problem, we use an endogenous discount factor.[2] We follow Ferrero et al. (2008) assuming that the endogeneity is not internalized by the agents so to have the following functional form for β_t:

$$\beta_t \equiv \frac{1}{1 + \psi \left(\log \overline{C}_t - \vartheta \right)} \tag{2}$$

where $\bar{C}_t$ is the average consumption and is taken as exogenous by the household in the optimization problem.[3]

The Home agent's consumption basket C consists of Home and Foreign goods with

[2] This method was developed originally by Uzawa (1969) and implemented by Mendoza (1991). Uribe and Schmitt-Grohe (2003) suggest this as one way to close small open economy models with incomplete markets. Bodenstein (2006) notes that while there are different methods of closing NOEM models with incomplete markets, the restrictions of an endogenous discount factor is suggested as the only one which always gives a unique solution to the model.

[3] Yet, at the steady state $C_t = \overline{C}_t$. As consequence in the first order conditions β_t depends on C_t and the system of equations becomes stationary (See Appendix A). The economic intuition of such a structure is that there is a positive spillover from average consumption to individual consumption. Higher average consumption induces individuals to want to consume more today relative to the future, meaning, β_t decreases. On the other hand, greater indebtness reduces borrower's consumption, raising the discount factor and inducing saving (Uzawa, 1968). This ensures that there is a determinate steady state in a model with international borrowing and lending.

elasticity of substitution of θ and takes the explicit form of a CES index:

$$C \equiv \left[\mu^{\frac{1}{\theta}} C_h^{\frac{\theta-1}{\theta}} + (1-\mu)^{\frac{1}{\theta}} C_f^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where μ is the degree of preference for the domestically produced goods, if $\mu > 0.5$, the country is characterized by home bias. C_h and C_f are the domestic goods and the imported goods components respectively of the Home consumption basket and are themselves indices. C_h is a CES index of Home goods monopolistically produced:

$$C_h \equiv \left[\int_0^1 \left(C_h(i)\right)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}}$$

where ϕ is the elasticity of substitution between varieties produced within the country; hence it is the parameter that governs the markup.

Analogously C_f is the index of Home imported goods and is given by:

$$C_f \equiv \left[\int_0^1 \left(C_f(i)\right)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}}$$

It can then be shown that under this formulation the aggregate price index is:

$$P = \left[\mu P_h^{1-\theta} + (1-\mu) P_f^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

where P_f and P_h are the price indices for Foreign and Home goods respectively.

The Home agent faces the following period budget constraint:

$$P_t C_t + P_t W_{t+1} = w_t L_t + P_t \Pi_t + P_{t-1} \left(\begin{array}{c} \alpha_{hb,t-1} r_{hb,t} \frac{P_t}{P_{t-1}} + \alpha_{fb,t-1} r_{fb,t} \frac{P_t}{P_{t-1}} + \\ \alpha_{he,t-1} e^{-\delta_{he}} r_{he,t} \frac{P_t}{P_{t-1}} + \alpha_{fe,t-1} e^{-\delta_{fe}} r_{fe,t} \frac{P_t}{P_{t-1}} \end{array} \right) \tag{4}$$

The right hand side includes the different sources of income. The term $w_t L_t$ is the nominal labour income and $P_t \Pi_t$ the profits on Home firms. Since the nominal Home output equals the sum of income accruing to labour and profits, it must hold that: $w_t L_t + P_t \Pi_t = P_{h,t} Y_t$. The Home agent is the default owner of Home firms, however international trade of Home equity transfers claims on Home profits to Foreign agents. Similarly, trade of Foreign equity transfers claims on Foreign firms profits to Home agents. The last term in parentheses represents the total return (in nominal terms) on the Home country portfolio including four tradable assets: Home and Foreign bonds (hb, fb) and Home and Foreign equities (he, fe). The term $r_{k,t}$ is the real return and $\alpha_{k,t-1}$ is the real holding of asset k purchased at the end of period $t-1$ and carried into period t. Holding equity entails a cost, which reduces their real return. We assume that the return on the equity purchased abroad is reduced by an iceberg cost of the form $e^{-\delta} < 1$. In particular, δ_{fe} is the cost that the Home agent has to pay for holding Foreign equities (symmetrically δ^*_{he} is the cost that the Foreign agent will have to pay for holding Home equities). Not only, we argue that when the Home market is regulated, the Home agent incurs a cost δ_{he} on Home equity holdings as well . The idea is that when competition among trading firms is low because of regulation, buyers are required to pay (banking) commissions and other variables fees. The costs δ are small enough (second order) to get well behaved portfolio at the steady state and to deliver incomplete market even when the number of shocks equals the number of internationally traded assets.

The left hand side of eq. (4) describes the allocation of resources between consumption and purchase of assets. As in Devereux and Suterland (2006), we define the *net* value of the real wealth (Net foreign asset) of the Home agent as follows:

$$W_t = \sum_{k=1}^{4} \alpha_{k,t-1}$$

In economic terms this means that the total investment in period $t-1$ must add up to wealth at the beginning of time t.

A step by step derivation of this model, together with all of the log-linearized conditions can be found in Appendix A. For the sake of brevity, we present here only the results of the optimization.

Optimal intratemporal allocation of expenditure yields:

$$C_h = \mu \left(\frac{P_h}{P}\right)^{-\theta} C \quad ; \quad C_f = (1-\mu)\left(\frac{P_f}{P}\right)^{-\theta} C$$

The first order conditions for labor, and the four Euler equations for bonds and equities are given, respectively, by

$$K = \frac{w_t}{P_t} C_t^{-\rho}$$

$$\begin{aligned}
C_t^{-\rho} &= \beta_{t+1} E_t(C_{t+1}^{-\rho} r_{hb,t+1}) \\
C_t^{-\rho} &= \beta_{t+1} E_t(C_{t+1}^{-\rho} r_{fb,t+1}) \\
C_t^{-\rho} &= \beta_{t+1} E_t(C_{t+1}^{-\rho} e^{-\delta_{he}} r_{he,t+1}) \\
C_t^{-\rho} &= \beta_{t+1} E_t(C_{t+1}^{-\rho} e^{-\delta_{fe}} r_{fe,t+1})
\end{aligned}$$

The solution method we employ requires rewriting the model in terms of excess returns. Without loss of generality, we thus designate the Foreign bond as the reference asset and we rewrite the Euler equations as below:

$$0 = E_t \beta_{t+1} C_{t+1}^{-\rho} \left(r_{hb,t+1} - r_{fb,t+1} \right) \tag{5a}$$

$$0 = E_t \beta_{t+1} C_{t+1}^{-\rho} \left(e^{-\delta_{he}} r_{he,t+1} - r_{fb,t+1} \right) \tag{5b}$$

$$0 = E_t \beta_{t+1} C_{t+1}^{-\rho} \left(e^{-\delta_{fe}} r_{fe,t+1} - r_{fb,t+1} \right) \tag{5c}$$

3.2 Firms

Each firm produces using only homogeneous labor, L. Let $Y(i)$ be the output of firm i and A_t the productivity factor that is common across firms within the country. We assume that production is linear in labor input:

$$Y(i) = A_t L(i)$$

with A_t evolving according to an autoregressive process of this form:

$$\log\ A_t = \varsigma_a \log\ A_{t-1} + u_t$$

where $0 \leq \varsigma_a \leq 1$ and u_t is an $i.i.d.$ shock with $E_{t-1}[u_t] = 0$ and $Var[u_t] = \sigma_u^2$.

We assume that each firm acts in an environment of monopolistic competition where it produces a differentiated good that it sells on both domestic and foreign markets where it sets the price according to producer currency pricing (PCP), i.e. in its own currency, regardless of the nationality of the market where the good is actually sold.[4] We also suppose that the firm's ability to set the price is constrained exogenously and prices evolve on a staggered basis. To account for this feature, we include price stickiness à la Calvo (1983), so that, every period, each firm faces the same probability $(1-\kappa)$ to reset its price. From profits maximization we can write out the dynamics of the newly set price as:

$$\widetilde{P}_{h,t} = \frac{\phi}{1-\phi} \frac{E_t \sum_{i=0}^{\infty} \Gamma_{t+i} \frac{w_{t+i}}{A_{t+i}} X_{h,t+i}}{E_t \sum_{i=0}^{\infty} \Gamma_{t+i} X_{h,t+i}}$$

where $X_{h,t+i}$ represents the demand a firm faces for its output[5] and Γ_{t+i} is the discount factor to evaluate future profits. Since financial markets are incomplete, there is an open question as to what exactly determines Γ_{t+i}. Here we follow the argument of Devereux and Sutherland (2007), according to which, since the non-portfolio part of the model is solved by first order approximation around a stationary steady state, the discount factor is simply the consumer discount factor, i.e. β.[6]

Given the Calvo price setting, the dynamics of the Home price index are governed by the following law of motion:

$$P_{h,t} = \left[(1-\kappa) \widetilde{P}_{h,t}^{1-\phi} + \kappa P_{h,t-1}^{1-\phi} \right]^{\frac{1}{1-\phi}}$$

Combining the last two equations and linearizing lead to the following New Keynesian Phillips curve:

$$\pi_{h,t} = \lambda^{-1} \left(\rho \widehat{C}_t + \widehat{P}_t - \widehat{u}_t - \widehat{P}_{h,t} \right) + \beta E_t \pi_{h,t+1}$$

[4] In this we follow Devereux and Sutherland (2007). LCP setting is probably more realistic but it would increase the complexity of the model delivering a much more complex steady state. Since the main focus of the paper is on international portfolio, we prefer to keep this part as simple a spossible.

[5] The total demand, Home and Foreign, for the Home firm output is defined as follows:

$$X_{h,t}(i) = \mu \left(\frac{\widetilde{P}_{h,t}}{P_{h,t}} \right)^{-\phi} \left(\frac{P_{h,t}}{P_t} \right)^{-\theta} C_t + (1-\mu) \left(\frac{\widetilde{P}_{h,t} S_t}{P_{h,t} S_t} \right)^{-\phi} \left(\frac{P^*_{h,t}}{P^*_t} \right)^{-\theta} C^*_t$$

[6] See Devereux and Sutherland (2007) for a detailed discussion on this point.

where $\lambda = \left(\frac{(1-\beta\kappa)(1-\kappa)}{\kappa}\right)^{-1}$ is a measure of price stickiness arising from Calvo price-adjustment restriction and the term in the parentheses is the marginal cost of production.

3.3 Monetary Authorities

We close the non-portfolio part of the model by supposing that monetary policy obeys a simple interest-rate rule according to which the nominal rate depends on the steady-state natural rate of interest in the frictionless zero inflation equilibrium and on the gross PPI inflation rate.[7] We also assume that the rate is subject to stochastic monetary shocks. Lastly, since in the model there is only one interest rate, we impose that the policy instrument corresponds to the interest rate on nominal bonds $r^n_{hb,t+1}$, so that the policy rule can be represented as follows:

$$r^n_{hb,t+1} = \beta^{-1}\left(\frac{P_{h,t}}{P_{h,t-1}}\right)^{\gamma} \exp m_t$$

where m_t is a stochastic i.i.d. shock to the interest rate such that $E_{t-1}[m_t] = 0$ and $Var[m_t] = \sigma^2_m$.

In what follows we introduce the elements of the portfolio part of the model.

3.4 Portfolio Assets

The model includes four assets, Home and Foreign bonds and equities. The Home equity is a claim on Home aggregate profits and a unit of it purchased in period t yields a real payoff equal to $\Pi_{t+1} + Z_{he,t+1}$, where Π_{t+1} are aggregate Home profits and $Z_{he,t+1}$ the real price of the Home equity. As a result, the gross real rate of return on the Home equity is the sum of capital gains and dividend yields:

$$r_{he,t+1} = \frac{\Pi_{t+1} + Z_{he,t+1}}{Z_{he,t}}.$$

[7] The formulation of the policy rule is borrowed from Devereux and Sutherland (2007). As they point out, the choice of PPI, inflation instead of CPI, follows the well known result in the literature that in (complete markets) open economy without "cost-push" shocks, it is optimal to stabilize PPI inflation.

The Home nominal bond is a zero-coupon bond paying 1 unit of Home currency in the following period. Hence, the real pay off of a bond purchased in t and carried into $t+1$ is $1/P_{t+1}$. Given the real price of the bond $Z_{hb,t}$, the gross real return on the Home bond is:

$$r_{hb,t+1} = \frac{1}{P_{t+1} Z_{hb,t}}$$

Since we assumed that the central bank controls the nominal interest rate $r^n_{hb,t+1}$, the following relationship holds:

$$r^n_{hb,t+1} = r_{hb,t+1} \frac{P_{t+1}}{P_t} = \frac{1}{P_t Z_{hb,t}}$$

3.5 Foreign Economy

The Foreign economy has an analogous representation as the Home economy. Consumers choose their labor effort, portfolio and consumption subject to the following budget constraint (in terms of Home consumption basket):

$$P^*_t C^*_t + \frac{P^*_t}{Q_t} W^*_{t+1} = w^*_t L^*_t + P^*_t \Pi^*_t + \frac{P^*_t}{Q_t} \left(\alpha^*_{hb,t-1} r_{hb,t} + \alpha^*_{fb,t-1} r_{fb,t} + \alpha^*_{he,t-1} e^{-\delta_{he}} r_{he,t} + \alpha^*_{fe,t-1} r_{fe,t} \right)$$

where $Q_t = P^*_t S_t / P_t$ is the real exchange rate and it enters the budget constraint because C^*_t and Y^*_t are measured in terms of Foreign aggregate consumption, while the returns are in terms of Home consumption basket. The first order conditions are analogous to the ones of the Home agent, but the portfolio holding cost falls now only on Home equity[8]:

$$\begin{aligned}
C_t^{*-\rho} &= E_t \beta^*_{t+1} C_{t+1}^{*-\rho} r_{hb,t+1} \\
C_t^{*-\rho} &= E_t \beta^*_{t+1} C_{t+1}^{*-\rho} r_{fb,t+1} \\
C_t^{*-\rho} &= E_t \beta^*_{t+1} C_{t+1}^{*-\rho} e^{-\delta^*_{he}} r_{he,t+1} \\
C_t^{*-\rho} &= E_t \beta^*_{t+1} C_{t+1}^{*-\rho} r_{fe,t+1}
\end{aligned}$$

[8] Since we focus on the Home economy, for semplicity, we assume that there is not cost for the Foreign agent on its domestic equity holdings.

As for the Home country, we can express everything in terms of excess return. Since α_k is net holdings of asset k in the Home country, the condition $\alpha_t = -\alpha_t^*$ must hold and the net debt of an agent is the net asset of the other so that:

$$W_t = -W_t^*$$

Foreign monetary authorities follow a rule analogous to the one of the Home country:

$$r_{fb,t+1}^{*n} = \beta^{-1} \left(\frac{P_{f,t}^*}{P_{f,t-1}^*} \right)^{\gamma} \exp m_t^*$$

3.6 Aggregate demand and goods market clearing

With identical producers within each region, at the equilibrium, production depends on preferences and the level of consumption in the two countries. The goods market clearing conditions are:

$$Y_t = \left[\mu \left(\frac{P_{h,t}}{P_t} \right)^{-\theta} C_t + (1-\mu) \left(\frac{P_{h,t}^*}{P_t^*} \right)^{-\theta} C_t^* \right] \tag{6}$$

$$Y_t^* = \left[(1-\mu) \left(\frac{P_{f,t}}{P_t} \right)^{-\theta} C_t + \mu \left(\frac{P_{f,t}^*}{P_t^*} \right)^{-\theta} C_t^* \right] \tag{7}$$

3.7 Model Solution

The standard solution approach of two-country DSGE models consists in linearizing the model around a non-stochastic steady state and then apply a solution algorithm, like the one suggested by Klein, yielding model's reaction to a certain shock. However, as mentioned above, in this model, portfolio choice is endogenous and two problems arise. First, the concept of portfolio choice is meaningless in a non-stochastic steady state by definition. Second, to a first order approximation all assets are perfect substitute and the portfolio composition is undetermined. Since assets vary in their riskiness with incomplete markets, it is necessary to take a second order Taylor expansion in order to obtain information about the covariances of individual returns with consumption. This does indeed raise the complexity of the solution. Yet, as shown by Devereux and

Sutherland (2006), one can solve the model including portfolio holdings by taking the second order expansion only of the portfolio part of the model, and in particular of the following equations:

$$
\begin{aligned}
0 &= E_t \beta_{t+1} C_{t+1}^{-\rho} \left(r_{hb,t+1} - r_{fb,t+1} \right) \\
0 &= E_t \beta_{t+1} C_{t+1}^{-\rho} \left(e^{-\delta_{he}} r_{he,t+1} - r_{fb,t+1} \right) \\
0 &= E_t \beta_{t+1} C_{t+1}^{-\rho} \left(e^{-\delta_{fe}} r_{fe,t+1} - r_{fb,t+1} \right)
\end{aligned}
$$

The rest of the model's equilibrium conditions only need to be approximated up to a first order.

3.7.1 Portfolio Solution

We describe the solution method intuitively here and put all of the derivations along with more detailed explanations in Appendix A. The solution algorithm we use consists of two stages. In the first stage we treat the portfolio holdings α_t as a stochastic unknown[9] and obtain a solution conditional on that assumption. Following the derivations in the Appendix A, it is then possible to calculate the implied value for each α_t. In the second stage we can then obtain the full solution, since α_t is now known.

Accordingly, we take a second order approximation of the portfolio selection equations, which yields:

$$
\begin{aligned}
0 &= E_t \left\{ \left(\widehat{r}_{hb,t+1} - \widehat{r}_{fb,t+1} \right) - \left(\rho + \psi \beta \right) \widehat{C}_{t+1} \left(\widehat{r}_{hb,t+1} - \widehat{r}_{fb,t+1} \right) + \frac{1}{2} \left(\widehat{r}_{hb,t+1}^2 - \widehat{r}_{fb,t+1}^2 \right) \right\} \\
\delta_{he} &= E_t \left\{ \left(\widehat{r}_{he,t+1} - \widehat{r}_{fb,t+1} \right) - \left(\rho + \psi \beta \right) \widehat{C}_{t+1} \left(\widehat{r}_{he,t+1} - \widehat{r}_{fb,t+1} \right) + \frac{1}{2} \left(\widehat{r}_{he,t+1}^2 - \widehat{r}_{fb,t+1}^2 \right) \right\} \\
\delta_{fe} &= E_t \left\{ \left(\widehat{r}_{fe,t+1} - \widehat{r}_{fb,t+1} \right) - \left(\rho + \psi \beta \right) \widehat{C}_{t+1} \left(\widehat{r}_{fe,t+1} - \widehat{r}_{fb,t+1} \right) + \frac{1}{2} \left(\widehat{r}_{fe,t+1}^2 - \widehat{r}_{fb,t+1}^2 \right) \right\}
\end{aligned}
$$

where all variables with a hat are in log-deviations from the steady state. As previously explained, δ_{fe} and δ_{he} are costs of second order magnitude, which therefore only appears at the second order of approximation. Defining the excess return for each asset with respect to the reference asset as: $\hat{r}_{x,i,t+1} = \widehat{r}_{i,t+1} - \widehat{r}_{fb,t+1}$, and $\hat{r}_{x,i,t+1}^2 = \hat{r}_{i,t+1}^2 - \hat{r}_{fb,t+1}^2$

[9] Up to first order of approximation.

the set of equivalent Euler equations for the Foreign country, becomes:

$$
\begin{aligned}
0 &= E_t\hat{r}_{x,hb,t+1} - (\rho + \psi\beta)\, E_t\hat{C}^*_{t+1}\hat{r}_{x,hb,t+1} + \frac{1}{2}E_t\hat{r}^2_{x,hb,t+1} - \hat{r}_{x,he,t+1}\left(\hat{Q}_{t+1} - \hat{Q}_t\right) \\
\delta^*_{he} &= E_t\hat{r}_{x,he,t+1} - (\rho + \psi\beta)\, E_t\hat{C}^*_{t+1}\hat{r}_{x,he,t+1} + \frac{1}{2}E_t\hat{r}^2_{x,he,t+1} - \hat{r}_{x,he,t+1}\left(\hat{Q}_{t+1} - \hat{Q}_t\right) \\
0 &= E_t\hat{r}_{x,fe,t+1} - (\rho + \psi\beta)\, E_t\hat{C}^*_{t+1}\hat{r}_{x,fe,t+1} + \frac{1}{2}E_t\hat{r}^2_{x,fe,t+1} - \hat{r}_{x,he,t+1}\left(\hat{Q}_{t+1} - \hat{Q}_t\right)
\end{aligned}
$$

Subtracting each of the Foreign country equation from the corresponding one for Home yields covariances between the excess return and the consumption difference that describe how well one can hedge consumption risk with the asset considered:

$$
0 = E_t\left(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1} - \hat{Q}_t}{\rho + \psi\beta}\right)\hat{r}_{x,hb,t+1} \tag{9}
$$

$$
\delta^*_{he} - \delta_{he} = E_t\left(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1} - \hat{Q}_t}{\rho + \psi\beta}\right)\hat{r}_{x,he,t+1} \tag{10}
$$

$$
-\delta_{fe} = E_t\left(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1} - \hat{Q}_t}{\rho + \psi\beta}\right)\hat{r}_{x,fe,t+1} \tag{11}
$$

Taking the sum of each of the Home Euler equations and corresponding Foreign equations (see equations (40) (41) (42) in Appendix A), a crucial property of the model shows up. All the terms in the equations are second order (products of first order and costs), except the expected excess return. Hence, it must be the case that, to a first order approximation, the following condition holds:

$$
E_t\widehat{r}_{x,i,t+1} = 0
$$

This says that up to a first order all assets are perfect substitutes in expectation and excess returns unpredictable, as a consequence the latter can be treated as shocks, i.e. $\hat{r}_{x,i,t+1}\forall\ i$ is a zero-mean i.i.d. process. We will exploit this property to derive the equilibrium solution for α_i. To do so, it is first necessary to expand the budget constraint. Since second order solutions for the second moments can be obtained from the realized values of the first order solution, the second order cost does not enter the budget constraint, which then only needs to be approximated to the first order.

This last point can be seen looking at the linearized budget constraint for the Home agent:

$$\hat{W}_{t+1} = \sum_{i=1}^{4} \widetilde{\alpha}_i(0)\,\hat{r}_{x,i,t} + \frac{1}{\beta}\hat{W}_t + \left[\hat{Y}_t + \widehat{P}_{H,t} - \hat{C}_t - \widehat{P}_t\right]$$

where $\tilde{\alpha}_i(0) = \frac{\alpha_i(0)}{C(0)\beta}$. Note that $\tilde{\alpha}_i(0)$ is defined relative to steady state GDP rather than a log deviation. We can see that there are no terms in $\widehat{\alpha}_i$,(deviations of gross holdings from their value at the approximation point), only the zero order portfolio allocation enters. Moreover, since $\hat{r}_{x,i,t}$ is a zero-mean i.i.d. process, it must be true that $\sum_{i=1}^{4} \widetilde{\alpha}_i(0)\,\hat{r}_{x,i,t}$ is also a zero-mean i.i.d. process. This property is exploited by temporarily replacing $\sum_{i=1}^{4} \widetilde{\alpha}_i(0)\,\hat{r}_{x,i,t}$ with exogenous i.i.d. processes $\sum_{i=1}^{4} \xi_{,i,t}$ which are treated as separate exogenous shocks in the model. It is now possible to cast the model in state space form to get the solution for predetermined state and control variables. As shown in Appendix A, by extracting from the solution matrix the relevant rows for the excess return and the relative consumption and using the results in equations (9), (10) (11), one can find the solution for the portfolio holdings $\tilde{\alpha}$:

$$\tilde{\alpha} = \left[D_1 R_2 \Sigma R_2' - R_2 \Sigma D_2' R_1'\right]^{-1} \left[\Delta - R_2 \Sigma D_2'\right]$$

where $\Delta = \begin{pmatrix} \delta_{he}^{*} - \delta_{he} \\ 0 \\ -\delta_{fe} \end{pmatrix}$ and D_1 D_2 R_1 R_2 and Σ are defined as in the appendix.

Since $\tilde{\alpha}$ is known now, we replace $\xi_{,i,t}$ with the corresponding $\widetilde{\alpha}_i(0)\,\hat{r}_{x,i,t}$. At this point we have a system, with portfolio choice without any second order terms, which can be solved, in the second step, with standard methods. To do so, we use the solution algorithm suggested by Klein (2000).

3.7.2 The Loglinear Non-portfolio Part of the Model

We now characterize the loglinear system of the Home and Foreign country. Combining the first order approximation of the Euler equations for the two countries yields the cross-country Euler:

$$\left(\rho+\psi\beta\right)E_t\hat{C}_{t+1}-\rho\hat{C}_t=\left(\rho+\psi\beta\right)E_t\hat{C}^*_{t+1}-\rho\hat{C}^*_t+E_t\left(\hat{Q}_{t+1}-\hat{Q}_t\right) \tag{12}$$

Home real output equals the total demand for locally produced goods, so that the goods-market clearing condition is:

$$\widehat{Y}_t=\mu\widehat{C}_t+(1-\mu)\widehat{C}^*_t+2\theta\mu(1-\mu)\widehat{\tau}_t \tag{13}$$

Similarly the Foreign goods-market clearing is given by:

$$\widehat{Y}^*_t=(1-\mu)\widehat{C}_t+\mu\widehat{C}^*_t-2\theta\mu(1-\mu)\widehat{\tau}_t \tag{14}$$

The combination of Calvo pricing equation, the Home goods price index and the definition of marginal costs leads to the open economy version of the New-Keynesian Phillips curve as shown above:

$$\pi_{h,t}=\lambda^{-1}\left(\rho\widehat{C}_t-\widehat{u}_t+(1-\mu)\,\widehat{\tau}_t\right)+\beta E_t\pi_{h,t+1} \tag{15}$$

By symmetry for the Foreign country, the NKPC is given by:

$$\pi^*_{f,t}=\lambda^{-1}\left(\rho\widehat{C}^*_t-\widehat{u}^*_t-(1-\mu)\,\widehat{\tau}_t\right)+\beta E_t\pi^*_{f,t+1} \tag{16}$$

We next turn to Home monetary policy:

$$\begin{aligned}\widehat{r}^n_{hb,t+1} &= \gamma\pi_{h,t}+m_t\\ E_t\pi_{t+1}+\left(\rho+\psi\beta\right)E_t\hat{C}_{t+1} &= \rho\widehat{C}_t+\gamma\pi_{h,t}+m_t\end{aligned} \tag{17}$$

and Foreign monetary policy:

$$\left(\rho+\psi\beta\right)E_t\hat{C}^*_{t+1}+E_t\widehat{\pi}^*_{t+1}=\rho\widehat{C}^*_t+\gamma\pi^*_{f,t}+m^*_t \tag{18}$$

We then add the definitions of the returns and the prices of the menu of assets existing in the model. The return on the Home equity is:

$$\hat{r}_{he,t+1} + \widehat{Z}_{he,t} = (1-\beta)\widehat{\Pi}_{t+1} + \beta\widehat{Z}_{he,t+1} \tag{19}$$

the return on the Foreign equity:

$$\hat{r}^*_{fe,t+1} + \widehat{Z}^*_{fe,t} = (1-\beta)\widehat{\Pi}^*_{t+1} + \beta\widehat{Z}^*_{fe,t+1} \tag{20}$$

the return on the Home bond:

$$\widehat{r}_{hb,t} = -\widehat{P}_t - \widehat{Z}_{hb,t-1} \tag{21}$$

the return on the Foreign Bond:

$$\begin{aligned} \widehat{r}_{fb,t} - \left(\widehat{Q}_t - \widehat{Q}_{t-1}\right) &= -\widehat{P}^*_t - \widehat{Z}^*_{fb,t-1} \\ \widehat{r}_{fb,t} &= -\widehat{\pi}^*_t - \widehat{Z}^{*n}_{fb,t-1} + \left(\widehat{Q}_t - \widehat{Q}_{t-1}\right) \end{aligned} \tag{22}$$

the price of the Home Bond (nominal):

$$\begin{aligned} -\rho\widehat{C}_t + \widehat{Z}_{hb,t} &= E_t\left\{-\rho\left(1+\frac{\psi\beta}{\rho}\right)\hat{C}_{t+1} - \widehat{P}_{t+1}\right\} \\ \widehat{Z}^n_{hb,t} &= \rho\widehat{C}_t - E_t\left\{(\rho+\psi\beta)\,\hat{C}_{t+1} + \widehat{\pi}_{t+1}\right\} \end{aligned} \tag{23}$$

the price of the Foreign Bond (nominal):

$$\begin{aligned} -\rho\widehat{C}^*_t + \widehat{Z}^*_{fb,t} &= E_t\left\{-\rho\left(1+\frac{\psi\beta}{\rho}\right)\hat{C}^*_{t+1} - \widehat{P}^*_{t+1}\right\} \\ \widehat{Z}^{*n}_{fb,t} &= \rho\widehat{C}^*_t - E_t\left\{(\rho+\psi\beta)\,\hat{C}^*_{t+1} + \widehat{\pi}^*_{t+1}\right\} \end{aligned} \tag{24}$$

the price of the Home Equity:

$$\widehat{Z}_{he,t} = -\left(\rho+\psi\beta\right)E_t\widehat{C}_{t+1} + \rho\widehat{C}_t + (1-\beta e^{-\tau})E_t\widehat{\Pi}_{t+1} + \beta e^{-\tau}E_t\widehat{Z}_{he,t+1} \tag{25}$$

the price of the Foreign Equity:

$$\widehat{Z}^*_{fe,t} = -\left(\rho + \psi\beta\right) E_t\widehat{C}^*_{t+1} + \rho\widehat{C}^*_t + (1-\beta)E_t\widehat{\Pi}^*_{t+1} + \beta E_t\widehat{Z}^*_{fe,t+1} \tag{26}$$

Lastly we add the predetermined endogenous variable, i.e. the state variable net portfolio wealth:

$$\hat{W}_{t+1} = \frac{1}{\beta}\hat{W}_t + \left[\hat{Y}_t + \widehat{P}_{h,t} - \hat{C}_t - \widehat{P}_t\right] + \widetilde{\alpha}_i\left(0\right)\hat{r}_{x,t} \tag{27}$$

In practice, to solve the model we include few other equations describing production, profits, CPI inflation for both countries and three excess returns. Finally we denote c_t the vector of the control variables and s_t as the state variables:

$c_t = [\widehat{C}_t,\ \widehat{C}^*_t,\ \widehat{Y}_t,\ \widehat{Y}^*_t,\ \widehat{\tau}_t,\ \widehat{\pi}_{h,t},\ \widehat{\pi}^*_{f,t},\ \widehat{\pi}_t,\ \widehat{\pi}^*_t,\ \widehat{\Pi}_t,\ \widehat{\Pi}^*_t,\ \widehat{r}_{x,he,t},\ \widehat{r}_{x,fe,t},\ \widehat{r}_{x,hb,t},\ \widehat{Z}_{he,t},\ \widehat{Z}^*_{fe,t},$ $\widehat{Z}^n_{hb,t},\ \widehat{Z}^{*n}_{fb,t},\ \widehat{r}_{hb,t},\ \widehat{r}_{fb,t},\ \widehat{r}_{he,t},\ \widehat{r}_{fe,t},\ \widehat{Q}_t,\ \widehat{C}^D_t,\ \widehat{L}_t,\ \widehat{L}^*_t]$

$s_t = [\widehat{W}_t,\ \widehat{\tau}_{t-1},\ \widehat{Z}_{he,t-1},\ \widehat{Z}^*_{fe,t-1},\ \widehat{Z}^n_{hb,t-1},\ \widehat{Z}^{*n}_{fb,t-1}]$

We will use this model to study the theoretical effect of equity market liberalization on the transmission mechanism of monetary policy.

3.8 Theoretical Results

We first describe how we calibrate the model. Then we explore the behavior of the model in response to a monetary policy contraction under falling portfolio holding cost.

3.8.1 Calibration

The model is assumed to be quarterly. The parameter that governs the open economy dimension of the model is the elasticity of substitution between Foreign and Home goods, θ, in most of the literature this takes a value between 1 and 2, here we set it at 1.2.

There are seven additional parameters, five of them are standard and calibrated in a standard way: the steady state discount factor β is set equal to 0.99, the elasticity of substitution between domestic goods ϕ= 6 so to deliver a steady state mark-up of 20%, and the risk aversion in the utility function $\rho = 1.2$. The probability that prices do not adjust, κ, is set at 0.75 so to imply a mean duration that a price is fixed for

4 quarters, as in large part of the literature. The parameter of monetary response to inflation in the policy rule γ is set to 1.5, according to the Taylor principle. The other two parameters are: another preference parameter ψ, which determines the effect of consumption on the discount factor and the costs on holding equities. For the first, we follow the same reasoning as in Ferrero et al. (2007) and ψ is set arbitrarily, but such that the model is stationary and medium term dynamics not altered. In this perspective, we choose 0.056.[10] Finally we assume that before financial liberalization the cost of holding equities purchased abroad is the same for both Home and Foreign agents and equal to: $\delta_{fe} = \delta^*_{he} = 0.45\%$. This value close to 0.419% used in the Tille and vanWincoop (2008), whereas the cost on Home equity holding is lower and given by and $\delta_{he} = 0.15\%$. We assume that once financial liberalization has taken place, the costs fall but in an asymmetric way so as to capture the idea that in the Home country this process has been faster and deeper than in the Foreign country. In order to do so, we assume that δ_{he} and δ^*_{he} drop to zero, while investing across border still entails a positive cost for the Home agent and δ_{fe} becomes 0.1%.[11]

Lastly, we turn to the parameter governing the process for productivity and money shocks. We assume that the money shock follows an autoregressive process:

$$m_t = \zeta_m m_{t-1} + \epsilon_{m,t}$$

with persistence ζ_m=0.85. The productivity shock is also an AR(1)

$$a_t = \zeta_a a_{t-1} + \epsilon_{a,t}$$

with $\zeta_a = 0.9$. $\epsilon_{m,t}$ and $\epsilon_{a,t}$ are zero mean.

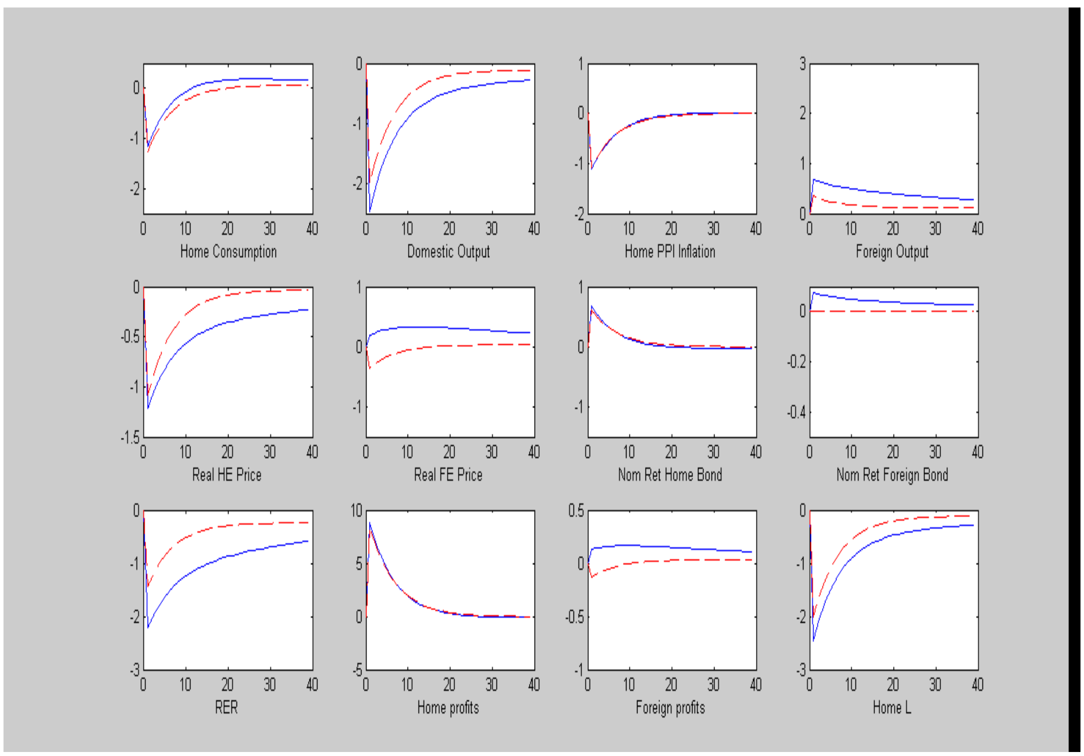
[10] This paramter interacts with the persistence of the shocks. Higher persistence of shocks require smaller ψ, like in the case presented in Ferrero et al.(2007). In our model, their calibration is such that the endogeniety of the discount factor does not solve the sationarity issue. Given absence of any other reference, we calibrate the parameter arbitrarily, like they do, but ensuring that the medium term dynamics are not affected in a significant way. In facts, if ψ is too large the dynamics of some variable reverts. For instance for ψ >0.11, foreign output reacts in the wrong way. Note that the fall in the persistence that characterizes the results under complete markets does not depend on our particular calibration of ψ.

[11] Note that here we assume full liberalization, as alternative scenario, to better appreciate the difference, but the same qualitative results hold simply by reducing the costs.

3.8.2 Impulse Responses

In this section we present the way our model reacts to a contractionary monetary policy shock. We assume that monetary policy is described by the Taylor rule presented in the model and that the money shock is characterized by a variance $\sigma_m^2 = 0.007$[12].

We show the effect of such a shock on the domestic variables and include only few Foreign variables that help to better understand the dynamics of the model. The solid line represent the impulse responses before liberalization, while the dotted line displays the scenario after liberalization has taken place and portfolio holding costs have fallen.



Responses to a contractionary money shock, before (solid line) and after (dotted line) liberalization.

We start from the first scenario, that is before liberalization. An increase in the interest rate generates standard negative responses for consumption, output and prices. After the official nominal rate rises, since prices are sticky, the real rate rises as well.

[12]The variance of the shock and the portfolio holding cost are calibrated so to be proportional, according to the definition of second order magnitude.

Agents are induced to postpone consumption which thus falls as well as output and prices. The real exchange rate appreciates, driven by the effect on the nominal rate. In addition, given the presence of sticky prices, the contractionary shock results in a fall in the firms' demand for labor. Under flexible wages this translates into a sharp fall in the real wage, which in turns lowers production cost and ultimately raises firms' profits. Since labor income cannot be traded and is largely affected by monetary policy, agents will set a portfolio that possibly can hedge this kind of risk. Home bonds seems to be the best option if one accounts for the absence of holding costs, their higher return induced by the policy shock and the negative effect on return on Foreign bond induced by the exchange rate appreciation. If long and short positions are allowed, the share of Home bond will tend to be larger than 1. A similar reasoning explains why Home equities are largely preferred to Foreign equities. The exchange rate appreciation lower the return on foreign equities in terms of Home consumption basket and Foreign equities carry a higher holding cost. The net portfolio holdings (relative to output) are consistent with this picture:

$$\alpha_{he} = 0.7575$$

$$\alpha_{hb} = 2.6868$$

$$\alpha_{fe} = -0.9797$$

Both agents exhibit strong preference for the domestic equities, in fact almost all Home equities are held by the Home agent and the Foreign equity by the Foreign agent. Once the costs fall, the portfolio composition changes as follows:

$$\alpha_{he} = 0.6161$$

$$\alpha_{hb} = 0.9494$$

$$\alpha_{fe} = -0.8585$$

The drop in the cost and the smaller effect of the shock on the real exchange rate have reduced the holdings of the Home equity in favour of the Foreign, yet because of the asymmetric decline in the holding costs, we still observe home bias in the portfolio composition. In addition, the large fall in the holdings of Home bond implies that the Home agent holds Foreign bond as well.

If one looks at the dynamics of the model she can see that they are almost analogous to those associated with the previous scenario. Yet, with falling holding costs, the impact of the monetary policy shock on the Home equity price, output and real exchange rate is definitely weaker and less persistent. In the case of Home equity price, a possible explanation is that the lower cost increases the market completeness and improves the risk-sharing. Better risk-sharing implies a more diversified portfolio and therefore a better hedging of averse risks on consumption, which reduces the effect of the monetary shock on the equity price compared to the previous scenario. The effect on output is a consequence of both the effect on the equity price and the muted effect on the exchange rate. In a situation of smaller costs on equities holdings, Home bonds are still attractive, but less than before. In addition, the effect of the monetary contraction on capital movements is smaller and so it is the effect on the nominal exchange rate that is driving the dynamics of the real rate.

In the next sections we estimate rolling VARs to compare the theoretical prediction of the model with impulse responses generated by the data. But before we progress to the comparison of impulse responses, we discuss our empirical framework and identification scheme first.

4 The Empirical Specification and Estimation Strategy

The empirical model used in this study consists is a reduced form VAR model. To this general setting, we add two features. First, we estimate rolling coefficients which allows us to control for monetary policy regimes and introduce the possibility of automatic changes in the monetary policy rule.[13] The second feature we embed is factor augmentation so to get Factor-Augmented VARs. In facts, the model we actually estimate is a FAVAR with rolling coefficients. FAVAR models allow for the inclusion of information from a large set of panel data in empirical models, without the subsequent increase in dimensionality and the associated loss of degrees of freedom present in traditional VARs.

[13] Rudebusch (1998) points out that neglecting the distinction between different monetary policy regimes implies the ignorance of important breaks in the parameters of the VAR. In contrast, Sims (1998) argues that these breaks are only of modest quantitative significance, at least for US data.

This methodology developed by Bernanke, Boivin and Eliasz (2005) (BBE hereafter) aims at improving the identification process of monetary policy shocks of a standard VAR in a data-rich environment, where traditional VARs are likely to omit sensible information and therefore suffer misspecification. BBE suggest two approaches of estimating FAVARs which differ in the technique of extracting a common information from a large panel of time series. The first one is a two-step principal components approach, while the second is a single-step Bayesian likelihood approach based on the estimation of a dynamic common factor with the Kalman filter. These approaches differ in various dimensions, however according to BBE the results do not change very much across the two methods and it is not clear *a priori* that one should be favored over the other. A clear advantage of the two-step approach is computational simplicity. On this ground, we choose to estimate the common factors through the principal component (see Stock and Watson, 2002).

4.1 The Empirical Model: FAVAR

We propose here a short description of the estimation of the common factor through the principal component.

The idea of common factor is that there is an unobservable common component F_t which underlies the behavior of many economic variables $X_{i,t}$ and that an observable, exogenous variable Y_t drives the common component F_t.[14]

One problem that may arise using this approach if the factors are extracted from all of the series together is that the factors lack of economic interpretation. Since in our application the economic interpretation behind each factor is crucial, we follow the approach presented in Belviso and Milani (2006) who propose a structural (S)FAVAR approach. They suggest to separate the data into economic categories and extract a factor from each category. In our approach this essentially means that we group all of our candidate variables into classes. So for instance: X_t^1 represents a vector with of different measures of total production and F_t^1 represents the underlying common factor

[14] In the model by BBE (2005), the observable variable is the federal funds rate, while the panel of economic variables $X_{i,t}$ consists of 120 panel data series describing the real and financial side of the economy.

of the whole real activity class. Similarly, X_t^2 represent all of the country's price indices and F_t^2 the common component underlying the price level and so on. In matrix form this can be represented as follows.

$$\begin{bmatrix} X_t^1 \\ X_t^2 \\ \vdots \\ X_t^I \end{bmatrix} = \begin{bmatrix} \Lambda_1^f & 0 & \dots & 0 \\ 0 & \Lambda_2^f & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \Lambda_I^f \end{bmatrix} \begin{bmatrix} F_t^1 \\ F_t^2 \\ \vdots \\ F_t^I \end{bmatrix}$$

Once the common factors from each class separately has been estimated, we arrange all of the variables into the following VAR:

$$\begin{bmatrix} F_t^1 \\ F_t^2 \\ \vdots \\ F_t^I \end{bmatrix} = A(L) \begin{bmatrix} F_{t-1}^1 \\ F_{t-1}^2 \\ \vdots \\ F_{t-1}^I \end{bmatrix} + e_t$$

For convenience, we rewrite the model then as:

$$F_t = A(L)F_{t-1} + e_t \qquad e_t \sim N(0, \Sigma_e) \tag{28}$$

For the reason mentioned above, in this application the common factor is estimated through principal component analysis. We now explain how we estimate F.

The first principal component F_1 of the first class of economic series X_1 can be obtained as the solution to the following maximization problem.

$$\max_{\mathbf{x}_1} \mathbf{x}_1' \widehat{\Omega} \mathbf{x}_1$$

subject to

$$\mathbf{x}_1' \mathbf{x}_1 = 1$$

where $\widehat{\Omega}$ is the estimated sample correlation matrix of the time series. The solution to the above maximization problem gives $\mathbf{x}_1^*$ which is the eigenvector with the largest eigenvalue of $\widehat{\Omega}$. The first principal component is then given by the formula:

$$F_1 = X_1 \mathbf{x}_1^*$$

Given that the theoretical model is calibrated to make predictions at the quarterly horizon, we use data at the same frequency. The data is from the OECD Main Economic Indicators 2008, while the real exchange rate is taken from the new database on real effective exchange rates of the BIS.[15] We now estimate a rolling VAR that uses the result of the principal component analysis as input data and such that each window includes 80 observations.[16] We choose to estimate the VAR in levels through the application of OLS to each equation and apply the lag length selection criteria to each one of rolling regressions (see Appendix B2 for an extensive discussion of the point).

As explained in the next section we use the sign restrictions approach to the identification of monetary policy shocks. This permits us to test our theoretical model using its prediction about the responses of macroeconomic variables following a monetary policy contraction. We will focus on the variables which better reflect the outcomes of a monetary policy shock as suggested by the theory and many VAR studies (see for instance Favero, 2001).

In order to estimate our model we define $F_t = [IP_t \ P_t \ R_t \ M_t \ E_t]$ where IP_t, P_t, R_t, M_t and E_t are the common factors, estimated by principle component analysis, in indicators of real activity, prices, interest rates, monetary aggregates and the natural logarithm of the real effective exchange rate.

4.2 The Sign Restriction Approach to Identification

The sign restriction approach to the identification of macroeconomic shocks pioneered by Canova and De Nicolo (2002) and Uhlig (2005) became popular in applied work in recent years as possible solution to the problem of bridging economic theory and empirics.

[15] The description of all series is available upon request.

[16] We choose a rather large sample for the window so to have enough degree of freedom and avoid small sample related issues.

We believe that the sign restriction is the approach that better allows the study of the effects of monetary policy contractions without imposing additional, possibly implausible restrictions. In this regard, it is important to point out a difference with the structural (S)SVAR as alternative identification technique. While the SVAR approach requires to model the reaction function of the monetary authority and therefore a precise knowledge of the structure of the economy, in the sign restriction approach identification is achieved by putting restrictions on the responses of variables following the shock. Implying that what matters is the variables reflecting outcomes rather than policy choices.

We will outline here the main building blocks behind this identification scheme. The technical details are in Appendix B.

The procedure consists of two separate steps. In the first step, a VAR of the following kind is estimated:

$$F_t = A(L)F_t + e_t \qquad e_t \sim N(0, \Sigma_e) \tag{29}$$

Where F_t is a $n \times 1$ vector and $A\ (L)$ a matrix polynomial in the lag operator. For any non-singular orthogonal matrix P, satisfying $\Sigma_e = P'P$, eq.(29) can be transformed to have contemporaneously uncorrelated innovations. Since there may be many orthogonal matrices P which satisfy the condition above, we use rotation matrices to find all possible uncorrelated innovations. We then estimate impulse response functions conditional on the shock.

In the second step, contemporaneous correlations among the impulse responses obtained in step 1 are computed. At this point we use our model's predictions to describe the behavior of the variables following a monetary policy contraction and we discard the relative impulse responses which are inconsistent with the path suggested by the theory.

In practice there may be many impulse responses which satisfy the sign restrictions imposed upon impact. Scholl and Uhlig (2005) point out that this may lead to spurious inference. In order to limit the possible number of the resulting impulse responses, we impose the sign restrictions across the time horizon as well.

Finally, we generate the confidence interval of the impulse responses using the Bayesian approach of Sims and Zha (1998), which makes statistical inference on the basis of a posterior distribution and therefore is independent of the sample size (Canova (2006)) and we pick the median as response to display.[17]

4.2.1 Identifying a Monetary Policy Contraction

Using the prediction of our theoretical model presented above we impose the sign restrictions to the VAR as summarized in the following table:

Variable	GDP_t	P_t	R_t	M_t	RER_t
Sign	≤ 0	≤ 0	≥ 0	≤ 0	≤ 0

In order to get the largest information from the data none of the inequalities is strict. In addition, as explained above, since it is very likely that many impulse response functions satisfy the sign restrictions upon impact, we impose the signs across the time horizon as well. In particular, we impose that the response of the (principal component of several) interest rates, which represents the outcome of the shock in the money market, stays non-negative for at least 2 quarters. We also require that the impulse response of the (principal component of the) monetary aggregates is non-positive for the first two quarters. Implicitly, these sign restrictions are based on the assumption of a liquidity effect lasting up to 2 quarters following the monetary policy contraction. We do not truly know whether the liquidity effect lasts this long in Australia.[18] Nonetheless, if the interest rate is used as a policy instrument by the central bank, then a persistent liquidity effect is most likely.

Unlike financial variables, variables describing the real side of the economy probably only react with a lag to a monetary policy contraction. We require that the non-positive sign restriction (of the principal components) of output and price level is only binding

[17] To implement this procedure, we use the programs in RATS kindly provided by Canova.

[18] Scholl and Uhlig (2005) argue that the liquidity effect could be up to 4 quarters following a monetary policy contraction. According to Christiano, Eichenbaum and Evans (1998) and some early work on VARs and monetary policy there may be no liquidity effect of monetary policy depending on the innovations in the monetary aggregate.

for 2 quarters, starting form the second quarter after the shock, to permit for a lagged reaction.[19]

We consider that the real exchange rate is a forward looking asset price and adjusts instantly. As a result we only impose the non-positive response upon impact.[20]

Finally, we do not impose any restriction on equity prices, so that the data are completely free to show the impact of the monetary policy contraction.

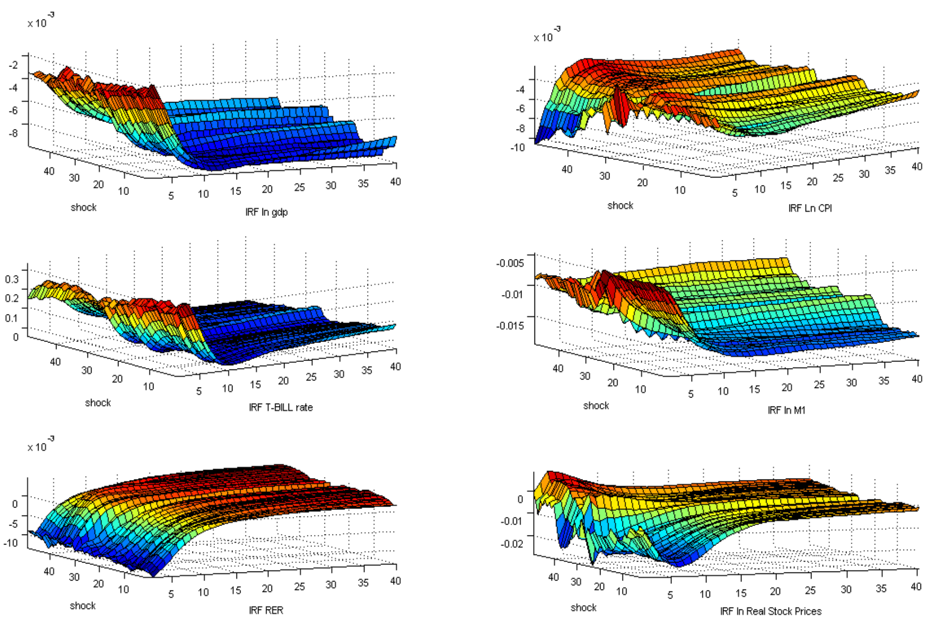
4.3 Empirical Results

We estimate rolling impulse responses by means of a FAVAR model over the whole time period which ranges from 1975Q1 to 2007Q2. We use a moving window of 80 quarters, with the first window starting in 1985 and the last one in 1997 so to have a sequence of 49 impulse response functions (IRF). Given the sign restriction approach the size of the shock is determined by the data in each sample (each window) and visible on the sequence of interest rate's IRF .

The Figure below displays the rolling IRFs for output, prices, interest rate, monetary base, real exchange rate and stock prices. All variables are in log with the exception of the interest rate and the real exchange rate. Several features stand out:

[19] Christiano, Eichenbaum and Evans(1998) document that real variables at quarterly horizon react after 2 quarters.

[20] To compare this scheme with other papers, we note that both Mountford (2005) and Farant and Peersman (2006) impose sign restrictions to be binding on the first 4 quarters. On a monthly horizon on the other hand Uhlig (2005) imposes his restrictions upon impact and for 5 month following the monetary policy contraction. Faust and Rogers (2003) on the other hand only use impact restrictions to study impulse responses in open economy VARs. Scholl and Uhlig (2005) argue that the restrictions imposed by Faust and Rogers (2003) may lead to the acceptance of many impulse responses and therefore to spurious inference. As a result Scholl and Uhlig (2005) impose their sign restrictions upon impact and for 11 months following the monetary policy contraction. A restriction horizon of 4 quarters does therefore not seem entirely unreasonable.



Response to monetary contraction rolling from 1985 (1 on the y-axis) to 1997 (49 on the y-axis)

The most clear-cut result is that the monetary policy contraction generates a reaction of equity prices that seems to be substantial even though it becomes smaller upon impact across sample of estimation. In other words, it seems that the reaction of stock prices to a monetary policy contraction was larger in the 80s than in the 90s. This seems to confirm the model's prediction generated by equity market liberalization and financial integration taking place via a reduction in the portfolio holding costs. Furthermore equity price dynamics display a lower degree of persistence over time, also as predicted by the model. We believe that once portfolio holding costs fall, there is more room for risk diversification prior to the shock, hence there is smaller incentive for portfolio reallocation once the shock arrives. This weakens the magnitude of the reaction upon impact.

A similar pattern towards smaller reaction and persistence, even if less distinct, is also visible in the output responses. This is also consistent with the prediction of our model. On the other hand, unlike what predicted by the model the responses of the exchange rate seems to be quite the same across samples. One possible explanation for this discrepancy relates to the fact that Australia is a commodity exporter and the real exchange rate is strongly affected by movements in commodity prices, not only by monetary policy. Unfortunately, our model cannot disentangle these different factors.

It is important to stress that, as mentioned above, sign restrictions identify monetary policy outcomes rather than the exact reaction function of the monetary policy authority. Therefore one can argue that the different reaction of the variables across samples could be solely a result of different sizes of monetary policy shocks. In particular, shocks are alleged to be smaller in the 90s than in the 80s.[21] However, a careful look at the interest rate responses suggests that the interest rate reaction does not change significantly over time, and that idea seems thus to be rejected.

To conclude, it is worthwhile to notice that even if we cannot draw a clear picture of how the asset prices channel of monetary policy changes in a financially integrated word, one message stands out from this paper. The reaction of assets prices to monetary policy shocks weakens with increasing market liberalization and integration. This likely to affect, at least on a second round, consumption and output. Inevitably this is has implications for the monetary policy decisions process and in particular, when monetary authorities are called to intervene in the face of special events on financial markets.

5 Concluding Remarks

Equity market liberalization has been rising during the 1980's across developed countries and led to substantial increments in integration thereafter. Falling portfolio holding costs permitted international investors to use foreign equity markets as a hedge against macroeconomic shocks. The implications of this phenomenon for macroeconomic policy outcomes should then be a key priority for research: it affects monetary policy transmission mechanisms and an increasing number of emerging markets liberalizes their exchanges. In this study we analyzed changes in the effect of monetary policy shock to equity prices during financial liberalization and test the model using data for Australia. In fact the country experienced gradual equity market liberalization during

[21] Recall that the last window starts in 1997 which implies that only the "Great Moderation" is only partialy included in the analysis.

the 1980's.

In order to guide our empirical results, we construct a New Open Economy Macroeconomics which predicts a substantially weaker response of equity prices to a monetary policy shock when financial liberalization takes place. Subsequently we estimate a FAVAR and identify monetary policy shocks with the sign restriction scheme.

The main results of this paper is to show the transmission of monetary policy shocks to equity prices is affected by equity market liberalization. Deeper integration weakens the effect of monetary policy. This result is predicted by the theoretical model and confirmed by the data. This result can have significant implications for the monetary policy setting in reaction to output and inflation since transmission mechanism of monetary policy are affected but for monetary policy intervention to face extreme (and not so rare) financial events like asset bubbles and crashes.

References

[1] Bodenstein, M. (2006), "Closing Open Economy Models", International Finance Discussion Papers 867, Board of Governors of the Federal Reserve System.

[2] Candelon, B. and Luekepohl, H. (2001) "On the reliability of Chow-type tests for parameter constancy in multivariate dynamic models", Economics Letters, pp.155-160

[3] Canova, F. (2006). Methods for Applied Macroeconomic Research. Princeton University Press, Princeton, N.J.

[4] Canova, F. and De Nicolo, G. (2002),"Money Matters for Business Cycle Fluctuations in the G7", Journal of Monetary Economics, 49, pp. 1131-1159.

[5] Canova, F. and Pina, J. (2004), "What VARs Tell us about DSGE Models?", forthcoming in C. Diebolt, and Krystou, C. (eds.), New Trends In Macroeconomics, Springer Verlag, New York, NY.

[6] Christiano, L., Eichenbaum, M. and Evans, C. (1996) "The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds". Review of Economics and Statistics, Vol. 78, pp. 16-34

[7] Cushman, D. O. and T. Zha, (1997). "Identifying monetary policy in a small open economy under flexible exchange rates," Journal of Monetary Economics, Elsevier, vol. 39 (3), pages 433-448, August.

[8] Devereux, M and A. Sutherland (2006) "Solving for Country Portfolios in Open Economy Macro Models" CEPR Discussion Paper No 5966.

[9] Devereux, M. B. and Sutherland, A. (2007). "Financial Globalization and Monetary Policy," CEPR Discussion Papers 6147, C.E.P.R. Discussion Papers.

[10] Eichenbaum, M. and C. L. Evans (1995), "Some Empirical Evidence of Shocks to Monetary Policy on Exchange Rates", Quarterly Journal of Economics, 1995, no. 110, 975-1010.

[11] Engel, C. and A. Matsumoto (2005), “Portfolio Choice and Home Bias in Equities in a Monetary Open-Economy DSGE Model,”working paper, University of Wisconsin.

[12] Evans, M. and V. Hnatkovska (2005) “International Capital Flows, Returns and World Financial Integration” NBER Working Paper 11701

[13] Faust, J. and J. H. Rogers (2003), "Monetary Policy's Role in Exchange Rate Behavior", Journal of Monetary Economics, 50 (7), pp. 1403-1622.

[14] Favero, C. (2001). Applied Macroeconomics, Oxford University Press, Oxford.

[15] Ferrero, A. Gertler, M. and Svensson, L. E.O., "Current Account Dynamics and Monetary Policy" (April 2008). NBER Working Paper No. W13906

[16] Grilli, V. and N. Roubini (1995), ”Liquidity and Exchange Rates: Puzzling Evidence from the G-7 countries”, mimeo, Yale University.

[17] Grilli, V. and N. Roubini (1996), ”Liquidity Models in Open Economies: Theory and Empirical Evidence”, European Economic Review, 1996, pp. 847-859.

[18] Kilian, L. and Ivanov, V. (2005), "A Practitioner's Guide to Lag Order Selection for VAR Impulse Response Analysis", Studies in Nonlinear Dynamics & Econometrics, Berkeley Electronic Press, vol. 9 (1), pp. 1219-1219.

[19] Kim, S. and N. Roubini (2000), ”Exchange Rate Anomalies in the Industrial Countries; A Solution with a Structural VAR Approach”, Journal of Monetary Economics 45, 2000, pp. 561-586.

[20] Klein, P. (2000), "Using the generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model", Journal of Economic Dynamics and Control 24(10), September 2000, pp. 1405-1423.

[21] Kollmann, R. (2006) “International Portfolio Equilibrium and the Current Account” CEPR discussion paper 3819.

[22] Kose, M. A., E. Prasad, K. Rogoff and S.-J.Wei (2006), "Financial Globalization: A Reappraisal" (August 2006). IMF Working Paper No. 06/189

[23] Lane, P. (2001), "The New Open Economy Macroeconomics: A Survey" Journal of International Economics, August 2001, pp. 235-66.

[24] Lane, P., and G-M.Milesi-Ferretti, (2006), "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004" IIIS Discussion Paper No.126.

[25] Luetkepohl, H. and M. Kratzig (2004). Applied Time Series Econometrics. Cambridge University Press: Cambridge, U.K.

[26] Martin, P.and H. Rey (2004), "Financial Super-Markets: Size Matters for Asset Trade", Journal of International Economics 64 pp. 335– 361

[27] Mountford, A. (2005). "Leaning into the Wind: A Structural VAR Investigation of UK Monetary Policy," Oxford Bulletin of Economics and Statistics, Department of Economics, University of Oxford, vol. 67 (5), pp. 597-621, October.

[28] Obstfeld, M. and K. Rogoff (1996), Foundations of International Macroeconomics, MIT Press, Cambridge, Massachusetts.

[29] Paulsen, J. (1984) "Order Determination of Multivariate Autoregressive Time Series with Unit Roots", Journal of Time Series Analysis, 5, 115 - 127

[30] Peersman G. (2005), "What Caused the Early Millennium Slowdown? Evidence Based on Vector Autoregressions", Journal of Applied Econometrics, Vol 20, pp. 185-207.

[31] Rogoff, K. (2004), "Globalization and Global Disinflation", paper presented at the 2003 Federal Reserve Bank of Kansas City conference on Monetary Policy and Uncertainty: Adapting to a Changing Economy, Jackson Hole, Wyoming.

[32] Rudebusch, G. D. (1998), "Do Measures of Monetary Policy in a VAR Make Sense?", International Economic Review, Vol. 39 (4), pp. 907–31.

[33] Scholl, A. & H. Uhlig, (2005). "New Evidence on the Puzzles. Results from Agnostic Identification on Monetary Policy and Exchange Rates," SFB 649 Discussion Papers SFB649DP2005-037, Sonderforschungsbereich 649, Humboldt University, Berlin, Germany.

[34] Sims, C. (1980), "Macroeconomics and Reality" Econometrica, vol. 48, no. 1, pp. 1-48.

[35] Sims, C. (1992), "Interpreting the Macroeconomic Time Series Facts, The Effects of Monetary Policy", European Economic Review, 1992, 36, pp. 975-1011.

[36] Sims, C. Stock, J. H. and Watson, M. (1990). "Inference in Linear Time Series Models with some Unit Roots", Econometrica, Vol. 58, pp. 113–144.

[37] Sims, C. and H. Uhlig (1991), "Understanding Unit Rooters: A Helicopter Tour", Econometrica, vol. 59, no. 6, pp. 1591-1599.

[38] Sims, C. and T. Zha (1999), "Error Bands for Impulse Responses", Econometrica, 67 (5), pp. 1113-1155.

[39] Sims, C. & T. Zha, (2006). "Does Monetary Policy Generate Recessions?", Macroeconomic Dynamics, Cambridge University Press, vol. 10 (02), pages 231-272, April.

[40] Tille C. and E. van Wincoop (2007) "International Capital Flows" NBER Working Paper 13820

[41] Uhlig, H. (1994), "What Macroeconomist Should Know About Unit Roots: A Bayesian Perspective", Econometric Theory, vol. 10, 1994, pp. 645-671.

[42] Uhlig, H. (2005), "What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure", Journal of Monetary Economics, Elsevier, vol. 52 (2), pp. 381-419, March.

[43] Uribe, M. and Schmitt-Grohe, (2003) S. "Closing Small Open Economy Models", Journal of International Economics, 61, October 2003, pp. 163-185

[44] Uzawa, H. (1968), "Time Preference, the Consumption Function and Optimum Asset Holdings", in James Wolfe (ed.), Value, Capital and Growth: Papers in Honor of Sir John Hicks, The University of Edinburgh Press, Edinburgh, Scotland.

A Derivation of the Theoretical Model

Demand

The Home representative agent's maximizes the following utility function:

$$U = E_0 \sum_{\tau=t}^{\infty} \eta_t \left[\frac{C_t^{1-\rho}}{1-\rho} - KL_t \right]$$

As explained in the body of the text, the discount factor η_t is assumed to be endogenous and defined by the following recursion:

$$\eta_t = \beta_t \eta_{t-1}$$

where

$$\beta_t \equiv \frac{1}{1+\psi\left(\log \overline{C}_t - \vartheta\right)} \tag{30}$$

The calibration is such that the steady state discount factor will pin down to the desired value of β. In terms of deviation from its steady state value we get:

$$\widehat{\beta}_t = -\psi\beta\widehat{C}_t$$

The presence of $\widehat{C}_t$ in the equation of $\widehat{\beta}_t$ ensures stationarity of the dynamics of the model.

Given the period budget constraint:

$$P_tC_t + P_tW_{t+1} = w_tL_t + P_t\Pi_t + P_{t-1}\left(\begin{array}{c} \alpha_{hb,t-1}r_{hb,t}\frac{P_t}{P_{t-1}} + \alpha_{fb,t-1}r_{fb,t}\frac{P_t}{P_{t-1}} + \\ \alpha_{he,t-1}e^{-\delta_{he}}r_{he,t}\frac{P_t}{P_{t-1}} + \alpha_{fe,t-1}e^{-\delta_{fe}}r_{fe,t}\frac{P_t}{P_{t-1}} \end{array} \right)$$

Intertemporal optimization problem is solved by setting the Lagrangian:

$$\mathcal{L} = E_0 \sum_{\tau=t}^{\infty} \eta_t \left\{ \left[\frac{C_t^{1-\rho}}{1-\rho} - KL_t \right] - \lambda_t \left(\begin{array}{c} P_tC_t + P_t\sum_{k=1}^{4}\alpha_{k,t} - w_tL_t - P_t\Pi_t \\ -P_t\left(\begin{array}{c} \alpha_{hb,t-1}r_{hb,t} + \alpha_{fb,t-1}r_{fb,t} + \\ \alpha_{he,t-1}e^{-\delta_{he}}r_{he,t} + \alpha_{fe,t-1}e^{-\delta_{fe}}r_{fe,t} \end{array} \right) \end{array} \right) \right\}$$

and maximization with respect to C_t, L_t and $\alpha_{k,t}$ results in the following first order conditions (FOCs):

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0$$

$$\lambda_t = \frac{C_t^{-\rho}}{P_t}$$

$\frac{\partial \mathcal{L}}{\partial L_t} = 0$ implies that the consumption-leisure trade-off can be written as:

$$K = \frac{w_t}{P_t} C_t^{-\rho}$$

$\frac{\partial \mathcal{L}}{\partial \alpha_{hb,t}} = 0$ gives the Euler equation for the Home bond:

$$\begin{aligned} -\lambda_t \eta_t P_t + E_t \left\{ \lambda_{t+1} \eta_{t+1} P_{t+1} r_{b,t+1} \right\} &= 0 \\ \frac{C_t^{-\rho}}{P_t} \eta_t P_t &= E_t \left\{ P_{t+1} \beta_{t+1} \eta_t \frac{C_{t+1}^{-\rho}}{P_{t+1}} r_{hb,t+1} \right\} \\ C_t^{-\rho} &= E_t \left\{ \beta_{t+1} C_{t+1}^{-\rho} r_{hb,t+1} \right\} \end{aligned} \tag{31}$$

$\frac{\partial \mathcal{L}}{\partial \alpha_{fb,t}} = 0$ gives the Euler equation for the Foreign bond:

$$\begin{aligned} -\lambda_t \eta_t P_t + E_t \left\{ \lambda_{t+1} \eta_{t+1} P_{t+1} r_{b,t+1} \right\} &= 0 \\ C_t^{-\rho} &= E_t \left\{ \beta_{t+1} C_{t+1}^{-\rho} r_{b,t+1} \right\} \end{aligned}$$

$\frac{\partial \mathcal{L}}{\partial \alpha_{he,t}} = 0$ gives the Euler equation for the Home equity:

$$\begin{aligned} -\lambda_t \eta_t P_t + E_t \left\{ \lambda_{t+1} \eta_{t+1} P_{t+1} e^{-\delta_{he}} r_{he,t+1} \right\} &= 0 \\ C_t^{-\rho} &= E_t \left\{ \beta_{t+1} C_{t+1}^{-\rho} e^{-\delta_{he}} r_{he,t+1} \right\} \end{aligned}$$

$\frac{\partial \mathcal{L}}{\partial \alpha_{fe,t}} = 0$ gives the Euler equation for the Foreign equity:

$$\begin{aligned} -\lambda_t \eta_t P_t + E_t \left\{ \lambda_{t+1} \eta_{t+1} P_{t+1} e^{-\delta_{fe}} r_{fe,t+1} \right\} &= 0 \\ C_t^{-\rho} &= E_t \left\{ \beta_{t+1} C_{t+1}^{-\rho} e^{-\delta_{fe}} r_{fe,t+1} \right\} \end{aligned}$$

The optimal allocation of any given expenditure within each category of goods yields the demand functions for the specific variety:

$$C_h(i) = \left(\frac{P_h(i)}{P_h}\right)^{-\phi} C_h \qquad ; \qquad C_f(i) = \left(\frac{P_f(i)}{P_f}\right)^{-\phi} C_f$$

where $P_h \equiv \left[\int\limits_0^1 (P_h(i))^{1-\phi}\, di\right]^{\frac{1}{1-\phi}}$ is the Home domestic price index and $P_f \equiv \left[\int\limits_0^1 (P_f(i))^{1-\phi}\, di\right]^{\frac{1}{1-\phi}}$ is Home price index of imported goods.

Finally the optimal allocation of the expenditure between domestic and imported goods is given by:

$$C_h = \mu \left(\frac{P_h}{P}\right)^{-\theta} C \qquad ; \qquad C_f = (1-\mu)\left(\frac{P_f}{P}\right)^{-\phi} C$$

where

$$P = \left[\mu P_h^{1-\theta} + (1-\mu) P_f^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

is the Home consumer price index (CPI).

Supply

Firms choose the price that maximize the following expected discounted profits:

$$E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \widetilde{P}_{h,t} Y_{h,t}(i) - w_{t+i} \frac{Y_{h,t}(i)}{A_t} \tag{32}$$

Given the Calvo price setting, the dynamics of the Home price index are governed by the following low of motion:

$$P_{h,t} = \left[(1-\kappa)\widetilde{P}_{h,t}^{1-\phi} + \kappa P_{h,t-1}^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

Form profits maximization and loglinearization of the first order condition we derive the equation for the NKPC:

$$\begin{aligned}
\sum_{i=0}^{\infty} (\beta\kappa)^i \widetilde{P}_{h,t} &= \sum_{i=0}^{\infty} (\beta\kappa)^i E_t \left(\widehat{w}_{t+i} - \widehat{A}_{t+i}\right) \\
\widetilde{P}_{h,t} &= (1-\beta\kappa)\left(\widehat{w}_t - \widehat{A}_t\right) + \beta\kappa\widetilde{P}_{h,t+1} \\
\kappa\widehat{P}_{h,t} - \kappa\widehat{P}_{h,t-1} &= (1-\beta\kappa)(1-\kappa)\left(\rho\widehat{C}_t + \widehat{P}_t - \widehat{A}_t\right) + \beta\kappa\left(\widehat{P}_{h,t+1} - \kappa\widehat{P}_{h,t}\right) - \widehat{P}_{h,t}(1-\kappa) \\
\pi_{h,t} &= \lambda^{-1}\left(\rho\widehat{C}_t + \widehat{P}_t - \widehat{u}_t - \widehat{P}_{h,t}\right) + \beta E_t \pi_{h,t+1} \qquad (33)
\end{aligned}$$

where $\lambda = \left(\frac{(1-\beta\kappa)(1-\kappa)}{\kappa}\right)^{-1}$.

The assumption of PCP implies a complete exchange rate pass-through and the "law of one price" will hold all the time. In addition, it will be the case that the terms of trade are equal (one the inverse of the other) in the two countries and given by:

$$\tau = \frac{P_f}{P^*_{h,t} S_t} = \frac{P^*_{f,t} S_t}{P_{h,t}}$$

where S_t is the nominal exchange rate. This implies that the real exchange rate Q_t can be written in loglinear terms as:

$$\begin{aligned}
\widehat{Q}_t &= \widehat{P}^*_t + \widehat{S}_t - \widehat{P}_t \\
&= \mu\widehat{P}^*_{f,t} + (1-\mu)\widehat{P}^*_{h,t} + \widehat{S}_t - \mu\widehat{P}_{h,t} - (1-\mu)\widehat{P}_{f,t} \\
&= (2\mu - 1)\widehat{\tau}_t
\end{aligned}$$

Moreover, it can been shown that:

$$\begin{aligned}
\widehat{P}_t &= \mu\widehat{P}_{ht} + (1-\mu)\widehat{P}_{ft} \\
\widehat{P}_t - \widehat{P}_{h,t} &= (1-\mu)\widehat{\tau}_t
\end{aligned}$$

which taking the first difference on both sides also implies that the Home CPI inflation can also be written as:

$$\hat{\pi}_t = \hat{\pi}_{h,t} + (1-\mu)\Delta\widehat{\tau}_t$$

From the same set up we can also write an equation for imported inflation:

$$\begin{aligned} \widehat{P}_t &= \mu \widehat{P}_{h,t} + (1-\mu)\,\widehat{P}_{ft} \\ \widehat{\pi}_{f,t} &= \hat{\pi}_t + \mu \Delta \widehat{\tau}_t \end{aligned} \tag{34}$$

Using the relation above we can rewrite the NKPC as function of the terms of trade:

$$\pi_{h,t} = \lambda^{-1}\left(\rho \widehat{C}_t - \widehat{u}_t + (1-\mu)\,\widehat{\tau}_t\right) + \beta E_t \pi_{h,t+1}$$

At aggregate level, nominal profits are determined in a residual way as follows:

$$P_t \Pi_t = P_{h,t} Y_t - w_t L_t$$

where Y_t is total output. The aggregate production function is linear in labor and given by:

$$Y_t = A_t L_t$$

where A_t is the stochastic productivity shock, which is assumed to be: $A_t = \zeta A_{t-1} + u_t$ so that $\overline{A} = 1$ and $\widehat{A}_t = u_t$

Using consumption-leisure trade-off condition profits can be written as:

$$P_{h,t} Y_t = P_t \Pi_t + C_t^{\rho} K P_t \frac{Y_t}{A_t}$$

Recalling that at the steady state $P_h = P$ and the price is set as markup over the marginal cost $P = \frac{\phi}{\phi-1} w$, the log-linearized profit condition yields:

$$\phi(\widehat{P}_{h,t} + \widehat{Y}_t) = \widehat{P}_t + \widehat{\Pi}_t + (\phi - 1)(\rho \widehat{C}_t + \widehat{P}_t + \widehat{Y}_t - \widehat{A}_t) \tag{35}$$

Rearranging to solve for profits, this gives the final expression for profits:

$$\widehat{\Pi}_t = \phi(\widehat{P}_{h,t} - \widehat{P}_t) + \widehat{Y}_t - (\phi - 1)\rho \widehat{C}_t + (\phi - 1) u_t \tag{36}$$

Monetary Policy

The monetary policy stance is described by the following rule:

$$r^n_{hb,t+1} = \beta^{-1}\left(\frac{P_{h,t}}{P_{h,t-1}}\right)^{\gamma} \exp m_t$$

can be expressed in log-deviation for the steady state in a more familiar form (recall $r_{hb,t+1}$ is gross rate) as:

$$\log\left(r^n_{hb,t+1}\right) = \log\left(\bar{r}\right) + \gamma\pi_{h,t} + m_t$$

$$\widehat{r}^n_{hb,t+1} = \gamma\pi_{h,t} + m_t$$

Foreign Economy

We assume two symmetric countries, so that the Foreign economy has an analogous representation to the Home country. The Foreign agent's consumption basket is defined as:

$$C^* \equiv \left[\mu^{\frac{1}{\theta}}\left(C^*_f\right)^{\frac{\theta-1}{\theta}} + (1-\mu)^{\frac{1}{\theta}}\left(C^*_h\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{37}$$

C^*_f, is the domestic consumption of Foreign goods monopolistically produced within the country is:

$$C^*_f \equiv \left[\int_0^1 \left(C^*_f(i)\right)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}}$$

C^*_h is the basket of imported goods:

$$C^*_h \equiv \left[\int_0^1 \left(C_h(i)\right)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}}$$

The demand functions:

$$C^*_h(i) = \left(\frac{P^*_h(i)}{P^*_f}\right)^{-\phi} C_f \quad ; \quad C^*_f(i) = \left(\frac{P^*_f(i)}{P^*_f}\right)^{-\phi} C_f$$

where $P^*_h \equiv \left[\int_0^1 \left(P_h(i)\right)^{1-\phi} di\right]^{\frac{1}{1-\phi}}$ is the Foreign price index of imported goods and

$P_f^* \equiv \left[\int_0^1 \left(P_f \left(i \right) \right)^{1-\phi} di \right]^{\frac{1}{1-\phi}}$ the Foreign domestic price index.

Finally the optimal allocation of the expenditure between domestic and imported goods is given by:

$$C_h^* = (1-\mu) \left(\frac{P_h^*}{P^*} \right)^{-\theta} C^* \quad ; \quad C_f^* = \mu \left(\frac{P_f^*}{P^*} \right)^{-\theta} C^*$$

where

$$P^* = \left[(1-\mu) P_h^{*1-\theta} + \mu P_f^{*1-\theta} \right]^{\frac{1}{1-\theta}}$$

is the Foreign consumer price index (CPI).

It can be shown that the Foreign CPI can be written in loglinear form as:

$$\begin{aligned} \widehat{P}_t^* &= (1-\mu) \widehat{P}_{h,t}^* + \mu \widehat{P}_{f,t}^* \\ \widehat{P}_t^* - \widehat{P}_{f,t}^* &= -(1-\mu) \widehat{\tau}_t \end{aligned}$$

Taking the first difference on both sides gives us the same relation in terms of CPI inflation and domestic inflation:

$$\hat{\pi}_t^* = \widehat{\pi}_{f,t}^* - (1-\mu) \Delta \widehat{\tau}_t$$

or imported inflation:

$$\begin{aligned} \widehat{P}_t^* &= (1-\mu) \widehat{P}_{h,t}^* + \mu \widehat{P}_{f,t}^* \\ \widehat{\pi}_{h,t}^* &= \hat{\pi}_t^* + \mu \Delta \widehat{\tau}_t \end{aligned}$$

Analogously to the Home the production function is linear in labour:

$$Y_t^* = A_t^* L_t^*$$

and firms maximize profits operating in an environment with monopolistic competition.

The optimization implies a NKPC of the following form:

$$\pi^*_{f,t} = \lambda^{-1}\left(\rho\widehat{C}^*_t - \widehat{u}^*_t - (1-\mu)\,\widehat{\tau}_t\right) + \beta E_t \pi^*_{f,t+1}$$

and the loglinear equation for aggregate profits is given by:

$$\widehat{\Pi}^*_t = \phi\,(1-\mu)\,\widehat{\tau}_t + \widehat{Y}^*_t - (\phi-1)\rho\widehat{C}^*_t + (\phi-1)u^*_t$$

Moving on to portfolio holdings and returns, the real return on asset k is $r^*_{k,t}$. The nominal Home currency return $r_{k,t}$ is the same for both investors. (Recall that $r_{k,t+1}$ is a growth rate):

$$r_{k,t+1} = r_{k,t+1}\frac{P_t}{P_{t+1}} \qquad\qquad r^*_{k,t+1} = r_{k,t+1}\frac{P^*_t}{P^*_{t+1}}\frac{S_t}{S_{t+1}}$$

$$\Rightarrow r^*_{k,t+1} = r_{k,t+1}\frac{P_{t+1}}{P_t}\frac{P^*_t}{P^*_{t+1}}\frac{S_t}{S_{t+1}} = r_{k,t+1}\frac{Q_t}{Q_{t+1}}$$

Given this relation the Euler equations are derived in a analogous way as for the Home agent.

Monetary policy rule is analogous to the one of the Home country:

$$E_t\hat{r}^{*n}_{fb,t+1} = \gamma\pi^*_{f,t} + m^*_t$$

The portfolio part of the model

According to Devereux and Sutherland (2006) method, the solution of the model requires a first order approximation of all the non-portfolio conditions and a second order of the portfolio equations. This allows us to get around the problem of portfolio indeterminacy in a non-stochastic steady state and to have a meaningful portfolio choice problem.[22]

We start taking the second order approximation of equations (31):

$$E_t\beta_{t+1}C^{-\rho}_{t+1}r_{fb,t+1} = E_t\beta_{t+1}C^{-\rho}_{t+1}r_{hb,t+1}$$

[22] For details see Devereux and Sutherland (2006).

We first focus on the left hand side:

$$
\begin{aligned}
E_t \beta_{t+1} C_{t+1}^{-\rho} r_{fb,t+1} &= \\
&= \beta C^{-\rho} r_{fb} \left[\begin{array}{c} 1 + E_t \left\{ -\left(\rho + \psi\beta\right) \widehat{C}_{t+1} + \widehat{r}_{fb,t+1} \right\} \\ +\frac{1}{2} E_t \left\{ \begin{array}{c} \left(\rho + \psi\beta\right)^2 \widehat{C}_{t+1}^2 \\ -2\left(\rho + \psi\beta\right) \widehat{C}_{t+1} \widehat{r}_{fb,t+1} \\ +\widehat{r}_{fb,t+1}^2 \end{array} \right\} \end{array} \right]
\end{aligned}
$$

Since the right hand side is identical except for the type asset, in terms of excess return we can write:

$$
0 = E_t \left\{ \left(\widehat{r}_{hb,t+1} - \widehat{r}_{fb,t+1}\right) - \left(\rho + \psi\beta\right) \widehat{C}_{t+1} \left(\widehat{r}_{hb,t+1} - \widehat{r}_{fb,t+1}\right) + \frac{1}{2} \left(\widehat{r}_{hb,t+1}^2 - \widehat{r}_{fb,t+1}^2\right) \right\} \tag{38}
$$

When considering the Euler equation for the Home equity, which carries a cost, the expansion implies:

$$
E_t \beta_{t+1} C_{t+1}^{-\rho} e^{-\delta_{he}} r_{he,t+1} = \beta C^{-\rho} r_{he} \left[\begin{array}{c} 1 + E_t \left\{ \begin{array}{c} -\left(\rho + \psi\beta\right) \widehat{C}_{t+1} \\ +\widehat{r}_{he,t+1} \end{array} \right\} - \delta_{he} \\ +\frac{1}{2} E_t \left\{ \begin{array}{c} \left(\rho + \psi\beta\right)^2 \widehat{C}_{t+1}^2 \\ -2\left(\rho + \psi\beta\right) \widehat{C}_{t+1} \widehat{r}_{he,t+1} \\ +\widehat{r}_{he,t+1}^2 \end{array} \right\} \end{array} \right]
$$

Putting together the equation above and the second order expansion for the reference asset yields:

$$
E_t \left\{ \begin{array}{c} \widehat{r}_{he,t+1} - \delta_{he} + \\ \frac{1}{2} \left(\begin{array}{c} -2\left(\rho + \psi\beta\right) \widehat{C}_{t+1} \widehat{r}_{he,t+1} \\ +\widehat{r}_{he,t+1}^2 \end{array} \right) \end{array} \right\} = E_t \left\{ \begin{array}{c} \widehat{r}_{fb,t+1} + \\ \frac{1}{2} \left(\begin{array}{c} -2\left(\rho + \psi\beta\right) \widehat{C}_{t+1} \widehat{r}_{fb,t+1} \\ +\widehat{r}_{fb,t+1}^2 \end{array} \right) \end{array} \right\}
$$

which in terms of excess return yields:

$$
\delta_{he} = E_t \left\{ \left(\widehat{r}_{he,t+1} - \widehat{r}_{fb,t+1}\right) - \left(\rho + \psi\beta\right) \widehat{C}_{t+1} \left(\widehat{r}_{he,t+1} - \widehat{r}_{fb,t+1}\right) + \frac{1}{2} \left(\widehat{r}_{he,t+1}^2 - \widehat{r}_{fb,t+1}^2\right) \right\}
$$

We now repeat the exercise for the Foreign equity and get:

$$\delta_{fe} = E_t \left\{ \left(\widehat{r}_{fe,t+1} - \widehat{r}_{fb,t+1} \right) - \left(\rho + \psi\beta \right) \widehat{C}_{t+1} \left(\widehat{r}_{fe,t+1} - \widehat{r}_{fb,t+1} \right) + \frac{1}{2} \left[\left(\widehat{r}^2_{fe,t+1} - \widehat{r}^2_{fb,t+1} \right) \right] \right\}$$

In what follows we apply the same procedure as above to the Foreign FOCs and express the Foreign returns in terms of the Home consumption basket.

The second order approximation for the Foreign Euler equation of the Home bond (in terms of excess return) is given by:

$$0 = E_t \left\{ \left(\widehat{r}^*_{hb,t+1} - \widehat{r}^*_{fb,t+1} \right) - \left(\rho + \psi\beta \right) \widehat{C}^*_{t+1} \left(\widehat{r}^*_{hb,t+1} - \widehat{r}^*_{fb,t+1} \right) + \frac{1}{2} \left(\widehat{r}^{*2}_{hb,t+1} - \widehat{r}^{*2}_{fb,t+1} \right) \right\}$$

Recalling that:

$$\hat{r}^*_{k,t+1} = \hat{r}_{k,t+1} - \left(\widehat{Q}_{t+1} - \widehat{Q}_t \right)$$

we get that excess returns are the same regardless of the currency in which the return is expressed:

$$\begin{aligned} \hat{r}^*_{hb,t+1} - \hat{r}^*_{fb,t+1} &= \left(\hat{r}_{hb,t+1} + \widehat{Q}_t - \widehat{Q}_{t+1} \right) - \left(\hat{r}_{fb,t+1} + \widehat{Q}_t - \widehat{Q}_{t+1} \right) \\ &= \hat{r}_{hb,t+1} - \hat{r}_{fb,t+1} \end{aligned}$$

In addition:

$$\begin{aligned} & \left(\hat{r}^*_{hb,t+1} \right)^2 - \left(\hat{r}^*_{fb,t+1} \right)^2 \\ = \; & \left(\hat{r}_{hb,t+1} \right)^2 + \left(\hat{Q}_t - \hat{Q}_{t+1} \right)^2 + 2\hat{r}_{hb,t+1} \left(\hat{Q}_t - \hat{Q}_{t+1} \right) \\ & - \left(\hat{r}_{fb,t+1} \right)^2 - \left(\hat{Q}_t - \hat{Q}_{t+1} \right)^2 - 2\hat{r}_{fb,t+1} \left(\hat{Q}_t - \hat{Q}_{t+1} \right) \\ = \; & \left(\hat{r}_{hb,t+1} \right)^2 - \left(\hat{r}_{fb,t+1} \right)^2 + 2 \left(\hat{r}_{hb,t+1} - \hat{r}_{fb,t+1} \right) \left(\hat{Q}_t - \hat{Q}_{t+1} \right) \end{aligned}$$

Hence the expansion of the Foreign FOC for the Foreign bond in terms of the Home consumption basket becomes:

$$\begin{aligned} 0 \;=\; & E_t \left(\hat{r}_{hb,t+1} - \hat{r}_{fb,t+1} \right) - \left(\rho + \psi\beta \right) E_t \hat{C}^*_{t+1} \left(\hat{r}_{hb,t+1} - \hat{r}_{fb,t+1} \right) + \\ & + \frac{1}{2} E_t \left[\left(\hat{r}_{hb,t+1} \right)^2 - \left(\hat{r}_{fb,t+1} \right)^2 - 2 \left(\hat{r}_{hb,t+1} - \hat{r}_{fb,t+1} \right) \left(\hat{Q}_{t+1} - \hat{Q}_t \right) \right] \end{aligned}$$

Similarly when we consider the Foreign equity:

$$\begin{aligned}0 &= E_t\left(\hat{r}_{fe,t+1}-\hat{r}_{fb,t+1}\right)-\left(\rho+\psi\beta\right)E_t\hat{C}^*_{t+1}\left(\hat{r}_{fe,t+1}-\hat{r}_{fb,t+1}\right)+\\ &+\frac{1}{2}E_t\left[\left(\hat{r}_{fe,t+1}\right)^2-\left(\hat{r}_{fb,t+1}\right)^2+2\left(\hat{r}_{fe,t+1}-\hat{r}_{fb,t+1}\right)\left(\hat{Q}_t-\hat{Q}_{t+1}\right)\right]\end{aligned}$$

Since the Foreign agent incurs a cost when investing in Home equity, the excess return of the Home equity over the Foreign bond to the second order approximation is:

$$0 = E_t\left\{\begin{array}{c}\left(\widehat{r}_{he,t+1}-\widehat{r}_{fb,t+1}\right)-\delta^*_{he}-\left(\rho+\psi\beta\right)\widehat{C}_{t+1}\left(\widehat{r}_{he,t+1}-\widehat{r}_{fb,t+1}\right)\\ +\frac{1}{2}\left(\left(\hat{r}_{he,t+1}\right)^2-\left(\hat{r}_{fb,t+1}\right)^2+2\left(\hat{r}_{he,t+1}-\hat{r}_{fb,t+1}\right)\left(\hat{Q}_t-\hat{Q}_{t+1}\right)\right)\end{array}\right\}$$

Following Devereux and Sutherland (2006) solution method, we compute now cross-country and average Euler equations.

To get the cross-country Euler equations, define the excess return for the generic assets i, $\hat{r}_{x,i,t+1}$ as $\widehat{r}_{i,t+1}-\widehat{r}_{fb,t+1}$ and $\hat{r}^2_{x,i,t+1}$ as $\left(\hat{r}_{i,t+1}\right)^2-\left(\hat{r}_{fb,t+1}\right)^2$ and take the difference between each Home Euler and the corresponding one for the Foreign country:

$$\begin{aligned}0 &= -\delta_{he}-E_t\left(\rho+\psi\beta\right)\widehat{C}_{t+1}\widehat{r}_{x,he,t+1}\\ &\quad+\left(\rho+\psi\beta\right)E_t\hat{C}^*_{t+1}\hat{r}_{x,he,t+1}+\hat{r}_{x,he,t+1}\left(\hat{Q}_{t+1}-\hat{Q}_t\right)+\delta^*_{he} \qquad (39)\\ 0 &= E_t\left(\hat{C}_{t+1}-\hat{C}^*_{t+1}-\frac{\hat{Q}_{t+1}-\hat{Q}_t}{\rho+\psi\beta}\right)\hat{r}_{x,he,t+1}+\delta_{he}-\delta^*_{he}\end{aligned}$$

$$\begin{aligned}0 &= E_t\hat{r}_{x,hb,t+1}-\left(\rho+\psi\beta\right)E_t\hat{C}_{t+1}\hat{r}_{x,hb,t+1}+\frac{1}{2}E_t\hat{r}^2_{x,hb,t+1}\\ &\quad-E_t\hat{r}_{x,hb,t+1}+\left(\rho+\psi\beta\right)E_t\hat{C}^*_{t+1}\hat{r}_{x,hb,t+1}-\frac{1}{2}E_t\hat{r}^2_{x,hb,t+1}+E_t\hat{r}_{x,hb,t+1}\left(\hat{Q}_{t+1}-\hat{Q}_t\right)\\ 0 &= E_t\left(\hat{C}_{t+1}-\hat{C}^*_{t+1}-\frac{\hat{Q}_{t+1}-\hat{Q}_t}{\rho+\psi\beta}\right)\hat{r}_{x,hb,t+1}\end{aligned}$$

$$
\begin{aligned}
0 &= E_t\hat{r}_{x,fe,t+1} - (\rho+\psi\beta) E_t\hat{C}_{t+1}\hat{r}_{x,fe,t+1} + \frac{1}{2}E_t\hat{r}^2_{x,fe,t+1} - \delta_{fe} \\
&\quad -E_t\hat{r}_{x,fe,t+1} + (\rho+\psi\beta) E_t\hat{C}^*_{t+1}\hat{r}_{x,fe,t+1} - \frac{1}{2}E_t\hat{r}^2_{x,fe,t+1} + \hat{r}_{x,fe,t+1}\left(\hat{Q}_{t+1} - \hat{Q}_t\right) \\
0 &= E_t\left(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1} - \hat{Q}_t}{\rho+\psi\beta}\right)\hat{r}_{x,fe,t+1} + \delta_{fe}
\end{aligned}
$$

Taking the sum of each of the Euler equations for the two countries gives:

$$
E_t\hat{r}_{x,he,t+1} = (\rho+\psi\beta) E_t\left(\frac{\hat{C}_{t+1} + \hat{C}^*_{t+1}}{2} + \frac{\hat{Q}_{t+1} - \hat{Q}_t}{2(\rho+\psi\beta)}\right)\hat{r}_{x,he,t+1} - \frac{1}{2}E_t\widehat{r}^2_{x,he,t+1} + \frac{\delta_{he} + \delta^*_{he}}{2} \tag{40}
$$

$$
E_t\hat{r}_{x,hb,t+1} = (\rho+\psi\beta) E_t\left(\frac{\hat{C}_{t+1} + \hat{C}^*_{t+1}}{2} + \frac{\hat{Q}_{t+1} - \hat{Q}_t}{2(\rho+\psi\beta)}\right)\hat{r}_{x,hb,t+1} - \frac{1}{2}E_t\widehat{r}^2_{x,hb,t+1} \tag{41}
$$

$$
E_t\hat{r}_{x,fe,t+1} = (\rho+\psi\beta) E_t\left(\frac{\hat{C}_{t+1} + \hat{C}^*_{t+1}}{2} + \frac{\hat{Q}_{t+1} - \hat{Q}_t}{2(\rho+\psi\beta)}\right)\hat{r}_{x,fe,t+1} - \frac{1}{2}E_t\widehat{r}^2_{x,fe,t+1} + \frac{\delta_{fe}}{2} \tag{42}
$$

Note that all the terms in the three equations above are of second order (products of first order terms and the costs), except the expected excess return.

We can now rewrite the cross-country Euler equations in a more convenient form to derive the general equilibrium solution for the model.

$$
\begin{aligned}
\Delta &= E_t\left(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1} - \hat{Q}_t}{\rho+\psi\beta}\right)\hat{r}_{x,i,t+1} \\
&\Downarrow \\
\Delta &= \begin{pmatrix} \delta^*_{he} - \delta_{he} \\ 0 \\ -\delta_{fe} \end{pmatrix} = cov\left(\hat{r}_{x,t+1}; \hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1} - \hat{Q}_t}{\rho+\psi\beta}\right)
\end{aligned}
$$

Before moving on to the portfolio solution, note that combining the FOC for the reference asset of the Home and Foreign agent, we have (the cross Euler equation):

$$
E_t\beta_{t+1}\frac{C^{-\rho}_{t+1}}{C^{-\rho}_t}r_{fb,t+1} = E_t\beta^*_{t+1}\frac{C^{*-\rho}_{t+1}}{C^{*-\rho}_t}r_{fb,t+1}\frac{Q_t}{Q_{t+1}}
$$

Therefore the first order dynamics for consumption are the following:

$$\begin{aligned}
-\rho E_t\left(\hat{C}_{t+1}-\hat{C}_t\right)-\psi\beta E_t\widehat{C}_{t+1} &= -E_t\left(\hat{C}^*_{t+1}-\hat{C}^*_t\right)-\psi\beta E_t\widehat{C}^*_{t+1}+E_t\left(\hat{Q}_t-\hat{Q}_{t+1}\right) \\
(\rho+\psi\beta)\,E_t\left(\hat{C}_{t+1}-\hat{C}^*_{t+1}\right) &= \rho\left(\hat{C}_t-\hat{C}^*_t\right)-E_t\left(\hat{Q}_t-\hat{Q}_{t+1}\right) \qquad (43)
\end{aligned}$$

$$(\rho+\psi\beta)\,E_t\hat{C}_{t+1}-\rho\hat{C}_t=(\rho+\psi\beta)\,E_t\hat{C}^*_{t+1}-\rho\hat{C}^*_t+E_t\left(\hat{Q}_{t+1}-\hat{Q}_t\right)$$

Recall that the budget constraint is:

$$P_tC_t+P_t\sum_{k=1}^{4}\alpha_{k,t}=P_{h,t}Y_t-P_t\begin{pmatrix}\alpha_{hb,t-1}r_{hb,t}+\alpha_{fb,t-1}r_{fb,t}+\\ \alpha_{he,t-1}r_{he,t}+\alpha_{fe,t-1}e^{-\tau}r_{fe,t}\end{pmatrix}$$

and that $W_t=\sum_{k=1}^{4}\alpha_{k,t-1}$; in addition define $\widehat{W}_{t+1}=\left(W_{t+1}-W\left(0\right)\right)/\bar{Y}$ and note that at the steady state $W\left(0\right)=0$ (under perfect symmetry) (i.e. $\sum_{k=1}^{4}\alpha_k\left(0\right)=0$), $r\left(0\right)=1/\beta$, $P_h\left(0\right)=P\left(0\right)$ and $Y\left(0\right)=C\left(0\right)$.

As in Devereux and Sutherland (2007) we get:

$$\begin{aligned}
Y\left(0\right)\hat{W}_{t+1} &= \left[r\left(0\right)\sum_{i=1}^{4}\alpha_i\left(0\right)\hat{r}_{i,t}+r\left(0\right)\sum_{i=1}^{4}\alpha_{i,t-1}\right]+Y\left(0\right)\left[\hat{Y}_t+\widehat{P}_{h,t}-\hat{C}_t-\widehat{P}_t\right] \\
\hat{W}_{t+1} &= \frac{r\left(0\right)}{Y\left(0\right)}\sum_{i=1}^{4}\alpha_i\left(0\right)\hat{r}_{i,t}+\frac{r\left(0\right)}{Y\left(0\right)}\sum_{i=1}^{4}\alpha_{i,t}+\left[\hat{Y}_t+\widehat{P}_{h,t}-\hat{C}_t-\widehat{P}_t\right] \\
\hat{W}_{t+1} &= \sum_{i=1}^{4}\tilde{\alpha}_i\left(0\right)\hat{r}_{i,t}+\frac{1}{\beta}\hat{W}_t+\left[\hat{Y}_t+\widehat{P}_{h,t}-\hat{C}_t-\widehat{P}_t\right] \qquad (44)
\end{aligned}$$

where $\tilde{\alpha}_i\left(0\right)=\frac{r(0)\alpha_i(0)}{C(0)}$.

In terms of vectors of excess returns we get:

$$\begin{aligned}
\sum_{i=1}^{n}\tilde{\alpha}_i\left(0\right)\hat{r}_{i,t} &= \sum_{i=1}^{n}\tilde{\alpha}_i\left(0\right)\hat{r}_{x,i,t}+\hat{r}_{fb,t}\sum_{i=1}^{n}\tilde{\alpha}_i\left(0\right) \\
&= \tilde{\alpha}'\left(0\right)\hat{r}_{x,t}+\hat{r}_{fb,t}W\left(0\right) \\
&= \tilde{\alpha}'\left(0\right)\hat{r}_{x,t}
\end{aligned}$$

Therefore:

$$\hat{W}_{t+1} = \widetilde{\alpha}_i(0)\,\hat{r}_{x,t} + \frac{1}{\beta}\hat{W}_t + \left[\hat{Y}_t + \widehat{P}_{h,t} - \hat{C}_t - \widehat{P}_t\right] \tag{45}$$

Similarly for the Foreign agent, using the asset market clearing conditions $\alpha_t = -\alpha_t^*$ and $W_t = -W_t^*$ (and $Q(0) = 1$, PPP holds at the steady state), the first order budget constraint is given by:

$$\begin{aligned} W_{t+1}^* &= \alpha_{t-1}^{*\prime} r_{x,t} + r_{fb,t} W_t^* + Q_t\left(Y_t^* - C_t^*\right) \\ -\hat{W}_{t+1} &= -\sum_{i=1}^{n} \tilde{\alpha}_i(0)\,\hat{r}_{i,t} - \frac{1}{\beta}\hat{W}_t + \hat{Y}_t^* + \widehat{P}_{f,t}^* - \hat{C}_t^* - \widehat{P}_t^* \end{aligned} \tag{46}$$

The full solution of the model

As explained in the main text we can treat the realized excess return on the portfolio as an exogenous independent mean-zero i.i.d. random variable (i.e. like a shock) denoted ξ_t., Given the zero-order (which can be interpreted as the steady state in models with only first order approximation) of the portfolio, we are back to a standard linear model. The home country budget constraint can therefore be written in the form:

$$\hat{W}_{t+1}(1) = \xi_t(1) + \frac{1}{\beta}\hat{W}_t(1) + \hat{Y}_{t+1}(1) - \hat{C}_{t+1}(1) + \widehat{P}_{h,t}(1) - \widehat{P}(1)$$

where:

$$\xi_t(1) = \sum_{i=1}^{n} \tilde{\alpha}_i(0)\,\hat{r}_{x,i,t}(1) = \tilde{\alpha}'\hat{r}_{x,t}(1)$$

The model combines predetermined states variables s, control variables c and exogenous processes x:

$$\begin{aligned} A_1 \begin{vmatrix} s_{t+1} \\ E_t c_{t+1} \end{vmatrix} &= A_2 \begin{vmatrix} s_t \\ c_t \end{vmatrix} + A_3 x_t + B\xi_t \\ x_t &= N x_{t-1} + \varepsilon_t \end{aligned}$$

The state space solution is of the form:

$$\begin{aligned} s_{t+1} &= F_1 x_t + F_2 s_t + F_3 \xi_t \\ c_t &= P_1 x_t + P_2 s_t + P_3 \xi_t \end{aligned}$$

By extracting the appropriate row of the matrix, it is easy to see that the solution for c includes the excess returns, which do not depend on predetermined variables:

$$\hat{r}_{x,t+1} = R_1 \xi_{t+1} + R_2 \varepsilon_{t+1}$$

This implies:

$$\xi_{t+1} = \tilde{\alpha}' \hat{r}_{x,t+1} = \tilde{\alpha}' R_1 \xi_{t+1} + \tilde{\alpha}' R_2 \varepsilon_{t+1}$$

$$\begin{aligned} \Rightarrow \quad \xi_{t+1} &= \frac{\tilde{\alpha}' R_2}{1 - \tilde{\alpha}' R_1} \varepsilon_{t+1} = \tilde{H} \varepsilon_{t+1} \\ \Rightarrow \quad \hat{r}_{x,t+1} &= \left(R_1 \tilde{H} + R_2 \right) \varepsilon_{t+1} = \tilde{R} \varepsilon_{t+1} \end{aligned}$$

which means that realized excess returns only depend on the exogenous innovation of the model.

As above, by extracting the appropriate rows, one can see that the solution for c also includes relative consumption (adjusted by the real exchange rate):

$$\begin{aligned} \hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\left(\hat{Q}_{t+1} - \hat{Q}_t \right)}{(\rho + \psi \beta)} &= D_1 \xi_{t+1} + D_2 \varepsilon_{t+1} + D_3 \left| \begin{array}{c} x_t \\ s_{t+1} \end{array} \right| \\ &= \left(D_1 \tilde{H} + D_2 \right) \varepsilon_{t+1} + D_3 \left| \begin{array}{c} x_t \\ s_{t+1} \end{array} \right| \\ &= \tilde{D} \varepsilon_{t+1} + D_3 \left| \begin{array}{c} x_t \\ s_{t+1} \end{array} \right| \end{aligned}$$

Using previous results we write:

$$cov \left(\hat{r}_{x,t+1}; \hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\left(\hat{Q}_{t+1} - \hat{Q}_t \right)}{(\rho + \psi \beta)} \right) = cov \left(\tilde{R} \varepsilon_{t+1}; \tilde{D} \varepsilon_{t+1} \right) = \tilde{R} \Sigma \tilde{D}'$$

Recall that:

$$\Delta = cov \left(\hat{r}_{x,t+1}; \hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\left(\hat{Q}_{t+1} - \hat{Q}_t \right)}{(\rho + \psi \beta)} \right) = \tilde{R} \Sigma \tilde{D}'$$

which is an implicit solution in $\tilde{\alpha}'$.

$$\begin{aligned}
\Delta &= \tilde{R}\Sigma\tilde{D}' \\
\Delta &= \left(R_1\tilde{H}+R_2\right)\Sigma\left(D_1\tilde{H}+D_2\right)' \\
\Delta &= R_1\tilde{H}\Sigma\tilde{H}'D_1'+R_2\Sigma\tilde{H}'D_1'+R_1\tilde{H}\Sigma D_2'+R_2\Sigma D_2' \\
\Delta &= R_1\frac{\tilde{\alpha}'R_2}{1-\tilde{\alpha}'R_1}\Sigma\frac{(\tilde{\alpha}'R_2)'}{1-\tilde{\alpha}'R_1}D_1'+R_2\Sigma\frac{(\tilde{\alpha}'R_2)'}{1-\tilde{\alpha}'R_1}D_1' \\
&\quad +R_1\frac{\tilde{\alpha}'R_2}{1-\tilde{\alpha}'R_1}\Sigma D_2'+R_2\Sigma D_2' \\
\Delta &= R_1\frac{\tilde{\alpha}'R_2}{1-\tilde{\alpha}'R_1}\Sigma\frac{R_2'\tilde{\alpha}}{1-\tilde{\alpha}'R_1}D_1'+R_2\Sigma\frac{R_2'\tilde{\alpha}}{1-\tilde{\alpha}'R_1}D_1' \\
&\quad +R_1\frac{\tilde{\alpha}'R_2}{1-\tilde{\alpha}'R_1}\Sigma D_2'+R_2\Sigma D_2' \\
\Delta &= R_1\tilde{\alpha}'R_2\Sigma R_2'\tilde{\alpha}D_1'+R_2\Sigma R_2'\tilde{\alpha}D_1'\left(1-\tilde{\alpha}'R_1\right) \\
&\quad +R_1\tilde{\alpha}'R_2\Sigma D_2'\left(1-\tilde{\alpha}'R_1\right)+R_2\Sigma D_2'\left(1-\tilde{\alpha}'R_1\right)^2
\end{aligned}$$

$\tilde{\alpha}'R_1$, $(1-\tilde{\alpha}'R_1)$ and D_1 are scalar, equal to their transposed values. So we write:

$$\begin{aligned}
\Delta &= \left(R_1\tilde{\alpha}'\right)R_2\Sigma R_2'\tilde{\alpha}D_1'-R_2\Sigma R_2'\tilde{\alpha}D_1'\left(\tilde{\alpha}'R_1\right) \\
&\quad +R_2\Sigma R_2'\tilde{\alpha}D_1'+R_1\tilde{\alpha}'R_2\Sigma D_2' \\
&\quad -\left(R_1\tilde{\alpha}'\right)R_2\Sigma D_2'\left(\tilde{\alpha}'R_1\right)+R_2\Sigma D_2' \\
&\quad -2R_2\Sigma D_2'\left(\tilde{\alpha}'R_1\right)+R_2\Sigma D_2'\left(\tilde{\alpha}'R_1\right)^2 \\
\Delta &= D_1R_2\Sigma R_2'\tilde{\alpha}+R_2\Sigma D_2'-R_2\Sigma D_2'R_1'\tilde{\alpha} \\
\Delta &= \left[D_1R_2\Sigma R_2'-R_2\Sigma D_2'R_1'\right]\tilde{\alpha}+R_2\Sigma D_2'
\end{aligned}$$

which implies that the solution for the equilibrium $\tilde{\alpha}$:

$$\tilde{\alpha}=\left[D_1R_2\Sigma R_2'-R_2\Sigma D_2'R_1'\right]^{-1}\left[\Delta-R_2\Sigma D_2'\right]$$

As previously noted the solution for $\overline{\alpha}/\overline{Y}=\tilde{\alpha}\beta$

The asset market

Equities.

The gross real rate of return on the Home equity is given by the dividend yield plus the capital gain/loss. Let $Z_{he,t}$ be the Home equity price in real terms, the ex-post return is:

$$r_{he,t+1} = \frac{\Pi_{t+1} + Z_{he,t+1}}{Z_{he,t}}$$

and gross real rate of return on the Foreign equity:

$$r^*_{fe,t+1} = \frac{\Pi^*_{t+1} + Z^*_{fe,t+1}}{Z^*_{fe,t}}$$

From the steady state condition we know that $r_{fe} = r_{he} = 1/\beta$ so that $\frac{\overline{Z}^*_{fe}}{\overline{Z}^*_{fe}+\overline{\Pi}^*} = \frac{\overline{Z}_{he}}{\overline{Z}_{he}+\overline{\Pi}} = \beta$, and $\frac{\overline{\Pi}^*}{\overline{Z}_{fe}+\overline{\Pi}^*} = \frac{\overline{\Pi}}{\overline{Z}_{he}+\overline{\Pi}} = 1-\beta$, hence log-linearization implies:

$$\begin{aligned} \hat{r}_{he,t+1} + \widehat{Z}_{he,t} &= (1-\beta)\widehat{\Pi}_{t+1} + \beta\widehat{Z}_{he,t+1} && (47) \\ \hat{r}^*_{fe,t+1} + \widehat{Z}^*_{fe,t} &= (1-\beta)\widehat{\Pi}^*_{t+1} + \beta\widehat{Z}^*_{fe,t+1} && (48) \end{aligned}$$

Substituting the equation of the return in the Euler condition yields:

$$Z_{he,t}C_t^{-\rho} = E_t\left\{\beta_{t+1}C_{t+1}^{-\rho}\left(\Pi_{t+1} + Z_{he,t+1}\right)\right\}$$

which in log-linear form becomes:

$$-\rho\widehat{C}_t + \widehat{Z}_{he,t} = -\left(\rho + \psi\beta\right)E_t\widehat{C}_{t+1} + \frac{\overline{\Pi}}{\overline{Z}_{he} + \overline{\Pi}}E_t\widehat{\Pi}_{t+1} + \frac{\overline{Z}_{he}}{\overline{Z}_{he} + \overline{\Pi}}E_t\widehat{Z}_{he,t+1}$$

After some rearrangement we can write the following equation for the Home equity price:

$$\widehat{Z}_{he,t} = -\left(\rho + \psi\beta\right)E_t\widehat{C}_{t+1} + \rho\widehat{C}_t + (1-\beta)E_t\widehat{\Pi}_{t+1} + \beta E_t\widehat{Z}_{he,t+1} \quad (49)$$

By symmetry for the Foreign country we have:

$$\widehat{Z}^*_{fe,t} = \rho\widehat{C}^*_t - \left(\rho + \psi\beta\right)E_t\widehat{C}^*_{t+1} + (1-\beta)E_t\widehat{\Pi}^*_{t+1} + \beta E_t\widehat{Z}^*_{fe,t+1} \quad (50)$$

Therefore:

$$
\begin{aligned}
(1-\beta)\widehat{\Pi}_{t+1}+\beta\widehat{Z}_{he,t+1} &= \left[\hat{r}_{he,t+1}-\left[\rho\left(1+\frac{\psi\beta}{\rho}\right)E_t\widehat{C}_{t+1}-\widehat{C}_t\right]+(1-\beta)E_t\widehat{\Pi}_{t+1}+\beta E_t\widehat{Z}_{he,t+1}\right] \\
\hat{r}_{he,t+1} &= \left[\begin{array}{c} (1-\beta)\widehat{\Pi}_{t+1}+\beta\widehat{Z}_{he,t+1}-E_t(1-\beta)\widehat{\Pi}_{,t+1} \\ +\beta\widehat{Z}_{he,t+1}+(\rho+\psi\beta)\,E_t\widehat{C}_{t+1}-\rho\widehat{C}_t \end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
\hat{r}^*_{fe,t+1} &= (1-\beta)\widehat{\Pi}^*_{t+1}+\beta\widehat{Z}^*_{fe,t+1}+\rho\left[\left(1+\frac{\psi\beta}{\rho}\right)E_t\widehat{C}^*_{t+1}-\widehat{C}^*_t\right]-E_t\left[(1-\beta)\widehat{\Pi}^*_{t+1}+\beta\widehat{Z}^*_{fe,t+1}\right] \\
\widehat{r}_{fe,t+1} &= \left[\begin{array}{c} (1-\beta)\widehat{\Pi}^*_{t+1}+\beta\widehat{Z}^*_{fe,t+1}+\rho\left[\left(1+\frac{\psi\beta}{\rho}\right)E_t\widehat{C}^*_{t+1}-\widehat{C}^*_t\right] \\ -E_t\left[(1-\beta)\widehat{\Pi}^*_{t+1}+\beta\widehat{Z}^*_{fe,t+1}\right]+\left(\widehat{Q}_{t+1}-\widehat{Q}_t\right) \end{array}\right]
\end{aligned}
$$

Bonds. Given the real price of the bond $Z_{hb,t}$, the gross real return on the Home bond is:

$$r_{hb,t+1}=\frac{1}{P_{t+1}Z_{hb,t}}$$

and the Foreign equivalent:

$$r_{fb,t+1}=\frac{1}{P^*_{t+1}Z^*_{fb,t}}$$

In linear terms:

$$
\begin{aligned}
\widehat{r}_{hb,t+1} &= -\widehat{P}_{t+1}-\widehat{Z}_{hb,t} \\
\widehat{r}^*_{fb,t+1} &= -\widehat{P}^*_{t+1}-\widehat{Z}^*_{fb,t}
\end{aligned}
$$

Combining each of them with the relevant Euler Equation yields:

$$C_t^{-\rho}=E_t\left\{\beta_{t+1}C_{t+1}^{-\rho}\frac{1}{P_{t+1}Z_{hb,t}}\right\} \qquad C_t^{*-\rho}=E_t\left\{\beta^*_{t+1}C_{t+1}^{*-\rho}\frac{1}{P^*_{t+1}Z^*_{fb,t}}\right\}$$

Log-linearizing both conditions yields:

$$-\rho\widehat{C}_t+\widehat{Z}_{hb,t}=E_t\left\{-\rho\left(1+\frac{\psi\beta}{\rho}\right)\hat{C}_{t+1}-\widehat{P}_{t+1}\right\}$$

$$-\rho\widehat{C}^*_t+\widehat{Z}^*_{fb,t}=E_t\left\{-\rho\left(1+\frac{\psi\beta}{\rho}\right)\hat{C}^*_{t+1}-\widehat{P}^*_{t+1}\right\}$$

These conditions can be used to rewrite the ex-post return:

$$
\begin{aligned}
\widehat{r}_{hb,t+1} &= -\widehat{P}_{t+1} - \rho\widehat{C}_t - E_t\left\{-\left(\rho+\psi\beta\right)\hat{C}_{t+1} - \widehat{P}_{t+1}\right\} + \widehat{P}_t - \widehat{P}_t \\
&= -\pi_{t+1} - \rho\widehat{C}_t + E_t\left(\rho+\psi\beta\right)\hat{C}_{t+1} + E_t\pi_{t+1} \\
\widehat{r}^*_{fb,t+1} &= -\widehat{P}^*_{t+1} - \rho\widehat{C}^*_t - E_t\left\{-\rho\left(1+\frac{\psi\beta}{\rho}\right)\hat{C}^*_{t+1} - \widehat{P}^*_{t+1}\right\} \\
\widehat{r}_{fb,t+1} &= \widehat{r}^*_{fb,t+1} - \left(Q_t - Q_{t+1}\right) = -\widehat{P}^*_{t+1} - \rho\widehat{C}^*_t - E_t\left\{\begin{array}{c} -\rho\left(1+\frac{\psi\beta}{\rho}\right)\hat{C}^*_{t+1} \\ -\widehat{P}^*_{t+1} \end{array}\right\} + \left(\widehat{Q}_{t+1} - \widehat{Q}_t\right)
\end{aligned}
$$

Recalling that the Foreign bond is the reference asset, the excess returns are:

$$
\begin{aligned}
\widehat{r}_{x,he,t+1} &= \widehat{r}_{he,t+1} - \widehat{r}_{fb,t+1} \\
\widehat{r}_{x,he,t+1} &= (1-\beta)\widehat{\Pi}_{t+1} + \beta\widehat{Z}_{he,t+1} - E_t\left[\begin{array}{c} (1-\beta e^{-\tau})\widehat{\Pi}_{t+1} \\ +\beta e^{-\tau}\widehat{Z}_{he,t+1} \end{array}\right] + E_t\left(\hat{Q}_t - \hat{Q}_{t+1}\right) + \widehat{P}^*_{t+1} - E_t\widehat{P}^*_{t+1} \\
&= (1-\beta)\widehat{\Pi}_{t+1} + \left[\begin{array}{c} \beta\widehat{Z}_{he,t+1} - E_t\left[(1-\beta e^{-\tau})\widehat{\Pi}_{t+1} + \beta e^{-\tau}\widehat{Z}_{he,t+1}\right] \\ +E_t\left(\hat{Q}_t - \hat{Q}_{t+1}\right) + \left(\widehat{P}^*_{t+1} - \widehat{P}^*_t\right) - E_t\left(\widehat{P}^*_{t+1} - \widehat{P}^*_t\right) \end{array}\right] \\
&= (1-\beta)\widehat{\Pi}_{t+1} + \beta\widehat{Z}_{he,t+1} - E_t\left[\begin{array}{c} (1-\beta e^{-\tau})\widehat{\Pi}_{t+1} \\ +\beta e^{-\tau}\widehat{Z}_{he,t+1} \end{array}\right] + E_t\left(\hat{Q}_t - \hat{Q}_{t+1}\right) + \pi^*_{t+1} - E_t\pi^*_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
\widehat{r}_{x,fe,t+1} &= \widehat{r}_{fe,t+1} - \widehat{r}_{fb,t+1} = \\
\widehat{r}_{x,fe,t+1} &= (1-\beta)\widehat{\Pi}^*_{t+1} + \beta\widehat{Z}^*_{fe,t+1} - E_t\left[(1-\beta)\widehat{\Pi}^*_{t+1} + \beta\widehat{Z}^*_{fe,t+1}\right] + \widehat{P}^*_{t+1} - E\widehat{P}^*_{t+1} \\
\widehat{r}_{x,fe,t+1} &= (1-\beta)\widehat{\Pi}^*_{t+1} + \beta\widehat{Z}^*_{fe,t+1} - E_t\left[(1-\beta)\widehat{\Pi}^*_{t+1} + \beta\widehat{Z}^*_{fe,t+1}\right] - \left(E_t\widehat{\pi}^*_{t+1} - \widehat{\pi}^*_{t+1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\widehat{r}_{x,hb,t+1} &= \widehat{r}_{hb,t+1} - \widehat{r}_{fb,t+1} \\
&= -\rho\widehat{C}_t + \rho\widehat{C}^*_t + \rho E_t\left\{\begin{array}{c} -\left(1+\frac{\psi\beta}{\rho}\right)\hat{C}_{t+1} \\ -\left(1+\frac{\psi\beta}{\rho}\right)\hat{C}^*_{t+1} \end{array}\right\} - \widehat{P}_{t+1} + \left[\begin{array}{c} E_t\widehat{P}_{t+1} + \widehat{P}^*_{t+1} \\ -E_t\widehat{P}^*_{t+1} + \left(Q_t - Q_{t+1}\right) \end{array}\right] \\
&= \rho E_t\left\{\begin{array}{c} \left(1+\frac{\psi\beta}{\rho}\right)\hat{C}_{t+1} \\ -\widehat{C}_t \end{array}\right\} - \rho E_t\left\{\begin{array}{c} \left(1+\frac{\psi\beta}{\rho}\right)\hat{C}^*_{t+1} \\ -\widehat{C}^*_t \end{array}\right\} + \left[\begin{array}{c} \left(Q_t - Q_{t+1}\right) - \widehat{P}_{t+1} + \\ E_t\widehat{P}_{t+1} + \widehat{P}^*_{t+1} - E_t\widehat{P}^*_{t+1} \end{array}\right] \\
&= E_t\hat{Q}_{t+1} - Q_{t+1} - \widehat{\pi}_{t+1} + E_t\widehat{\pi}_{t+1} + \widehat{\pi}^*_{t+1} - E_t\widehat{\pi}^*_{t+1}
\end{aligned}
$$

Where the computation uses the equation for the dynamics of consumption:

$$\rho E_t \left(\hat{C}_{t+1} \left(1 + \frac{\psi\beta}{\rho} \right) - \hat{C}_t \right) - \rho \left(\hat{C}^*_{t+1} \left(1 + \frac{\psi\beta}{\rho} \right) - \hat{C}^*_t \right) = E_t \left(\hat{Q}_t - \hat{Q}_{t+1} \right)$$

The steady state

In the symmetric long run equilibrium, we assume no growth and trade balance is zero. For each of the two countries we have a set of equations defining the behavior of real quantities.

Given that trade balance is zero national output equals national consumption:

$$Y = C$$

Market clearing for the output implies:

$$Y = \mu C + (1 - \mu) C^*$$

The level of production is determined by the amount of labor:

$$Y = AL$$

In addition from the consumption-leisure trade-off the real wage is:

$$\frac{w}{P} = A = KC^{-\rho}$$

as the long run equilibrium implies that

$$P_h = P$$

Finally for the portfolio part we have that under perfect symmetry:

$$W = 0$$

and all the interest rates are the same:

$$r = \frac{1}{\beta}$$

B The Empirical Methodology

B.1 Sign restriction identification

Let the reduced-form VAR be represented as:

$$Y_t = A(L)Y_{t-1} + BX_t + e_t \qquad e_t \sim N(0, \Sigma_e) \tag{51}$$

Where Y_t is a $n \times 1$ vector and $A(L)$ a matrix polynomial in the lag operator. Then, for any non-singular orthogonal matrix P, satisfying $\Sigma_e = P'P$, eq.(51) can be transformed to have contemporaneously uncorrelated innovations. One general orthogonalization which achieves this purpose is the eigenvalue-eigenvector decomposition $\Sigma_e = P'P = V'DV$ where V is a matrix of eigenvectors, D is a diagonal matrix with eigenvalues on the main diagonal and . Given this decomposition , eq.(51) can be easily transformed into:

$$\widetilde{Y}_t = A(L)\widetilde{Y}_{t-1} + B\widetilde{X}_t + \widetilde{e}_t \qquad \widetilde{e}_t \sim NID(0, I)$$

Where $\widetilde{Y}_t = P^{-1}Y_t$, $\widetilde{e}_t = P^{-1}e_t \sim NID\ (0, I)$, $\widetilde{X}_t = P^{-1}X_t$. Let the moving average representation then be:

$$\widetilde{Y}_t = C\widetilde{X}_t + D(L)\widetilde{e}_t \tag{52}$$

Where $C = \ (I - A\ (L))^{-1}B$, and $D\ (L) = \ (I - A\ (L))^{-1}$.

Economic theory provides important information on the pair-wise dynamic correlations to shocks and this information can be used for empirical identification of underlying shocks. Using eq.(52), the pair-wise dynamic cross correlations conditional on a shock can be calculated as:

$$\rho_{ij|k}(r) \equiv Corr(\widetilde{Y}_{i,t}, \widetilde{Y}_{i,t+r} | \widetilde{e}_{kt} = 1)$$

Where k indicates the shock, i, j the variables under consideration and r the horizon of the responses. The idea behind this identification scheme is to investigate whether for some k and for certain variables i, j, the correlations $\rho_{ij|k}\ (r)$ among the impulse responses correspond to the pattern predicted by a theoretical model as reaction to a given monetary policy shock for different values of r. If $\rho_{ij|k}(r)$ is not interpretable for some k, then for any orthogonal matrix Q such that $Q'Q = I$, $\Sigma_e = P'P = P'Q'QP$ is an admissible decomposition of the covariance matrix of the VAR residuals. Therefore it is possible to calculate the correlations once more in order to verify whether a given set of impulse responses fits the description of a monetary policy shock provided by the model.

A class of such orthogonal matrices (see for instance Canova and De Nicolo (2002), Canova (2007)) are rotation matrices which have a representation in terms of sine, cosine functions and ones of the form:

$$Q_{m,n} = \begin{bmatrix} 1 & 0 & & \cdots & & 1 & 0 \\ 0 & 1 & & & & 0 & 1 \\ & & \cos(\theta) & \cdots & -\sin(\theta) & & \\ \vdots & & \vdots & & \vdots & & \vdots \\ & & \sin(\theta) & \cdots & \cos(\theta) & & \\ 1 & 0 & & & & 1 & 0 \\ 0 & 1 & & \cdots & & 0 & 1 \end{bmatrix} \tag{5}$$

Where the subscript m, n indicates that only rows m and n and are rotated by the angle θ. In our case with 5 variables, there are 15 possible rotations for every angle θ in our system. In order to search for all possible representations we follow closely the algorithm in Canova and De Nicolo (2002). First we divide space of $\theta \in [0, 2\pi]$ into a fine grid. For each grid point we use the sign of the correlation among the impulse responses at time 0, $\rho_{ij|k}(r = 0)$ to identify the shocks. Finally, if there is more than one decomposition which produces the same number of identifiable shocks, we eliminate the "wrong" ones by requiring the correlation to hold at several horizons as well, i.e. by gradually increasing r. As described in the main text, we set r to 5 for the interest rate and the money stock. For the price level and the output impulse response, we set r to be binding for 3 out of 6 periods. Finally, we only impose the sign restriction on the exchange rate at $\rho_{ij|k}(r = 0)$.

B.2 Stationary and Lag Length Selection

Many macroeconomic time series may be non-stationary and since it is known that classical distribution theory in the classical linear regression model breaks down in this case, the issue of estimating a VAR in levels with OLS under possible non-stationarity deserves an extensive discussion.

The inference drawn from a monetary policy shock in a VAR is of a different nature than inferences drawn in the classical regression framework. The interest is not on t-statistics, which are affected by the presence of non-stationarity, but on the dynamic impulse responses. To this regard the consistency of parameter estimates is crucial. Sims, Stock and Watson (1990) show parameters estimated via OLS in a VAR in the presence of non-stationarity to be super consistent. Given that macroeconomists using VARs need to estimate many parameters, but have only limited data availability, this might be a useful property and an argument to estimate the VAR through the application of OLS to each equation in levels. Furthermore, Sims and Uhlig (1991) show that inference from the posterior distribution which we use to generate the confidence intervals around our impulse responses is independent of the presence of non-stationarity in the data.

Nevertheless the distribution of statistics used for the selection of the lags and testing for structural breaks could be affected by non-stationarity. Sims, Stock and Watson (1990) derive the distribution for a Wald test for lag length selection in a VAR system with some unit roots. They find that the asymptotic χ-square distribution is preserved in the presence of Unit-roots. Since the Chow test for structural breaks is an application of a Wald-test, this result applies to it as well. Paulson (1984) finds that the Hanan-Quinn (HQC) and the Schwarz lag length selection criteria are consistent in the presence of unit roots. More recently, Killian and Ivanov (2005) find that the effectiveness of the HQC, Schwarz and Akaike Information (AIC) criteria does not seem to be affected by the presence of near unit roots in the data.

If all of the variables display non-stationary behavior, then one can take first differences of the whole VAR, whereas if only some variables are cointegrated, then one could formulate a Vector Error Correction Model. Yet the problem is that one needs to know the exact number of cointegrating relationships, as otherwise the imposition of invalid cointegrating relationships will lead to inconsistent estimates. Since in reality one can never know the true number of cointegrating relationships, we prefer to leave the VAR unrestricted.

For all of the reasons above therefore, we choose to estimate the VAR in levels through the application of OLS to each equation.

Going back to lag length selection, since there is unanimity in the literature about which criterion one should prefer (see Ivanov and Kilian (2005) and Canova (2006)), we choose the lag length based on AIC, HQC and ex-post analysis of the white noise assumption in the residuals.

Following the *a priori* selection of the lag length we analyze the estimated residuals, in order to verify that they are white noise. Unfortunately, the Portmanteau test can not be used with the residuals of a VAR once deterministic variables are introduced (Luetkepohl and Kratzig (2004))[23], moreover non-stationarity could change the distribution of the residuals. For these reasons, we visually check the behavior of the residuals prior to any testing, by computing autocorrelations and plot the autocorrelogram of each residual series in order to verify that the residuals are approximately uncorrelated. If the autocorrelogram shows that the estimated residuals are significantly correlated at lag order 1, it may be that the lag order of the VAR chosen is too small. Autocorrelated residuals at lag order 1 may mean that an additional autoregressive term is omitted and therefore causes the autocorrelation in the white noise residuals. In this case we increase the number of lags until the autocorrelogram is not statistically significantly correlated at lag length 1 anymore[24].

We apply the lag length selection criteria to each sample of our rolling regression. If at least 2 indicate the same number of lags, this is the number we choose.

[23] One could use the LM test for the verification of uncorrelatedness among the residuals. Nevertheless we abstain from this as Killian and Ivanov(2005) report that the LM test criterion of the residuals does not perform well in selecting the true lag order.

[24] Possibly another reason to worry about autocorrelated residuals at lag order one is the resulting inconsistency of OLS parameter estimates.

In general, as shown in Table 1, a lag length of 2 seems to be appropriate during all time-periods studied in this paper.

Table 1 – Lag Length Selection			
Time Period			
Suggested Lag Length	AIC	HQC	White Noise
1975	2	2	2
1980	2	2	2
1985	2	3	2
1990	2	3	2
1995	2	3	2
2000	2	2	2
2005	2	2	2